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10CS42

Fourth Semester B.E. Degree Examination, June/July 2018
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO full questions from each part.***

PART – A

1 a. Let G be a simple graph of order n, if the size of G is 56 and the size of $\overline{G}$ is 80. What is n? **(05 Marks)**

b. Show that the following two graphs are isomorphic. [Refer Fig.Q1(b)] **(05 Marks)**

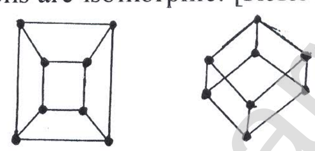

Fig.Q1(b)

c. Prove that if a graph has exactly two vertices of odd degree, then there must be a path connecting these vertices. **(05 Marks)**

d. Discuss Konigsberg Bridge problem. **(05 Marks)**

2 a. Define Hamilton path. Prove that the complete graph K_n, where $n \geq 3$, is a Hamilton graph. **(06 Marks)**

b. Define Planar graph. If G is a connected simple planar graph with $n(\geq 3)$ vertices, $m(>2)$ edges and r regions then

(i) $m \geq \frac{3}{2}r$ and (ii) $m \leq 3n - 6$ **(07 Marks)**

c. Define Chromatic number. Find the chromatic polynomial for the cycle C_4 of length 4. What is its chromatic number (Refer Fig.Q2(c)). **(07 Marks)**

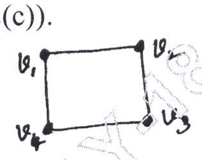

Fig.Q2(c)

3 a. Prove that tree with n vertices has $n-1$ edges. **(07 Marks)**

b. Prove that a graph with n vertices, $n-1$ edges and no cycles is connected. **(06 Marks)**

c. Define Prefix code. Construct an optimal prefix code for the letters of the word ENGINEERING. Hence deduce the code for this word. **(07 Marks)**

4 a. Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown below Fig.Q4(a) : **(07 Marks)**

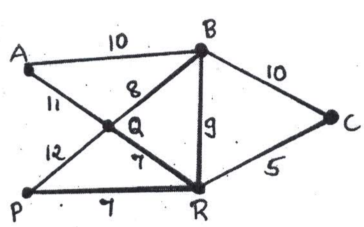

Fig.Q4(a)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Consider the bipartite graph shown below in Fig. Q4 (b). If four edges of this graph are chosen at random, what is the probability that they form a complete matching from V_1 to V_2?

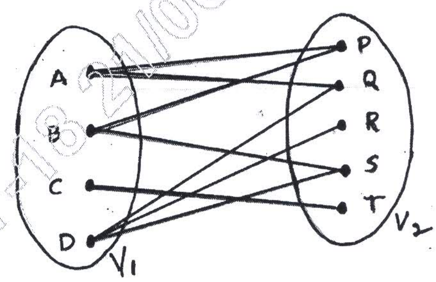

Fig.Q4(b) **(07 Marks)**

c. For the network shown in Fig.Q4(c) below, determine the maximum flow between the vertices A and D by identifying the cut-set of minimum capacity. **(06 Marks)**

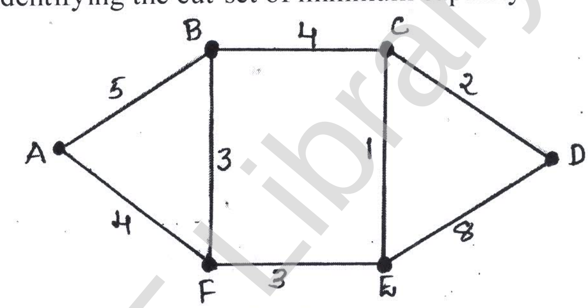

Fig.Q4(c)

PART – B

5 a. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000? **(07 Marks)**

b. A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in part C. It is required to answer seven questions selecting atleast two questions from each part. In how many different ways can a student select his seven questions for answering? **(07 Marks)**

c. In how many ways can 10 identical pencils be distributed among 5 children in the following cases:
(i) There are no restrictions.
(ii) Each child gets atleast one pencil.
(iii) The youngest child gets atleast two pencils. **(06 Marks)**

6 a. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(07 Marks)**

b. Define derangement. There are eight letters to eight different people to be placed in right different addressed envelopes. Find the number of ways of doing this, so that atleast one letter gets to the right person. **(06 Marks)**

c. Five teachers T_1, T_2, T_3, T_4, T_5 are to be made class teachers for five classes C_1, C_2, C_3, C_4, C_5 one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5, and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned the work? **(07 Marks)**

7 a. Find the generating functions for the following sequences: (06 Marks)
(i) $1^2, 2^2, 3^2, 4^2, \ldots\ldots$ (ii) $0^2, 1^2, 2^2, 3^2, \ldots\ldots$

b. Find the number of ways of forming a committee of 9 students drawn from 3 different classes so that students from the same class do not have an absolute majority in the committee. (07 Marks)

c. Define exponential generating function. Using this find the number of ways in which 5 of the letters in the word CALCULUS be arranged. (07 Marks)

8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. (06 Marks)

b. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$. Given that $a_0 = -1$ and $a_1 = 8$. (07 Marks)

c. Using the generating function method, solve the recurrence relation $a_n - 3a_{n-1} = n$, $n \geq 1$ given $a_0 = 1$. (07 Marks)

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