USN

17CS36

Third Semester B.E. Degree Examination, June/July 2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define tautology. Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology. **(06 Marks)**

b. If statement q has truth value 1, determine all truth value assignments for the primitive statements p, r, s for which the truth value of the statement :

$(q \to [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \to (\neg r \wedge q)]$ is 1. **(04 Marks)**

c. Establish the following logical equivalence :

i) $p \vee q \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow p \vee q \vee r$

ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$. **(10 Marks)**

OR

2 a. Establish the validity of following arguments :

i) $(\neg p \vee \neg q) \to (r \wedge s)$
$r \to t$
$\neg t$
$\therefore p$

ii) $u \to r$
$(r \wedge s) \to (p \vee t)$
$q \to (u \wedge s)$
$\neg t$
q
$\therefore p$ **(08 Marks)**

b. Let p(x), q(x) and r(x) be the following open statements :
$p(x) : x^2 - 7x + 10 = 0$ $q(x) : x^2 - 2x - 3 = 0$ $r(x) < 0$.
Determine truth or falsity of following statements, where universe is all integers. If a statement is false, provide a counter example.

i) $\forall x[p(x) \to \neg r(x)]$ ii) $\forall x[q(x) \to r(x)]$

iii) $\exists x[q(x) \to r(x)]$ iv) $\exists x[p(x) \to r(x)]$. **(08 Marks)**

c. Prove that for all integers 'k' and 'ℓ', if 'k' and 'ℓ' are both even, then $k + \ell$ is even and $k\ell$ is even by direct proof. **(04 Marks)**

Module-2

3 a. Define well ordering principle and prove the following by mathematical induction :

i) $1^2 + 3^2 + 5^2 + - - - - + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

ii) $1*3 + 2*4 + 3*5 + - - - - + n(n+2) = \frac{n(n+1)(2n+7)}{6}$. **(12 Marks)**

b. Find the coefficients of :

i. x^9y^3 in the expansion of $(2x - 3y)^{12}$

ii. $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(08 Marks)**

OR

4 a. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in following situations,
   i. There is no restriction on the choice
   ii. Two particular persons will not attend separately
   iii. Two particular persons will not attend together. **(06 Marks)**

b. How many arrangements are there for all letters in word SOCIOLOGICAL? In how many of these arrangements all vowels are adjacent. **(06 Marks)**

c. For the Fibonacci sequence $F_0, F_1, F_2 \ldots$ prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$.. **(08 Marks)**

Module-3

5 a. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5,6}.
   i. How many functions are there from A to B?
   ii. How many of these are one to one?
   iii. How many are onto?
   iv. How many functions are there from B to A?
   v. How many of these are onto?
   vi. How many are one to one? **(06 Marks)**

b. A computer operator is given a magnetic tape that contains 500,000 words of four or fewer lowercase letters. Can it be that the 500,000 words are all distinct? **(06 Marks)**

c. Let f, g, h : R → R where $f(x) = x^2$, $g(x) = x + 5$ and $h(x) = \sqrt{x^2+2}$. Show that (hog) of = ho(gof). **(08 Marks)**

OR

6 a. Let A = {1, 2, 3, 6, 9, 18} and define R on A by xRy if "x divides y", Draw the Hasse diagram for the poset (A, R). Also write the matrix of relation. **(08 Marks)**

b. Consider Poset whose Hasse diagram is given below. Consider B = {3, 4, 5}. Find upper and lower bounds of B, least upper bound and greatest lower bound of B. **(04 Marks)**
(Ref. Fig.Q6(b)).

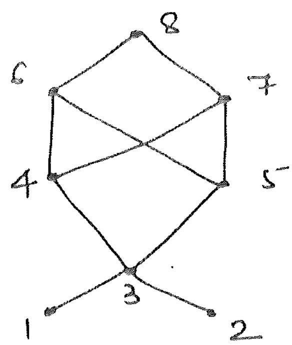

Fig.Q6(b)

c. Let A = {1, 2, 3, 4, 5} × {1, 2, 3, 4, 5} and define R on A by (x_1, y_1) R (x_2, y_2) if $x_1 + y_1 = x_2 + y_2$.
   i. Verify that R is an equivalence relation on A
   ii. Determine equivalence classes [(1, 3)], [(2, 4)] and [(1, 1)]
   iii. Determine partition of A induced by R. **(08 Marks)**

Module-4

7 a. In how many ways can the 26 letters of English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(08 Marks)**

b. There are eight letters to eight different people to be placed in eight different addressed envelops. Find the number of ways of doing this so that atleast one letter gets to right person. **(04 Marks)**

c. Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party find that only one chair at each of five table T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs. **(08 Marks**

OR

8 a. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42, ... Hence find the general term of the sequence. **(10 Marks)**

b. If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $n \geq 0$, determine the constants b and c and then solve the relation for a_n. **(10 Marks)**

Module-5

9 a. Merge sort the list –1, 7, 4, 11, 5, –8, 15, –3, –2, 6, 10, 3. **(06 Marks)**

b. Determine whether the following graphs are isomorphic or not. **(06 Marks)**

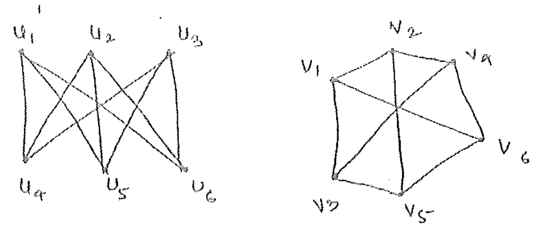

Fig.Q9(b)

c. Define the following with an example to each.
i) Simple graph ii) Complete graph iii) Regular graph iv) Spanning sub graph v) Induced subgraph vi) Complete Bipartite graph vii) Tree viii) Complement of graph. **(08 Marks)**

OR

10 a. Define trail, circuit, path, cycle. In the graph shown below determine : [Ref.Q10(a)]
  i. a walk from b to d that is not a trail
  ii. b-d trail that is not a path
  iii. a path from b to d
  iv. a closed walk from b to b that is not a circuit
  v. a circuit from b to b that is not cycle
  vi. a cycle form b to b. **(10 Marks)**

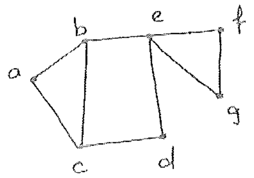

Fig.Q10(a)

b. Define optimal tree and construct an optimal tree for a given set of weights {4, 15, 25, 5, 8, 16}. Hence find the weight of optimal tree. **(06 Marks)**

c. Prove that in a graph. The sum of degrees of all vertices is an even number and is equal to twice the number of edges in the graph. **(04 Marks)**

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