USN | | | | | | | | | |

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Determine the sets A and B, given that $A - B = \{1, 3, 7, 11\}$, $B - A = \{2, 6, 8\}$ and $A \cap B = \{4, 9\}$. **(04 Marks)**

b. State and prove DeMorgan Laws. **(06 Marks)**

c. Using the laws of set theory, simplify $\overline{\overline{(A \cup B)} \cap C \cup \overline{B}}$. **(04 Marks)**

d. A problem is given to four students A, B, C, D whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability that the problem is solved. **(06 Marks)**

2 a. Prove that, for any propositions p, q, r the compound proposition, $[(p \vee q) \wedge \{(p \to r) \wedge (q \to r)\}] \to r$ is a tautology. **(06 Marks)**

b. Prove that, for any three propositions, p, q, r $[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$. **(07 Marks)**

c. Test the validity of the following argument:

If Ravi goes out with friends, he will not study.
If Ravi does not study, his father becomes angry.
His father is not angry

∴ Ravi has not gone out with friends. **(07 Marks)**

3 a. Suppose the universe consists of all integers. Consider the following open statements: Consider the following open statements : $p(x) : x \le 3$, $q(x)$: x+1 is odd, $r(x) : x>0$

Write down the truth values of the following:

(i) $p(2)$ (ii) $\neg q(4)$ (iii) $p(-1) \wedge q(1)$ (iv) $\neg p(3) \vee r(0)$
(v) $p(0) \to q(0)$ (vi) $p(1) \leftrightarrow \neg q(2)$ **(06 Marks)**

b. Find whether the following is a valid argument for which the universe is the set of all students.

No Engineering student is bad in studies
Anil is not bad in studies

∴ Anil is an Engineering student **(07 Marks)**

c. Prove that for all integers k and *l*, if

(i) k and *l* are both odd, then $k + l$ is even and kl is odd.
(ii) k and *l* are both even, then $k + l$ and kl are even. **(07 Marks)**

4 a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \ge 6$. **(06 Marks)**

b. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \ge 2$. Find a_n in explicit form. **(07 Marks)**

c. The Fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \ge 2$. Evaluate F_2 to F_{10}. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. For any non-empty sets A, B, C, prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$. **(05 Marks)**

b. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{w, x, y, z\}$. Find the number of onto functions from A to B. **(05 Marks)**

c. Show that if any 6 numbers from 1 to 10 are choosen, then two of them have their sum equal to 11. **(05 Marks)**

d. Let f, g, h be functions from R to R defined by $f(x) = x + 2$, $g(x) = x - 2$, $h(x) = 3x$ for all $x \in R$. Find gof, fog, foh, hog, hof. **(05 Marks)**

6 a. Consider the sets $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$ and the relations $R = \{(a,1),(b,1),(c,2),(c,3)\}$ and $S = \{(a,1),(a,2),(b,1),(b,2)\}$ from A to B determine $\overline{R}$, $\overline{S}$, $R \cup S$, $R \cap S$, R^C and S^C. **(06 Marks)**

b. Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by $(x_1, y_1) R (x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

(i) Verify R is an equivalence relation on $A \times A$.

(ii) Determine the equivalence classes $[(1, 3)]$, $[(2, 4)]$ and $[(1, 1)]$

(iii) Determine the partition of $A \times A$ induced by R. **(07 Marks)**

c. Let A = {1, 2, 3, 4, 6, 12} on A define the relation R by aRb if an only if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. **(07 Marks)**

7 a. If * is an operation on z defined by $x * y = x + y + 1$. Prove that $(z, *)$ is an abelian group. **(06 Marks)**

b. State and proof Lagrange's theorem. **(07 Marks)**

c. Prove that the intersection of two subgroups of a group is a subgroup of the group. **(07 Marks)**

8 a. The Parity-check matrix for an encoding function, $E : Z_2^3 \to Z_2^6$ is given by,

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

(i) Determine the associated generator matrix.

(ii) Does this code correct all single errors in transmission? **(06 Marks)**

b. Prove that the set z with binary operations $\oplus$ and $\odot$ defined by $x \oplus y = x + y - 1$ and $x \odot y = x + y - xy$ is a commutative ring with unity. **(07 Marks)**

c. Prove that every finite integral domain is a field. **(07 Marks)**

* * * * *