USN | | | | | | | | | |    **10MAT11**

## First Semester B.E. Degree Examination, Dec.2017/Jan.2018
# Engineering Mathematics – I

Time: 3 hrs.    Max. Marks:100

**Note:** ***Answer any FIVE full questions, choosing atleast TWO questions from each part.***

### PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The $n^{th}$ derivative of $\log_e(ax + b)$ is

A) $\frac{(-1)^{n-1}(n-1)!a^n}{ax+b}$ B) $\frac{(-1)^{n-1}n!a^n}{(ax+b)^n}$ C) $\frac{n!a^n}{(ax+b)^{n+1}}$ D) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$

ii) If f(x) = ex, g(x)=e--x statistics the conditions of Cauchy's mean value theorem in [3,7], then their exists atleast one point $C\in(3,7)$. The value of C is

A) 4 B) 5 C) 6 D) 3.5

iii) The $n^{th}$ derivative of $\cosh^2 ax$ is

A) $2^n a^n\{e^{2ax}+e^{-2ax}\}$ B) $2^{n-1}\{e^{2ax}+e^{-2ax}\}$

C) $2^{n-2}a^n\{e^{2ax}+(-1)^n e^{-2ax}\}$ D) $2^{n-1}\{e^{ax}+e^{-ax}\}$

iv) The Maclaurin's series expansion of cosh x is

A) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+----$ B) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+----$

C) $1-\frac{x^2}{2!}+\frac{x^4}{4!}+----$ D) $x-\frac{x^3}{3!}+\frac{x^5}{5!}+----$

b. If $\sin^{-1}y = 2\log(x + 1)$, then prove that $(x + 1)^2y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2+4)y_n = 0$. **(06 Marks)**

c. State and prove Cauchy's mean value theorem. **(06 Marks)**

d. By using Maclaurin's series, expand log(1+ cos x) up to the term containing $x^4$. **(04 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) $\lim_{x\to0}(\cos x)^{1/x^2}$ is

A) 1 B) 0 C) $e^{-1}$ D) $e^{-1/2}$

ii) The angle between the radius vector and tangent to the curve $r = a\,e^{\theta\cot\alpha}$ at every point is

A) θ B) $\frac{\pi}{2}+\theta$ C) α D) $\frac{\pi}{4}+\theta$

iii) The pedal equation to the curve r = a θ is

A) $\frac{r}{r^2+a^2}$ B) $\frac{a}{\sqrt{r^2+a^2}}$ C) $\frac{r^2}{r^2+a^2}$ D) $\frac{r^2}{\sqrt{r^2+a^2}}$

iv) The radius of curvature of a circle $x^2 + y^2 = 5$ is

A) 5 B) $\sqrt{5}$ C) 0 D) None of these

b. Evaluate : i) $\lim_{x\to0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$ ii) $\lim_{x\to a}\left(2-\frac{x}{a}\right)^{\tan(\pi x/2a)}$. **(06 Marks)**

c. Prove that the following polar curves intersect orthogonally : $r = a\sec^2\theta/2$, $r = b\,\mathrm{cosec}^2\,\theta/2$. **(06 Marks)**

d. Find the radius of curvature of the curve y = 4 sin x – sin 2x, at $x = \pi/2$. **(04 Marks)**

3 a. Choose the correct answers for the following : (04 Marks)

i) If $u = xy\, f(x/y)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

A) 1 B) 0 C) 2u D) 3u

ii) If $u = f(4y - 5z, 5z - 3x, 3x - 4y)$, then $\frac{1}{3}\frac{\partial u}{\partial x} + \frac{1}{4}\frac{\partial u}{\partial y} + \frac{1}{5}\frac{\partial u}{\partial z} =$

A) 0 B) 1 C) u D) 3u

iii) If $x = r\cos\theta$, $y = r\sin\theta$, then the Jacobian of $(r, \theta)$ with reference to $(x, y)$ is equal to

A) $\frac{1}{r}$ B) r C) 1 D) 0

iv) The percentage error in the area of a rectangle if an error of 1% is made while measuring its sides is

A) 1% B) 2% C) 3% D) 4%

b. If $u = At^{-1/2} e^{-x^2/4a^2t}$, prove that $a^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$. (06 Marks)

c. If $u = e^x \sin y$, $v = e^x \cos y$ and $w = f(u, v)$, prove that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 w}{\partial u^2} + \frac{\partial^2 w}{\partial v^2}\right)$. (06 Marks)

d. Given $x = a(u + v)$, $y = b(u - v)$ and $u = r^2\cos 2\theta$ and $v = r^2 \sin 2\theta$, find $\frac{\partial(x, y)}{\partial(r, \theta)}$. (04 Marks)

4 a. Choose the correct answers for the following : (04 Marks)

i) If $\vec{a}$ is a constant vector, then $\nabla(\vec{a} \cdot \vec{r})$ is

A) $\vec{r}$ B) $\vec{a}$ C) a D) $|\vec{r}|$

ii) If $\vec{f} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x - az)\hat{k}$ is solenoidal then a is

A) 1 B) 0 C) −1 D) 2

iii) If $\vec{v} = \vec{w} \times \vec{r}$, where $\vec{w}$ is a constant vector, then curl $\vec{v}$ is

A) 0 B) $2\vec{w}$ C) $\vec{w}$ D) $\vec{r}$

iv) The scale factor for spherical polar coordinates $(r, \theta, \phi)$ are

A) (1, 0, 1) B) (−r, 0, 1) C) $(1, r, r\sin\theta)$ D) $(\cos\theta, 1, r)$.

b. If $\vec{f} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that $\vec{f}$ is irrotational. Then find $\phi$ such that $\vec{f} = \nabla\phi$. (06 Marks)

c. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\nabla r^n = nr^{n-2}\vec{r}$. Hence deduce that $\nabla^2 r^n = n(n+1)r^{n-2}$. (06 Marks)

d. Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical polar coordinates. (04 Marks)

## PART – B

**5** a. Choose the correct answers for the following : **(04 Marks)**

i) Leibnitz's rule for differentiation under the integral sign is

A) $\phi'(y) = \int_a^b \frac{\partial}{\partial x \partial y} f(x,y)dx$ B) $\phi'(y) = \int_a^b \frac{\partial}{\partial y} f(x,y)dx$

C) $\phi(y) = \int_a^b \frac{\partial}{\partial x \partial y} f(x,y)$ D) None of these

ii) The value of $\int_0^1 x^6\sqrt{1-x^2}\, dx$ is

A) $\frac{12}{765}$ B) $\frac{13}{764}$ C) $\frac{14}{365}\pi$ D) $\frac{15}{768}\pi$

iii) Asymptote to the curve $y^2(a + x) = x^2(a - x)$ is

A) $x = 0$ B) $x = -a$ C) $x = a$ D) $y = 0$

iv) The surface are of the solid generated by the curve $y = f(x)$ about the y – axis is

A) $\int_{x=a}^{b} \pi y^2 dx$ B) $\int_{x=a}^{b} \pi x\, dx$ C) $\int_{y=c}^{d} 2\pi y\, ds$ D) $\int_{y=c}^{d} 2\pi x\, ds$.

b. By using the rule of differentiation under the integral sign, evaluate $\int_0^\infty \frac{\tan^{-1}\alpha x}{x(1+x^2)}dx$, where $\alpha \geq 0$. **(06 Marks)**

c. If n is a positive integer, show that $\int_0^{2a} x^n\sqrt{2ax - x^2}\, dx = \frac{(2n+1)!a^{n+2}}{(n+2)!n!2^n}\pi$. **(06 Marks)**

d. The cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, $0 \leq \theta \leq 2\pi$ rotates about its base. Find the volume of the solid generated. **(04 Marks)**

**6** a. Choose the correct answers for the following : **(04 Marks)**

i) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$ is

A) $\operatorname{cosec}(y/x) + \cot(y/x) = c$ B) $\operatorname{cosec}(y/x) - \cot(y/x) = cx$

C) $\operatorname{cosec}(y/x)\cot(y/x) = c$ D) $\tan(y/x) - \sec(y/x) = cx$

ii) The integrating factor of the differential equation $\frac{dy}{dx} - \frac{2}{x}y = x$ is

A) $\frac{1}{x}$ B) $x^2$ C) $\frac{1}{x^2}$ D) $\frac{2}{x^2}$

iii) The differential equation $(x + x^8 + ay^2)\, dx + (y^8 - y + bxy)dy = 0$ is exact, if b =

A) 4a B) 2a C) –3a D) None of these

iv) The orthogonal trajectories of the family of curves $y = ax^2$, where a is the parameter, is

A) $x^2 + y^2 = c$ B) $x^2 + 2y^2 = c$ C) $2x^2 + y^2 = c$ D) $x^2 - y^2 = c$.

b. Solve : $(x^2 - xy + y^2)\, dx - xy\, dy = 0$. **(06 Marks)**

c. Solve : $(x^2y^3 + xy)\frac{dy}{dx} = 1$. **(06 Marks)**

d. Find the orthogonal trajectories of the family of curves $y = x + c\, e^{-x}$, where c is the parameter. **(04 Marks)**

7 a. Choose the correct answers for the following : (04 Marks)

i) The rank of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$;

A) 3 B) 4 C) 2 D) 1

ii) If rank of the coefficient matrix is equal to the rank of the augmented matrix, then the system of equations :

A) is inconsistent B) have no solution
C) have infinite number of solutions D) is consistent

iii) In Gauss – Jordan method the coefficient matrix reduces to

A) Diagonal matrix B) Scalar matrix C) Identity matrix D) Symmetric matrix

iv) A square matrix is said to be symmetric, if

A) $a_{iJ} = -a_{Ji}$ B) $a_{iJ} > a_{Ji}$ C) $a_{iJ} = a_{Ji}$ D) $a_{iJ} < a_{Ji}$.

b. Test the following system for consistency and solve if it is consistent :

$5x + 3y + 7z = 4$
$3x + 26y + 2z = 9$
$7x + 2y + 10z = 5.$ (06 Marks)

c. Solve the following system of equations by using the Gauss – Jordan method :

$2x - 3y + z = -1$
$x + 4y + 5z = 25$
$3x - 4y + z = 2.$ (06 Marks)

d. Find the rank of the following matrix by reducing it to echelon form :

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}.$$ (04 Marks

8 a. Choose the correct answers for the following : (04 Marks)

i) The product of the eigenvalues of the matrix : $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 3 & 1 \\ 0 & 2 & 5 \end{bmatrix}$

A) 9 B) 15 C) 88 D) 48

ii) If 3, 6 and – 9 are the eigenvalues of the square matrix A of the given quadratic form, then its signature is

A) 0 B) 1 C) 2 D) 3

iii) An orthogonal transformation preserves the

A) Dot product of vectors B) Cross product of vectors
C) Norm of a vector D) None of these

iv) For an orthogonal matrix A, if $\lambda$ is an eigenvalue then

A) $-\lambda$ is an eivenvalue B) $\lambda^2$ is an eigenvalue C) $1/\lambda$ is an eigenvalue D) $-\frac{1}{\lambda}$ is an eigenvalue.

b. Reduce the following matrix to diagonal form : $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$. (06 Marks)

c. Show that the linear transformation : $y_1 = 2x_1 + x_2 + x_3$ ; $y_2 = x_1 + x_2 + 2x_3$ ; $y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation. (06 Marks)

d. Obtain the canonical form of the quadratic form :
$2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ got by an orthogonal transformation. Indicate its rank and signature. (04 Marks)




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