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14MAT11

First Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, selecting ONE full question from each module.

Module-1

- 1 a. If $y = a \cos(\log_e x) + b \sin(\log_e x)$, show that $x^2 y_2 + xy_1 + y = 0$ and $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (07 Marks)
- b. Show that the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ cut each other orthogonally. (06 Marks)
- c. Find the radius of curvature at the point $(a, 0)$ on the curve $xy^2 = a^3 - x^3$. (07 Marks)

OR

- 2 a. Find the n^{th} derivative of $\cos x \cos 2x \cos 3x$. (06 Marks)
- b. Define curvature of a curve and derive an expression for the radius of curvature in the polar form. (07 Marks)
- c. Find the pedal equation of the curve $\frac{2a}{r} = 1 + \cos\theta$. (07 Marks)

Module-2

- 3 a. Obtain the Maclaurin's series for $e^x \cos x$ upto the term containing x^4 . (06 Marks)
- b. If $w = f(x, y)$, $x = r \cos\theta$, $y = r \sin\theta$, show that $\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$. (07 Marks)
- c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. (07 Marks)

OR

- 4 a. Find the constants 'a' and 'b' such that $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3}$ may be equal to unity. (07 Marks)
- b. If $u = \log_e \left(\frac{x^5 + y^5 + z^5}{x^2 + y^2 + z^2} \right)$, then show that by using Euler's theorem $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 3$. (06 Marks)
- c. If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r \cos\theta$, then find the value of $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. (07 Marks)

Module-3

- 5 a. Prove that the surfaces $4x^2 y + z^3 = 4$ and $5x^2 - 2yz = 9x$ intersect orthogonally at the point $(1, -1, 2)$. (07 Marks)
- b. Show that $\text{Div}(\text{curl } \vec{A}) = \vec{0}$. (06 Marks)
- c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$. (07 Marks)

1 of 2

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.

OR

- 6 a. A vector field is given by $\vec{f} = (x^2 + xy^2)\mathbf{i} + (y^2 + x^2)\mathbf{j}$. Show that the field is irrotational and find the scalar potential. (07 Marks)
- b. If $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, show that i) $\text{div } \vec{r} = 3$ ii) $\text{curl } \vec{r} = \vec{0}$ (06 Marks)
- c. Evaluate $\int_0^{\infty} \left(\frac{e^{-ax} \sin x}{x} \right) dx$ and hence show that $\int_0^{\infty} \left(\frac{\sin x}{x} \right) dx = \frac{\pi}{2}$, by using differentiation under integral sign. (07 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \sin^m x \cos^n x dx$, where m and n are positive integers. (06 Marks)
- b. Solve: $xy(1 + xy^2) \frac{dy}{dx} = 1$. (07 Marks)
- c. Find the orthogonal trajectories of a system of confocal and coaxial parabolas $y^2 = 4a(x + a)$. (07 Marks)

OR

- 8 a. Solve: $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$. (07 Marks)
- b. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2 - x^2}} dx$, using reduction formulae. (06 Marks)
- c. Water at temperature 100°C cools in 10 minutes to 88°C in a room of temperature 25°C . Find the temperature of water after 20 minutes. (07 Marks)

Module-5

- 9 a. Reduce the following matrix to Echelon form and hence find the Rank. (06 Marks)
- $$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$
- b. Solve by LU decomposition method $x + 2y + 3z = 14$, $-2x + 3y + 4z = 20$, $3x + 4y + z = 14$. (07 Marks)
- c. Determine the largest eigen-value and the corresponding eigen vector of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ with the initial eigen vector be $[1, 1, 0]^T$ using Rayleigh's power method. Perform six iterations. (07 Marks)

OR

- 10 a. Solve $3x + 8y + 29z = 71$, $83x + 11y - 4z = 95$, $7x + 52y + 13z = 104$ by using Gauss-Seidel method. Carryout 3 iterations. (06 Marks)
- b. Reduce the matrix $\begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$ to the diagonal form. (07 Marks)
- c. Reduce the quadratic form $x^2 + 5y^2 + z^2 + 6xz + 2xy + 2yz$ to the canonical form and specify the matrix of transformation. (07 Marks)