14MAT21

Engineering Mathematics – II

Time: 3 hrs.

Max. Marks:100

Note: *Answer any FIVE full questions, selecting ONE full question from each module.*

Module – 1

1 a. Solve: $\dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} + 13y = e^{3t}\cosh 2t + 2^t$. **(06 Marks)**

 b. Solve : $y'' - 4y' + 4y = 8\cos 2x$. **(07 Marks)**

 c. Solve : $y'' + 4y = x^2 + e^{-x}$ by the method of undetermined coefficients. **(07 Marks)**

OR

2 a. Solve : $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. **(06 Marks)**

 b. Solve : $y'' + 4y' - 12y = e^{2x} - 3\sin 2x$. **(07 Marks)**

 c. Solve by the method of variation of parameters $y'' + 2y' + 2y = e^{-x}\sec^3 x$. **(07 Marks)**

Module – 2

3 a. Solve : $\dfrac{dx}{dt} + 2y = -\sin t$, $\dfrac{dy}{dt} - 2x = \cos t$. **(06 Marks)**

 b. Solve : $x^4\dfrac{d^3y}{dx^3} + 2x^3\dfrac{d^2y}{dx^2} - x^2\dfrac{dy}{dx} + xy = \sin(\log x)$. **(07 Marks)**

 c. Solve : $xy\left(\dfrac{dy}{dx}\right)^2 - (x^2 + y^2)\dfrac{dy}{dx} + xy = 0$, using solvable for P. **(07 Marks)**

OR

4 a. Solve : $\dfrac{dy}{dx} + y = z + e^x$, $\dfrac{dz}{dx} + z = y + e^x$. **(06 Marks)**

 b. Solve : $(3x + 2)^2 y'' + 3(3x + 2)y' - 36y = 8x^2 + 4x + 1$. **(07 Marks)**

 c. Show that the equation, $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation. Hence obtain the general and singular solution. **(07 Marks)**

Module – 3

5 a. Form the partial differential equation by eliminating the arbitrary function in $\phi(x + y + z, \ x^2 + y^2 - z^2) = 0$ **(06 Marks)**

 b. Derive one dimensional wave equation in the form, $\dfrac{\partial^2 u}{\partial t^2} = C^2\dfrac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

 c. Evaluate : $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. **(07 Marks)**

OR

6 a. Solve $\dfrac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$ given that $u = 0$ when $t = 0$ and $\dfrac{\partial u}{\partial t} = 0$ at $x = 0$. Also show that
       $u \to \sin x$ as $t \to \infty$. **(06 Marks)**

   b. Derive one dimensional heat equation in the form, $\dfrac{\partial u}{\partial t} = C^2 \dfrac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

   c. Evaluate $\displaystyle\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ by changing the order of integration. **(07 Marks)**

Module – 4

7 a. Find the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ by double integration. **(06 Marks)**

   b. Show that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

   c. Express the vector $\vec{A} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates. **(07 Marks)**

OR

8 a. Find the volume generated by the revolution of the cardioide $r = a(1+\cos\theta)$ about the initial
       line. **(06 Marks)**

   b. Show that $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta = \pi$ **(07 Marks)**

   c. Show that spherical polar coordinate system is orthogonal. **(07 Marks)**

Module – 5

9 a. Find the Laplace transform of $2^t + \dfrac{\cos 2t - \cos 3t}{t} + t\sin t$. **(06 Marks)**

   b. If $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a, \ f(t+2a) = f(t) \end{cases}$
       then (i) Sketch the graph of $f(t)$ as a periodic function.
       (ii) Show that $L\{f(t)\} = \dfrac{1}{s^2}\tanh\left(\dfrac{as}{2}\right)$ **(07 Marks)**

   c. Solve $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using Laplace
       transform method. **(07 Marks)**

OR

10 a. Find the Laplace transform of $t^2 e^{-3t}\sin 2t$. **(06 Marks)**

    b. Express the following function interms of Heaviside unit step function and hence find its
        Laplace transform:
        $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ **(07 Marks)**

    c. Using convolution theorem obtain the inverse Laplace transform of :
        $\dfrac{s^2}{(s^2 + a^2)(s^2 + b^2)}$. **(07 Marks)**

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