

OR

- 6 a. Form the partial differential equation by eliminating the arbitrary function from $z = y\phi(x) + x\psi(y)$. (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial y^2} = z$; given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$. (07 Marks)
- c. Find the various possible solution for one dimensional heat equation by the method of separation of variables. (07 Marks)

Module-4

- 7 a. Prove that $\Gamma(1/2) = \sqrt{\pi}$. (06 Marks)
- b. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx$ (07 Marks)
- c. Evaluate $\iint xy(x+y) \, dx \, dy$ over the area between $y = x^2$ and $y = x$. (07 Marks)

OR

- 8 a. Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} \, dy \, dx$ by changing the order of integration. (06 Marks)
- b. Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. (07 Marks)
- c. Prove that with usual notations $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)

Module-5

- 9 a. Find the Laplace transform of $\frac{\cos 2t - \cos 3t}{t}$. (06 Marks)
- b. Express the function in terms of unit step function and hence find its Laplace transform
- $$f(t) = \begin{cases} \cos t & 0 < t < \pi \\ 1 & \pi < t < 2\pi \\ \sin t & t > 2\pi \end{cases}$$
- (07 Marks)
- c. Find $L^{-1} \left\{ \frac{s+3}{s^2-4s+13} + \log_e \left(\frac{s+1}{s-1} \right) \right\}$. (07 Marks)

OR

- 10 a. Find the Laplace transform of the periodic function
- $$f(t) = \begin{cases} t & 0 < t < \pi \\ \pi - t & \pi < t < 2\pi \end{cases}$$
- of period
- 2π
- . (06 Marks)
- b. Using convolution theorem obtain the inverse Laplace transform of $\frac{s}{(s+2)(s^2+9)}$. (07 Marks)
- c. Solve the equation $y'' - 3y' + 2y = e^{3t}$; $y(0) = 1$ and $y'(0) = 0$ using Laplace transform technique. (07 Marks)
