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10MAT21

Second Semester B.E. Degree Examination, June/July 2018

Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two from each part.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) If the differential equation is solving for x then it is of the form,

A) $x = f\left(\frac{P}{y}\right)$ B) $y = f(x, P)$ C) $x = f\left(\frac{y}{P}\right)$ D) $x = f(y, p)$

ii) The general solution of $P^2 - 7P + 12 = 0$,

A) $(y+3x-c)(y+4x-c)=0$ B) $(y-3x-c)(y-4x-c)=0$

C) $(y-4x)(y+3x)=0$ D) None of these

iii) $y = cx + f(c)$ is the general solution of the equation,

A) $x = py + f(p)$ B) $y = 3x + \log p$ C) $y = px + f(p)$ D) None of these

iv) The general solution of the equation, $(y-px)^2 = 4p^2 + 9$ is,

A) $y = Cx + \sqrt{4C^2 + 9}$ B) $y = C + \sqrt{4C^2 + 9}$

C) $y = Cx + \sqrt{4C^2 - 9}$ D) $y - Cx = 4C^2 + 9$

b. Solve $P^2 + 2Py\cot x = y^2$. **(05 Marks)**

c. Solve $(y-px)(p-1) = p$. **(05 Marks)**

d. Solve the equation $(px-y)(py+x) = 2p$ by reducing into Clairaut's form, taking the substitutions $X = x^2$, $Y = y^2$. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) $\frac{1}{f(D)}\left[e^{2x}.x^2\right] =$ ________,

A) $e^{2x}\frac{1}{f(D+2)}x^2$ B) $e^{2x}\frac{1}{f(D-2)}x^2$ C) $x^2\frac{1}{f(D-2)}e^{2x}$ D) $x^2\frac{1}{f(D+2)}e^{2x}$

ii) If 1, 1, −2 are the roots of auxillary equation of the differential equation then its solution is,

A) $e^x + e^x + e^{-2x}$ B) $(C_1 + C_2x)e^x + C_3e^{-2x}$

C) $C_1e^x + C_2e^x + C_3e^{-2x}$ D) None of these

iii) Particular integral of $(D+1)^2 y = e^{-x+3}$ is,

A) $\frac{x^2}{2}$ B) x^3e^x C) $\frac{x^3}{3}e^{-x+3}$ D) $-\frac{x^2}{2}e^{-x+3}$

iv) General solution of non homogeneous linear differential equation is,

A) Sum of complementary function and particular integral
B) Difference of complementary function and particular integral
C) Product of complementary function and particular integral
D) None of these

b. Solve : $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$. **(05 Marks)**

c. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2\sin x\cos x$. **(05 Marks)**

d. Solve the system of equations: $\frac{dx}{dt} - 2y = \cos 2t$, $\frac{dy}{dt} + 2x = \sin 2t$. **(06 Marks)**

3 a. Choose the correct answers for the following : (04 Marks)

i) The complementary function of $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$

A) $C_1e^{-x}+C_2e^{-3x}$ B) $C_1(-x)+C_2(-2x)$ C) $C_1e^{-2x}+C_2e^{2x}$ D) $\frac{C_1}{x}+\frac{C_2}{x^2}$

ii) In the method of variation of parameters the value of W is called ______,

A) Wronskian function B) Exponential function
C) DeMorgans function D) Euler's function

iii) The roots of the auxillary equation of the transformed equation of,

$(2x+1)^2\frac{d^2y}{dx^2}-2(2x+1)\frac{dy}{dx}-12y=6x+5$ are,

A) $3,-1$ B) $-3,1$ C) $12,-4$ D) None of these

iv) The solution of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$ is,

A) $y=C_1\cos x+C_2\sin x$ B) $y=C_1e^x+C_2e^{-x}$ C) $y=C_1\log x+C_2$ D) $y=C_1+6x^3$

b. Solve $y''-6y'+9y=\frac{e^{3x}}{x^2}$ by the method of variation of parameters. (05 Marks)

c. Solve $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=\sin(\log x)$. (05 Marks)

d. Solve $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ in series solution. (06 Marks)

4 a. Choose the correct answers for the following : (04 Marks)

i) The partial differential equation obtained by eliminating arbitrary constants from the relation $z=(x-a)^2+(y-b)^2$ is,

A) $p^2+q^2=4z$ B) $p^2-q^2=4z$ C) $p+q=z$ D) $p-q=2z$

ii) The auxillary equation of $Pp+Qq=R$ are,

A) $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$ B) $\frac{dx}{p}=\frac{dy}{q}=\frac{dz}{R}$ C) $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$ D) $\frac{dx}{x}+\frac{dy}{y}+\frac{dz}{z}=0$

iii) A solution of $(y-z)p+(z-x)q=x-y$ is ......

A) $x^2-y^2-z^2=f(x-y+z)$ B) $x^2+y^2+z^2=f(x+y+z)$
C) $x^2-y^2-z^2=f(x-y-z)$ D) None of these

iv) By the separation of variables, solution is in the form of,

A) $Z=X.Y$ B) $Z=X+Y$ C) $Z=X^2.Y^2$ D) $Z=\frac{X}{Y}$

b. Form a PDE from the relation $Z=y^2+2f\left[\frac{1}{x}+\log y\right]$. (05 Marks)

c. Solve the equation $(x^2-y^2-z^2)p+2xyq=2xz$. (05 Marks)

d. Use the method of separation of variables to solve $\frac{\partial u}{\partial x}=2\frac{\partial u}{\partial t}+u$, given that $u(x,0)=6e^{-3x}$. (06 Marks)

PART – B

5 a. Choose the correct answers for the following : (04 Marks)

i) The value of $\int_1^2\int_1^3 xy^2dxdy$ is ______,

A) 0 B) 1 C) 13/2 D) 13

ii) The integral $\int_0^1 \int_{\sqrt{y}}^1 dxdy$ after changing the order of integration,

A) $\int_0^2 \int_0^y dxdy$ B) $\int_0^1 \int_0^{y^2} dxdy$ C) $\int_0^1 \int_0^1 dxdy$ D) $\int_0^1 \int_0^{x^2} dydx$

iii) $\beta\left(\frac{1}{2}, \frac{1}{2}\right) =$ ______

A) $\sqrt{\pi}$ B) $\frac{2}{\sqrt{\pi}}$ C) $\pi\sqrt{2}$ D) 3.1416

iv) The value of $\Gamma(1/4)\Gamma(3/4) =$ ________

A) $2\sqrt{\pi}$ B) $\pi\sqrt{2}$ C) $\sqrt{2}$ D) $\frac{\sqrt{\pi}}{2}$

b. Change the order of integration in $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dxdy$ and hence evaluate the same. **(05 Marks)**

c. Show that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$. **(05 Marks)**

d. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\int_C \vec{F}.dr = 0$ then $\vec{F}$ is called,

A) Rotational B) Solenoidal C) Irrotational D) Dependent

ii) If f is a vector field over a region of volume V in three dimensional space then, $\int_V f.dv$ is called,

A) Scalar volume integral B) Vector volume integral
C) Scalar surface integral D) Vector surface integral

iii) In Green's theorem in the plane $\iint_A \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] dxdy$ is ________,

A) $\int_C [Mdx - Ndy]$ B) $\int_C [Mdx + Ndy]$ C) $\int_C [Mdx \times Ndy]$ D) $\int_C [Ndx - Mdy]$

iv) If V is the volume bounded by a surface S and $\vec{F}$ is a continuously differentiable vector function then, $\iiint_V \operatorname{div}\vec{F}\, dv =$ __________

A) 0 B) $\iint_S \vec{F} \times \hat{n}\, ds$ C) $\iint_S \vec{F}.\hat{n} ds$ D) None of these

b. Evaluate $\iint_S f.nds$ where $f = yzi + zxj + xyk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the 1st Octant. **(05 Marks)**

c. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2 dy$ where C is the closed curve made up of the line $y = x$ and the parabola $y = x^2$. **(05 Marks)**

d. Verify stoke's theorem for, $f=(2xy-y)i-yz^2j-y^2zk$ for the upper half of the sphere $x^2+y^2+z^2=1$ **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) If $L[f(t)] = \tilde{f}(s)$ then $L[e^{at} f(t)]$ is ________.

A) $\tilde{f}(s-a)$ B) $\tilde{f}(s+a)$ C) $\tilde{f}\left(\frac{s}{a}\right)$ D) None of these

ii) $L\left[\frac{\sin at}{t}\right] =$ ________

A) $\cot^{-1}\left(\frac{s}{a}\right)$ B) $\tan^{-1}\left(\frac{s}{a}\right)$ C) $\frac{\pi}{2} + \tan^{-1}\left(\frac{s}{a}\right)$ D) None of these

iii) $L[u(t+2)] =$ ________

A) $\frac{e^{-2s}}{s^2}$ B) e^{2s} C) $\frac{e^{2s}}{s}$ D) $\frac{e^{-2s}}{s}$

iv) $L[s(t+2)] =$ ________

A) e^{-as} B) e^{2s} C) e^{-2s} D) e^{as}

b. Find the value of $\int_0^\infty t^3 e^{-t} \sin t\, dt$ using Laplace Transforms. **(05 Marks)**

c. If $f(t) = \begin{cases} t, & 0 \le t \le a \\ 2a - t, & a \le t \le 2a \end{cases}$ where $f(t+2a) = f(t)$ show that $L[f(t)] = \frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$. **(05 Marks)**

d. Express $f(t) = \begin{cases} 1, & 0 < t \le 1 \\ t, & 1 < t \le 2 \\ t^2, & t > 2 \end{cases}$ in terms of unit step function and hence find its Laplace transform. **(06 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) $L^{-1}[\cos at] =$ ________

A) $\frac{s}{s^2+a^2}$ B) $\frac{s}{s^2-a^2}$ C) $\frac{1}{s^2-a^2}$ D) $\frac{1}{s^2+a^2}$

ii) $L^{-1}\left[\tilde{f}(s-a)\right] =$ ________

A) $e^{at} f(t)$ B) $e^{t} f(t)$ C) $e^{-at} f(t)$ D) None of these

iii) $L^{-1}\left[\cot^{-1}(s)\right] =$ ________

A) $\frac{\sinh t}{t}$ B) $\frac{\sin t}{t}$ C) $-\frac{\sinh t}{t}$ D) $-\frac{\sin t}{t}$

iv) $L\left[\int_0^t f(u) g(t-u)\, du\right] =$ ________

A) $f(s).g(s)$ B) $\frac{f(s)}{g(s)}$ C) $f(t) \cdot g(t)$ D) $f(s) + g(s)$

b. Find $L^{-1}\left[\log \frac{s+1}{s-1}\right]$. **(05 Marks)**

c. Find $L^{-1}\left[\frac{s}{(s-1)(s^2+4)}\right]$ using convolution theorem. **(05 Marks)**

d. Solve the initial value problem $y'' - 3y' + 2y = 1 - e^{2t}$, $y(0) = 1$, $y'(0) = 1$ using Laplace transform. **(06 Marks)**

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