USN

14MAT11

First Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, selecting ONE full question from each module.*

Module-1

1 a. Find the n^{th} derivative of $y = e^{ax}\sin(bx+c)$. **(06 Marks)**

b. Find the angle of intersection between the curves $r = a(1+\cos\theta)$, $r = b(1-\cos\theta)$. **(07 Marks)**

c. Find the radius of curvature of the curve $a^2y = x^3 - a^3$ at the point where the curve cuts the x-axis. **(07 Marks)**

2 a. If $y = \tan^{-1}x$ then prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. **(06 Marks)**

b. Find the pedal equation of $\frac{2a}{r} = 1+\cos\theta$. **(07 Marks)**

c. Find the radius of curvature of the curve $r = a(1-\cos\theta)$. **(07 Marks)**

Module-2

3 a. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots\ldots$ **(06 Marks)**

b. If $u = \sin^{-1}\left[\frac{x^3+y^3}{x+y}\right]$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$. **(07 Marks)**

c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ find $J\left(\frac{u,v,w}{x,y,z}\right)$. **(07 Marks)**

4 a. If $Z = e^{ax+by}f(ax-by)$ prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. **(06 Marks)**

b. Evaluate $\lim_{x\to 0}\left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$ **(07 Marks)**

c. Find the extreme values of the function $f(x,y) = x^2 + 2xy + 2y^2 + 2x + y$. **(07 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where 't' is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$. **(06 Marks)**

b. Using differention under integral sign, evaluate $\int_0^\infty \frac{e^{-ax}\sin x}{x}dx$. **(07 Marks)**

c. Show that $\operatorname{div}(\operatorname{curl}\vec{A}) = 0$ **(07 Marks)**

6 a. If $\vec{v} = \vec{w}\times\vec{r}$, prove that $\operatorname{curl}\vec{v} = 2\vec{w}$ where $\vec{w}$ is a constant vector. **(06 Marks)**

b. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. Trace the curve $y^2(a-x) = x^3$, $a > 0$. **(07 Marks)**

Module-4

7 a. Obtain reduction formula for $\int \sin^n x dx$. **(06 Marks)**

b. Solve $(e^y + y\cos xy)dx + (xe^y + x\cos xy)dy = 0$. **(07 Marks)**

c. Find orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ where λ is the parameter. **(07 Marks)**

8 a. Evaluate $\int_0^1 x^5(1-x^2)^{5/2}dx$. **(06 Marks)**

b. Solve $x\frac{dy}{dx} + y = x^3y^6$. **(07 Marks)**

c. A body originally at 80°C cools down at 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(06 Marks)

b. Diagonalize the matrix $A = \begin{bmatrix} -5 & 9 \\ -6 & 10 \end{bmatrix}$. **(07 Marks)**

c. Reduce the quadratic form $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ to canonical form. Hence find its rank, index and signature. **(07 Marks)**

10 a. Solve $x + y + z = 9$, $2x + y - z = 0$, $2x + 5y + 7z = 52$ by Gauss elimination method. **(06 Marks)**

b. Show that, the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular transformation and find the inverse transformation. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the following matrix by using power method

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Taking $[1\ 0\ 0]^T$ as initial eigen vector. Take five iterations. **(07 Marks)**

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