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10MAT11

First Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least TWO from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following :

i) The n^{th} derivative of coshax is

A) $\frac{a^n}{2}\left[e^{ax} + (-1)^n e^{-ax}\right]$ B) $\frac{a^n}{2}\left[e^{ax} - (-1)^n e^{-ax}\right]$

C) $\frac{a^n}{2}\left[e^{-ax} + (-1)^n e^{ax}\right]$ D) $\frac{a^n}{2}\left[e^{-ax} - (-1)^n e^{ax}\right]$

ii) If f(x) is continuous in [a, b], differentiable in (a, b) and f(a) = f(b), then there exists at least one value c of x in (a,b) such that $f'(c) =$

A) $\frac{f(b)-f(a)}{b-a}$ B) $\frac{f(a)+f(b)}{a+b}$ C) 0 D) None of these

iii) If f(x) is continuous in [a, b], differentiable in (a, b) and $f'(x) > 0$ for all x in (a, b), then f(x) is

A) Strictly decreasing B) Strictly increasing
C) Neither increasing nor decreasing D) 0

iv) Maclauirin's series expansion of cosx is

A) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-.....$ B) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....$

C) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-.....$ D) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+.....$ **(04 Marks)**

b. If $y = e^{\tan^{-1}x}$ prove that $(1 + x^2)\, y_{n+2} + (2n + 2x - 1)\, y_{n+1} + n(n+1)\, y_n = 0$. **(04 Marks)**

c. Using Lagrange's mean value theorem, prove that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$ where $a < b < 1$. **(06 Marks)**

d. Expand logcosx about $x = \pi/3$ using Taylor's series up to the fourth degree terms. **(06 Marks)**

2 a. Choose the correct answers for the following :

i) $\underset{x\to 0}{\text{Lt}} \frac{\sec x - 1}{\tan^2 x}$ is equal to

A) 2 B) −2 C) 0 D) 1/2

ii) The angle between radius vector and tangent of $r = ae^{\theta\cot\alpha}$ is

A) $\pi/2 - \theta$ B) α C) θ D) $\pi/2$

iii) The radius of curvature of the curve $pa^2 = r^3$ is

A) $\frac{a^2}{3r}$ B) $\frac{a^2b^2}{p^3}$ C) $\frac{a^3}{3}$ D) $\frac{ab}{p}$

iv) Pedal equation of the curve $r = a(1 + \cos\theta)$ is

A) $r^2 = 2ap^3$ B) $r = 3ap$ C) $r^3 = 2ap$ D) $r^3 = 2ap^2$ **(04 Marks)**

b. Evaluate $\underset{x \to 0}{\text{Lt}} \dfrac{\tan x - x}{x^2 \tan x}$. (04 Marks)

c. Show that the pedal equation of the curve $r^n = a^n \sin n\theta + b^n \cos n\theta$ is $P^2(a^{2n} + b^{2n}) = r^{2n+2}$. (06 Marks)

d. Find the radius of curvature for the curve $xy^2 = a^2(a - x)$ at (a, 0). (06 Marks)

3 a. Choose the correct answers for the following : (04 Marks)

i) If $f(x, y) = \dfrac{1}{x^3} + \dfrac{1}{y^3} + \dfrac{1}{x^3 + y^3}$, then $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ is

A) 0 B) 3f C) f D) −3f

ii) If $x = r\cos\theta$, $y = r\sin\theta$ then $\dfrac{\partial(x, y)}{\partial(r, \theta)}$ is

A) 1 B) r C) $\dfrac{1}{r}$ D) 0

iii) If an error of 1% is made in measuring its base and height, the percentage error in the area of a triangle is

A) 0.2% B) 1% C) 2% D) 0

iv) If $r = f_{xx}(a, b)$, $s = f_{xy}(a, b)$, $t = f_{yy}(a, b)$ then f(x, y) will have maximum at (a, b) if

A) $f_x = 0, f_y = 0, rt - s^2 > 0$ and $r > 0$ B) $f_x = 0, f_y = 0, rt - s^2 > 0$ and $r < 0$

C) $f_x = 0, f_y = 0, rt - s^2 = 0, r > 0$ D) $f_x = 0, f_y = 0, rt - s^2 > 0$ and $r = 0$

b. If $u = \tan^{-1}(y/x)$ where $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$ find $\dfrac{du}{dt}$. (04 Marks)

c. A rectangular box, open at the top, is to have a volume of 32 cubic units. Find the dimensions of the box requiring least material for its construction. (06 Marks)

d. The focal length f of a lens is given by $\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q}$, where p and q are the distances of the lens from the object and the image respectively. For a certain lens, p and q are each 20cms with a possible maximum error of 0.5cm. Find the maximum error in f. (06 Marks)

4 a. Choose the correct answers for the following :

i) If div $\vec{F} = 0$ then $\vec{F}$ is called

A) Solenoidal vector B) Irrotational vector

C) Rotational vector D) None of these.

ii) The curl of gradient of $\vec{F}$ is

A) 1 B) $\nabla^2 \vec{F}$ C) $\nabla \vec{F}$ D) 0

iii) If $\vec{F} = 2x^2 i - 3yzj + xz^2 k$ then $\nabla \cdot \vec{F}$ is

A) $4x - 3z + 2xz$ B) $2x - 3y + 2xz$ C) $x + y - z$ D) $x^2 - 3y + z^2$

iv) Spherical polar coordinate system is

A) Orthogonal B) Coplanar C) Collinear D) Not orthogonal (04 Marks)

b. Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F}$ = grad $(x^3 + y^3 + z^3 - 3xyz)$ at the point (1, 2, 3) (04 Marks)

c. Prove that div curl $\vec{F} = 0$ (06 Marks)

d. Show that cylindrical coordinate system is orthogonal. (06 Marks)

PART – B

5 a. Choose the correct answers for the following :

i) The value of $\int_0^{\pi/4} \cos^3 2x \, dx$ is

A) 1 B) 1/3 C) – 1 D) $\pi/2$

ii) The value of $\int_0^1 x^6\sqrt{1-x^2} \, dx$ is

A) $\frac{8\pi}{135}$ B) $\frac{\pi}{16}$ C) $\frac{5\pi}{256}$ D) $\frac{5\pi}{126}$

iii) The asymptote to the curve $y^2(a - x) = x^3$ is

A) $y = 0$ B) $x = 0$ C) $x = a$ D) $y = a$

iv) The entire length of the cardioid $r = a(1 + \cos\theta)$ is

A) 7a B) 8a C) 10a D) 9a **(04 Marks)**

b. Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$ **(04 Marks)**

c. Using differentiation under integral sign evaluate $\int_0^\infty e^{-x} \frac{\sin \alpha x}{x} dx$ **(06 Marks)**

d. Find the volume of the solid obtained by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x – axis **(06 Marks)**

6 a. Choose the correct answers for the following :

i) The solution of the differential equation $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$ is

A) $\tan x \tan y = c$ B) $\sec x \tan y = c$ C) $\sec y \tan x = c$ D) $\sec y \tan y = c$

ii) The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \sin(y/x)$ is

A) $\tan cx = y$ B) $y = 2x \tan^{-1}(cx)$ C) $y = 2\tan^{-1} cx$ D) $\sin cx = y$

iii) The integrating factor for the differential equation $\cos^2 x \frac{dy}{dx} + y = \tan x$ is

A) tanx B) cotx C) sinx D) cosx

iv) By replacing $\frac{dr}{d\theta}$ by $-r^2 \frac{d\theta}{dr}$ in the differential equation $f\left(r, \theta, \frac{dr}{d\theta}\right) = 0$ we get the differential equation of

A) polar trajectory B) parametric trajectory
C) orthogonal trajectory D) parallel trajectory **(04 Marks)**

b. Solve $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$ **(04 Marks)**

c. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4) dy = 0$ **(06 Marks)**

d. Find the orthogonal trajectories of the family of curves $y = x + Ce^{-x}$, where C is the parameter. **(06 Marks)**

7 a. Choose the correct answers for the following :

i) The rank of a unit matrix of order 4 is

A) 4 B) 3 C) 1 D) 0

ii) The equation $x + 2y = 1$; $7x + 14y = 12$ are

A) Consistent B) Inconsistent
C) Consistent having unique solution D) Consistent having infinite solution.

iii) The homogeneous system of equations has
A) Trivial solution B) Non trivial solution
C) No solution D) None of these.

iv) Matrix has a value this statement
A) is always true B) Depends upon the matrix
C) is false D) None of these. **(04 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ in to normal form and hence find its rank. **(04 Marks)**

c. Test for consistency and solve
$5x + 3y + 7z = 4$; $3x + 26y + 2z = 9$; $7x + 2y + 10z = 5$ **(06 Marks)**

d. Solve the system of equations,
$2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$ by Gauss – Jordan method. **(06 Marks)**

8 a. Choose the correct answers for the following :

i) The matrix of the linear transformation that transforms the pair (x_1, x_2) to the pair $(2x_1 - 5x_2, 5x_1 + 4x_2)$ is
A) $\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 2 & -5 \\ 5 & 4 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

ii) The eigen values of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are
A) 0, 2, 15 B) 0, 3, 15 C) 0, 1, 2 D) 1, 3, 15

iii) The matrix of the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4zx$ is
A) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ B) $\begin{bmatrix} 8 & 0 & 3 \\ 6 & -7 & 4 \\ 2 & 3 & 5 \end{bmatrix}$ C) $\begin{bmatrix} -8 & 2 & 6 \\ 6 & 7 & -4 \\ 2 & -3 & 4 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 3 & 5 \\ -2 & 6 & 7 \\ 2 & 3 & 8 \end{bmatrix}$

iv) A real quadratic form $X'AX$ in n variates is said to be indefinite if
A) All the eigen values of A are positive
B) All the eigen values of A are negative
C) All the eigen values of A are zero
D) Some of the eigen values are positive and other negative. **(04 Marks)**

b. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}$ **(04 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ to the diagonal form. **(06 Marks)**

d. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation. **(06 Marks)**

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