CBCS SCHEME

USN

18HCE/ECS/ELD/EIE/ESP/EVE11

First Semester M.Tech. Degree Examination, June/July 2019
Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Show that set $W = \{(a, b, c) : a - 3b + 4c = 0\}$ is a subspace of the 3-tuple space $R^3(R)$. **(06 Marks)**

b. Show that three vectors $(1, 1, -1)$, $(2, -3, 5)$ and $(-2, 1, 4)$ of R^3 are linearly independent. **(07 Marks)**

c. Let $\alpha = (1, 2, 1)$, $\beta = (2, 9, 0)$ and $\nu = (3, 3, 4)$. Show that the set $S = \{\alpha, \beta, \nu\}$ is a basis of R^3. **(07 Marks)**

OR

2 a. Show that the mapping $T: V_2(R) \to V_2(R)$ defined as $T(x, y) = (x \cos\alpha - y \sin\alpha, x \sin\alpha + y \cos\alpha)$ is a linear transformation from $V_2(R) \to V_2(R)$. **(06 Marks)**

b. Show that the system of three vectors $(1, 3, 2)$, $(1, -7, -8)$, $(2, 1, -1)$ of $V_3(R)$ is linearly dependent. **(07 Marks)**

c. Find the matrix of the linear transformation T on $V_3(R)$ defined as $T(a, b, c) = (2b + c, a - 4b, 3a)$ with respect to the ordered basis $\{(1, 0, 0), (0, 1, 0), (0, 0,1)\}$. **(07 Marks)**

Module-2

3 a. Transform the matrix

$$M = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

to tridiagonal form by the Givens method. Also find the eigen values of the tridiagonal matrix. **(10 Marks)**

b. Apply Gram-Schmidt orthogonalization process to find an orthonormal basis from subspace of IR^4 spanned by vectors $X_1 = (1\ 1\ 1\ 1)^1$, $X_2 = (1\ 2\ 4\ 5)^1$, $X_3 = (1, -3, -4, -2)^1$ **(10 Marks)**

OR

4 a. Transform the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$$

to tridiagonal form by the Givens method. Obtain the intervals of unit length. **(10 Marks)**

b. Suppose $\{\overleftrightarrow{x_1}, \overrightarrow{x_2}, \overrightarrow{x_3}\}$ where $\overrightarrow{x_1} = (1\ 2\ 3\ 0)^1$, $\overrightarrow{x_2} = (1\ 2\ 0\ 0)^1$, $\overrightarrow{x_3} = (1\ 0\ 0\ 0)^1$ is a basis for subspace W of IR^4. Construct an orthonormal basis for W by applying Gram-Schmidt orthogonalization process. **(10 Marks)**

Module-3

5 a. Derive Euler's equation in the form

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

(07 Marks)

b. Find the extremal of the functional $\int_0^{\pi/2}\left[y^2 - (y')^2 - 2y\sin x\right]dx$ under the end conditions $y(0) = y(\pi/2) = 0$. **(07 Marks)**

c. Find the extremal of the functional $\int_0^1\left[1 + (y'')^2\right]dx$ **(06 Marks)**

OR

6 a. Find the extremal of the functional

$$\int_{x_1}^{x_2}\left[y^2 + (y')^2 + 2ye^x\right]dx$$

(06 Marks)

b. Find the extremal of the functional $\int_0^1\left[\dot{y}^2 + \dot{x}^2 + 2y\right]dt$ under the end conditions $x(0) = 0$, $x(1) = 1$, $y(0) = 1$ and $y(1) = 3/2$. **(07 Marks)**

c. Find the extremal of the functional $\int_0^1 (y')^2\,dx$ under the end conditions $y(0) = 1$, $y(1) = 0$ and subject to constraint $\int_0^1 y\,dx = 1$ **(07 Marks)**

Module-4

7 a. The p.d.f of a variate X is given by the following table:

X	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

For what value of K, this represents a valid probability distribution? Also find $P(x \geq 5)$ and $P(3 < x \leq 6)$. **(06 Marks)**

b. Derive the expressions for mean and variance in case of Poisson distribution. **(07 Marks)**

c. The marks of 100 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75. **(07 Marks)**

OR

8 a. If x is a random variable with probability generating function $P_x(t)$, find the probability generating function of (i) $x + 2$ (ii) $2x$. **(06 Marks)**

b. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction. **(07 Marks)**

c. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation of the distribution. **(07 Marks)**

Module-5

9 a. Define : (i) Stochastic process (ii) Stationary process. **(06 Marks)**

b. A stochastic process with its ensemble functions are assumed to have equal probabilities are given by : $x_1(t) = 3$, $x_2(t) = 3 \sin t$, $x_3(t) = -3 \sin t$, $x_4(t) = 3 \cos t$, $x_5(t) = -3 \cos t$, $x_6(t) = -3$. Show that the process is WSS. **(07 Marks)**

c. Find the autocorrelation $R(t_1, t_2)$ of the stochastic process defined by $x(t) = A \cos(wt + \alpha)$, where the random variables A and α are independent and α is uniform in the interval $[-\pi, \pi]$. **(07 Marks)**

OR

10 a. Time averaged mean values of an ensemble of a random process x(t) are given by $\{-K, -2K, -3K, K, 2K, 3K\}$, $K > 0$ show that the process is not mean ergodic. **(06 Marks)**

b. A random process X(t) is based on the outcome of tossing a die and observing the number on the face. The waveforms corresponding to each of the outcomes is given below:

Outcome	1	2	3	4	5	6
X(t)	–2	–1	1	2	–t	t

Compute the following probabilities:

(i) $P[X(1) = -1]$ (ii) $P[X(1) \leq 0]$ **(07 Marks)**

c. Define a stochastic process and explain

(i) Wide-Sense Stationarity (WSS)

(ii) Ergodicity. **(07 Marks)**

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