USN

First Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two from each part.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) Sixth derivative of $(3x+2)^{-5}$ is

A) 0 B) $\frac{-(10!)}{4!}\cdot\frac{3^6}{(3x+2)^{11}}$ C) $\frac{10!}{4!}\cdot\frac{3^5}{(3x+2)^{11}}$ D) $\frac{10!}{4!}\frac{3^6}{(3x+2)^{11}}$

ii) If $y = ax^{n+1} + \frac{b}{x^n}$ then y_2 is

A) $\frac{n(n-1)y}{x^2}$ B) $\frac{n(n+1)y}{x^2}$ C) $\frac{-n(n+1)y}{x^2}$ D) None of these.

iii) By Rolle's theorem for $f(x) = |x|$ in $[-1, 1]$, the value of c is

A) 1 B) Does not exist C) 0 D) None of these.

iv) Expansion of e^x about $x = 1$ is

A) $1+(x-1)+\frac{(x-1)^2}{2!}+\ldots\ldots$ B) $e^x\left\{1+(x-1)+\frac{(x-1)^2}{2!}+\ldots\ldots\right\}$

C) $e\left\{1+(x-1)+\frac{(x-1)^2}{2!}+\ldots\ldots\right\}$ D) $e\left\{1+e^{x-1}+\frac{e^{2(x-1)}}{2!}+\ldots\ldots\right\}$

b. If $x = \sin t$, $y = \cos pt$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(p^2-n^2)y_n = 0$ **(06 Marks)**

c. State and prove Lagrange's Mean Value theorem. **(04 Marks)**

d. Expand $\tan^{-1}x$ by Maclaurin's series upto first four non-zero terms. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\lim_{y\to\infty}\left(y+\frac{1}{y}\right)^{1/y}$ is

A) e B) ∞ C) 0 D) None of these

ii) The angle between the curves $r^n = a^n\cos n\theta$ and $r^n = b^n\tan n\theta$ is

A) 0 B) $\pi/2$ C) Does not exist D) None of these

iii) The radius of curvature of $\log(\cos x)$ is

A) $\sec x$ B) $-\sec x$ C) $\sec^2 x$ D) $\sec^3 x$

iv) The pedal equation of $m\theta = \log\left(\frac{r}{a}\right)$ is

A) $r^2 = p(1+m^2)$ B) $r = ae^{m\theta}$ C) $r = p^2(1+m^2)$ D) $r^2 = p^2(1+m^2)$

b. Find the angle between the radius vector and the tangent to the curve $2a = r(1-\cos\theta)$ **(04 Marks)**

c. Find the points on $y^2 = 8x$ at which the radius of curvature is 125/16. **(06 Marks)**

d. Evaluate $\lim_{x\to 0}\left(\frac{1}{x^2}-\cot^2 x\right)$ **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $f = f(x, y),\ x = x(t),\ y = y(t)$ then $\frac{df}{dt} =$

A) $\frac{\partial x}{\partial x}\cdot\frac{\partial f}{\partial t}+\frac{\partial y}{\partial t}\cdot\frac{\partial f}{\partial x}$ B) $\frac{\partial f}{\partial x}\cdot\frac{dt}{dx}+\frac{\partial f}{\partial y}\cdot\frac{dy}{dt}$

C) $\frac{\partial f}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial f}{\partial y}\cdot\frac{dy}{dt}$ D) $\frac{\partial f}{\partial y}\cdot\frac{dx}{dt}+\frac{\partial f}{\partial y}\cdot\frac{dy}{dt}$

ii) In $N = 10\pi L / R^4$, if L is decreased by 2% and R is increased by 2% then the value of N is

A) increased by 10% B) decreased by 10%

C) increased by 6% D) decreased by 6%

iii) If $u = x + y,\quad v = \frac{1}{x+y},\quad J = \frac{\partial(u,v)}{\partial(x,y)},\quad J^* = \frac{\partial(x,y)}{\partial(u,v)}$ then

A) $J^* = 0$ B) $J^* = 1$ C) J does not exist D) J^* does not exist

iv) If (a, b) is a critical point of f(x, y) and at (a, b), $f_{xx} = A,\ f_{xy} = B,\ f_{yy} = C$ then f(x, y) has a minimum at (a, b) if

A) $AC - B^2 > 0,\ A < 0$ B) $AC - B^2 > 0,\ A = 0$

C) $B^2 - AC > 0,\ A > 0$ D) None of these

b. If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, then find $\frac{\partial^2 u}{\partial x\partial y}$. **(04 Marks)**

c. If $u = x + y + z,\ uv = y + z,\ uvw = z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. **(06 Marks)**

d. Find the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) The operator $\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$ has the magnitude

A) 0 B) $|i + j + k| = 3$

C) Not defined D) $\left[\left(\frac{\partial}{\partial x}\right)^2+\left(\frac{\partial}{\partial y}\right)^2+\left(\frac{\partial}{\partial z}\right)^2\right]^{1/2}$

ii) $\mathrm{Div}(2x^2z\,i - xy^2z\,j + 3yz^2\,k)$ at (1, 1, 1) is

A) 12 B) 8 C) 10 D) − 8

iii) The velocity field is obtained by "Stirring coffee in a cup" is related to

A) Divergence B) curl C) Mixing sugar in it D) Cold coffee.

iv) The unit vectors e_1, e_2, e_3 are called

A) fundamental vectors B) base vectors

C) dependent vectors D) both (A) and (B)

b. Find the directional derivative of $\phi = x^2 - y^2 + 2z^2 = 0$ at P(1, 2, 3) in the direction of the line PQ were Q(5, 0, 4). In what direction it will be maximum and what is its value? **(04 Marks)**

c. Prove that $\nabla\times(\overline{A}\times\overline{B}) = (\overline{B}\cdot\nabla)\overline{A} - (\nabla\cdot\overline{A})\overline{B} + (\nabla\cdot\overline{B})\overline{A} - (\overline{A}\cdot\nabla)\overline{B}$ **(06 Marks)**

d. Prove that a spherical coordinate system is orthogonal. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) $\int_0^{\pi} \sin^n x \, dx = 2\int_0^{\pi/2} \sin^n x \, dx$ is

A) True only when n is even B) true only when n is odd
C) True always D) False always

ii) The area bounded by the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$, $0 \le t \le 2\pi$ and its base x-axis is

A) πa^2 B) $2\pi a^2$ C) $3\pi a^2$ D) $4\pi a^2$

iii) In a curve $f(x, y) = 0$, if only even powers of x and y appears in the equation then it is symmetric about the line

A) x – axis B) y – axis C) origin D) All of these.

iv) The value of $\int_0^{\pi/2} \sin^3 x \cos^4 x \, dx$ is

A) 2/35 B) $\pi/35$ C) $6\pi/105$ D) π

b. Evaluate $\int_0^{a} x^2 (a^2 - x^2)^{3/2} dx$ **(04 Marks)**

c. Find the entire length of $x^{2/3} + y^{2/3} = a^{2/3}$ **(06 Marks)**

d. Evaluate $\int_0^{\pi/2} \frac{\log(1 + a\sin^2 x)}{\sin^2 x} dx$, a is a parameter, by using the method of differentiation under integral sign. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) If $y' = \frac{1}{x}\left(y + x\tan\left(\frac{y}{x}\right)\right)$ then $\sin\left(\frac{y}{x}\right) =$

A) cx^2 B) cx C) cx^3 D) cx^4

ii) For $M\,dx + N\,dy = 0$, the integrating factor is of the form

A) $\frac{1}{N}\left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right) = f(x)$ B) $\frac{1}{N}\left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right) = g(y)$

C) $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = g(y)$ D) $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$

iii) If $(ax^2 + bxy^2)dx + (px^2y + qy^3)dy = 0$ is exact then

A) a = p, b = q B) a = q, b = p C) a = 2b = p = q D) b = p

iv) The orthogonal trajectories of $x^2 + y^2 = a^2$ are

A) y = x B) y = kx C) $x^2 + y^2 = k^2$ D) None of these

b. Find the orthogonal trajectories of $x^2 + y^2 = 2cx$ **(04 Marks)**

c. Solve $(1 + x^2)\frac{dy}{dx} + xy = \sinh^{-1} x$ **(06 Marks)**

d. Solve $(x^2 + y^3 + 6x)dx + xy^2 dy = 0$ **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) The statement "A matrix has a value" is

A) always false B) always true
C) true only if the matrix is square D) true only if the matrix is orthogonal.

ii) $2x - 3y = 5$ and $ax + by = -10$ can have many solutions if (a, b) =
A) (2, 3) B) (2, –3) C) (4, –6) D) (–4, 6)

iii) Rank of $\begin{pmatrix} 1 & 2 & -3 & 2 & 4 \\ 4 & 8 & -12 & 8 & 16 \\ 3 & 6 & -9 & 6 & 12 \end{pmatrix}$ is
A) 0 B) 1 C) 2 D) 3

iv) In a system of n linear equations, r and r_1 are the ranks of the coefficient matrix and the augmented matrix respectively and $r = r_1$ then there
A) is a unique solution if r < n B) is a unique solution if r > n
C) are infinitely many solutions if r > n D) are infinitely many solutions if r < n

b. Find the non-trivial solution of the equations
$x + 2y + 3z = 0$, $3x + 4y + 4z = 0$, $7x + 10y + 11z = 0$ if it exists. **(04 Marks)**

c. Find the condition satisfied by a, b, c so that the following system of equations may have a solution:
$x + 2y - 3z = a$, $3x - y + 2z = b$, $x - 5y + 8z = c$ **(06 Marks)**

d. Solve the system by Gauss Jordan method:
$x + 2y + z = 3$, $2x + 3y + 2z = 5$, $3x - 5y + 5z = 2$, $3x + 9y - z = 4$. **(06 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) The roots of $|A - \lambda I| = 0$ are called
A) Latent roots B) Characteristic roots
C) Both (A) and (B) D) None of these

ii) A matrix A is orthogonal if
A) $|A| = 0$ B) $A = A^T$ C) $A = A^{-1}$ D) $A^{-1} = A^T$

iii) If $|A| = 0$ then there atleast one eigenvalue of A is
A) 0 B) 1 C) 2 D) λ

iv) If r is the rank of A and p is the index of X^TAX in n variables then the Quadratic form X^TAX is called negative definite if
A) r = n, p = n B) r = n, p = 0 C) r < n, p = 0 D) r < n, p = r

b. Determine the nature of the Quadratic form without reducing to the canonical form:
$2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_3x_1$ **(04 Marks)**

c. Find the eigenvalues and eigenvector corresponding to the numerically largest eigenvalue of
$$A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$ **(06 Marks)**

d. Reduce the quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_3$ to canonical form by orthogonal reduction. **(06 Marks)**

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