USN

14ELD11

First Semester M.Tech. Degree Examination, June/July 2019

Advanced Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Apply shifted QR algorithm to compute eigen values of

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$

(10 Marks)

b. Find the singular value decomposition of

$$A = \begin{bmatrix} -3 & 1 \\ -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

(10 Marks)

2 a. Find the generalized inverse of A.

$$A = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{bmatrix}$$

(10 Marks)

b. Solve the following system of equations in least square sense.

$$2x_1 + 2x_2 - 2x_3 = 1$$
$$2x_1 + 2x_2 - 2x_3 = 3$$
$$-2x_1 - 2x_2 + 6x_3 = 2$$

(10 Marks)

3 a. Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity. **(10 Marks)**

b. Solve the variational problem $\int_1^2 [x^2(y')^2 + 2y(x+y)]dx = 0$ given $y(1) = y(2) = 0$. **(10 Marks)**

4 a. Prove that the sphere is the solid figure of revolution which for a given surface area has maximum volume. **(10 Marks)**

b. Show that the functional $\int_0^{\pi/2} \left\{2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\} dt$ such that $x(0) = 0$, $x(\pi/2) = -1$, $y(0) = 0$, $y(\pi/2) = 1$ is stationary for $x = -\sin t$ $y = \sin t$. **(10 Marks)**

5 a. Solve the following initial boundary value problem corresponding to wave equation by Laplace transform method.

$u_{tt} = u_{xx} \quad 0 < x < 1 \quad t > 0$

BC's $u(0, t) = u(1, t) = 0$

IC's $u(x, 0) = \sin\pi x \quad u_t(x, 0) = -\sin\pi x$ **(10 Marks)**

b. A string is stretched and fixed between two points (0, 0) and (ℓ, 0) motion is initiated by displacing string in the form $u = \lambda \sin\left(\frac{\pi x}{\ell}\right)$ and released from rest at time t = 0. Using Laplace transform find the displacement of the string at any time 't'. **(10 Marks)**

6 a. Solve the heat conduction in the infinite rod as given below using Fourier transform method.

$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad -\infty < x < \infty \quad t > 0$

BC's $u(x, t) \quad u_x(x, t) \to 0$ as $|x| \to \infty$

IC $u(x, 0) = f(x) \quad -\infty < x < \infty$ **(10 Marks)**

b. Determine the temperature distribution in the semi-infinite medium $x \geq 0$ when the end $x = 0$ is maintained at zero temperature and the initial temperature distribution is f(x), applying Fourier transform techniques. **(10 Marks)**

7 a. Prove that if a harmonic function vanishes everywhere on the boundary then its identically zero everywhere. **(10 Marks)**

b. Solve the Laplace equation with boundary conditions as given below by Fourier transform method.

$u_{xx} + u_{yy} = 0 \quad -\infty < x < \infty \quad y > 0$

BC $u(x, 0) = f(x) \quad -\infty < x < \infty$

u is bounded as $y \to \infty$, u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$. **(10 Marks)**

8 a. Use Big M method to

Maximize $Z = 2x_1 + 3x_2$

Subject to the constraints $x_1 + 2x_2 \leq 4$ and $x_1 + x_2 \leq 3$; $x_1 \geq 0$, $x_2 \geq 0$. **(10 Marks)**

b. Find the minimum point using Lagrangean method for the nonlinear function

$f(x) = x_1^2 + x_2^2 + x_3^2$

Subject to $g_1(x) = x_1 + x_2 + 3x_3 - 2 = 0$

$g_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0$ **(10 Marks)**

* * * * *