USN | | | | | | | | | | | **14MAT11**

First Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting ONE full question from each module.***

Module – 1

1 a. Find the n^{th} derivative of $\dfrac{x}{(x-1)(x^2+x-2)}$. **(07 Marks)**

b. Find the angle of interaction of the curves, $r = a(1+\cos\theta)$ and $r^2 = a^2\cos 2\theta$. **(06 Marks)**

c. Derive an expression for the radius of curvature in polar form. **(07 Marks)**

OR

2 a. If $\sin^{-1} y = 2\log(x+1)$, prove that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$. **(07 Marks)**

b. Find the pedal equation, $r^m = a^m(\cos m\theta + \sin m\theta)$. **(06 Marks)**

c. If ρ be the radius of curvature at any point P(x, y) on the parabola $y^2 = 4ax$ show that ρ^2 varies as $(SP)^3$ where S is the focus of the parabola. **(07 Marks)**

Module – 2

3 a. Obtain Taylor's series expansion of $\log(\cos x)$ about the point $x = \dfrac{\pi}{3}$ upto the fourth degree term. **(07 Marks)**

b. If $u = f(r)$ where $r = \sqrt{x^2+y^2+z^2}$, show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = f''(r) + \dfrac{2}{r}f'(r)$. **(06 Marks)**

c. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left[\dfrac{1}{x^2} - \cot^2 x\right]$. **(07 Marks)**

b. If u be a homogeneous function of degree n in x and y, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = nu$. **(06 Marks)**

c. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

Module – 3

5 a. Find the unit tangent vector and unit normal vector to the curve $\vec{r} = \cos 2t\, i + \sin 2t\, j + tk$ at $x = \dfrac{1}{\sqrt{2}}$. **(07 Marks)**

b. Using differentiation under the integral sign, show that $\displaystyle\int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}dx = \pi\sin^{-1} a$. **(06 Marks)**

c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$ **(07 Marks)**

OR

6 a. Show that $\vec{F} = \frac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

b. Show that $\text{curl}\left(\phi\vec{A}\right) = \phi(\text{curl}\vec{A}) + \text{grad}\phi \times \vec{A}$. **(06 Marks)**

c. Use general rules to trace the curve, $r = a\cos 2\theta$ (four leaved rose). **(07 Marks)**

Module – 4

7 a. Obtain the reduction formula for $\int \sin^m x \cos^n x dx$, where m and n are positive integers. **(07 Marks)**

b. Solve $y(1 + xy + x^2y^2)dx + x(1 - xy + x^2y^2)dy = 0$. **(06 Marks)**

c. Find the orthogonal trajectories of the family of curves $r = 4a\sec\theta\tan\theta$. **(07 Marks)**

OR

8 a. Evaluate (i) $\int_0^{2a} x^2\sqrt{2ax - x^2}dx$ (ii) $\int_0^{2a} \frac{x^2}{\sqrt{2ax - x^2}}dx$. **(07 Marks)**

b. Solve $\frac{dy}{dx} - y\tan x = \frac{\sin x\cos^2 x}{y^2}$ **(06 Marks)**

c. The R-L series circuit differential equation acted on by an electromotive force $E\sin\omega t$ satisfies the differential equation, $L\frac{di}{dt} + Ri = E\sin\omega t$. If there is no current in the circuit initially, obtain the value of current at any time 't'. **(07 Marks)**

Module – 5

9 a. Solve $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$ by Gauss elimination method. **(07 Marks)**

b. Diagonalize the matrix, $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(06 Marks)**

c. Determine the largest eigen value and the corresponding eigen vector of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ using Rayleigh's power method. **(07 Marks)**

OR

10 a. Solve by LU decomposition method, $4x_1 + x_2 + x_3 = 4$, $x_1 + 4x_2 - 2x_3 = 4$, $3x_1 + 2x_2 - 4x_3 = 6$. **(07 Marks)**

b. Show that the transformation, $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(06 Marks)**

c. Reduce the quadratic form, $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 6x_1x_3 + 2x_2x_3$ into canonical form by orthogonal transformation. Indicate the orthogonal transformation. **(07 Marks)**

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