USN | | | | | | | | | | **10CS34**

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. For any two sets A and B, prove that
$A-(A-B)=A\cap B$ **(05 Marks)**

b. If A, B, C are finite sets, then prove that
$|A-B-C|=|A|-|A\cap B|-|A\cap C|+|A\cap B\cap C|$ **(05 Marks)**

c. A professor has two dozen introductory text books on computer science and is concerned about their coverage of the topics (i) Compilers, (ii) Data structures, and (iii) Operating systems. The following is the data on the number of books that contain material on these topics.
$|A|=8,\ |B|=13,\ |C|=13,\ |A\cap B|=5, |A\cap C|=3,\ |B\cap C|=6,\ |A\cap B\cap C|=2.$
Find :
i) How many have no material on compilers.
ii) How many do not deal with any of the topics. **(05 Marks)**

d. A girl rolls (throws) a fair die three times. What is the probability that her second and third rolls are both larger than her first roll? **(05 Marks)**

2 a. Define Tautology. Prove that, for any propositions p, q, r the compound proposition.
$[(p\vee q)\wedge\{(p\to r)\wedge(q\to r)\}]\to r$ is a tautology. **(05 Marks)**

b. Without using truth tables, prove the following logical equivalence :
$[(p\vee q)\wedge(p\vee\neg q)]\vee q\Leftrightarrow p\vee q$ **(05 Marks)**

c. For any propositions p, q, r prove the following:
i) $p\uparrow(q\uparrow r)\Leftrightarrow\neg p\vee(q\wedge r)$
ii) $p\downarrow(q\downarrow r)\Leftrightarrow\neg p\wedge(q\vee r)$ **(05 Marks)**

d. Test the validity of the following argument : **(05 Marks)**
If I study, I will not fail in the examination.
If I do not watch TV in the evenings, I will study.
I failed in the examination.
$\therefore$ I must have watched TV in the evenings

3 a. Consider the following open statements with the set of all real numbers as the universe.
$p(x): x\geq 0,$ $q(x): x^2\geq 0$
$r(x): x^2-3x-4=0,$ $s(x): x^2-3>0.$
determine the truth values of the following statements :
i) $\forall x, p(x)\to q(x)$
ii) $\forall x, q(x)\to s(x)$
iii) $\forall x, r(x)\to p(x)$ **(06 Marks)**

b. Prove that the following argument is valid : **(07 Marks)**

$$\begin{array}{l} \forall x,[p(x) \rightarrow \{q(x) \wedge r(x)\}] \\ \underline{\forall x,[p(x) \wedge s(x)]} \\ \therefore \forall x,[r(x) \wedge s(x)] \end{array}$$

c. Give (i) a direct proof (ii) an indirect proof, and (iii) proof by contradiction, for the following statement :
"if n is an odd integer, then n + 9 is an even integer" **(07 Marks)**

4 a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(06 Marks)**

b. Find an explicit definition of the sequence defined recursively by
$a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$. **(07 Marks)**

c. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers, prove that

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$$

For all positive integers n. **(07 Marks)**

PART – B

5 a. Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. **(05 Marks)**
If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a, b.

b. Let A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6}.
Find :
i) The number of one-one function from A to B
ii) The number of onto functions from A to B.
iii) The number of onto functions from B to A. **(05 Marks)**

c. Let A = B = R the set of all real numbers, and the functions $f: A \rightarrow B$ and $g: B \rightarrow A$ be defined by $f(x) = 2x^3 - 1, \forall x \in A$; $g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}, \forall y \in B$.
Show that each of f and g is the inverse of the other. **(05 Marks)**

d. If 5 colours are used to paint 26 doors, prove that at least 6 doors will have the same colour. **(05 Marks)**

6 a. Let A {1, 2, 3, 4, 5}. Define a relation R on A×A by $(x_1, y_1)\ R\ (x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.
i) Verify that R is an equivalence relation on A×A.
ii) Determine the equivalence classes [(1, 3)] and [(2, 4)]. **(06 Marks)**

b. Let A = {1, 2, 3, 4, 6, 8, 12}. On A, define the relation R by aRb if and only if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation. **(07 Marks)**

c. Consider the Hasse diagram of a poset (A, R) given below in Fig Q6(c).

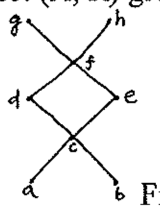

Fig Q6(c)

If B = {c, d, e}, find
i) All upper bounds of B
ii) all lower bounds of B
iii) the LUB of B
iv) the GLB of B. **(07 Marks)**

7 a. Define abelian group. Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$. **(06 Marks)**

b. Define homomorphism. Let f be a homomorphism from a group G_1 to a group G_2. Prove that

i) If e_1 is the identity in G_1, and e_2 is the identity in G_2, then $f(e_1) = e_2$

ii) $f(a^{-1}) = [f(a)]^{-1}$ for all $a \in G_1$ **(07 Marks)**

c. An encoding function $E : Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

i) Determine all the code words.

ii) Find the associated parity check matrix H. What about its error correction capability? **(07 Marks)**

8 a. Define group code. Consider the encoding function $E : Z_2^2 \rightarrow Z_2^6$ of the triple repetition code defined by E (00) = 000000, E(10) = 101010, E(01) = 010101, E(11) = 111111. Prove that $C = E\ (Z_2^2)$ is a group code. **(06 Marks)**

b. Define a Ring. If R is a ring with unity and a, b are units in R, prove that ab is a unit in R and that $(ab)^{-1} = b^{-1}a^{-1}$. **(07 Marks)**

c. Define integral domain. Let R be a commutative ring with unity. Prove that R is an integral domain if an only if, for all a, b, c, $\in$ R where $a \neq 0$, $ab = ac \Rightarrow b = c$. **(07 Marks)**

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