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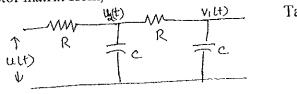
## Modern Control Theory

Max. Marks: 100

Note: Answer FIVE full questions, selecting at least TWO questions from each part.

- PART A

  What are the advantages of modern control theory over conventional control theory? 1 (05 Marks)
  - For the system shown, write the state equations satisfied by them. Bring these equations in (07 Marks) vector matrix form,



Take  $R = 1 M\Omega$  and  $C = 1 \mu F$ 

c. A Feedback system is characterized by the closed loop transfer function,

$$T(s) = \frac{s^2 + 3s + 3}{s^3 + 2s^2 + 3s + 1}$$

Draw the signal flow graph and obtain the state model in second companion form. (08 Marks)

Obtain the state space representation of the given system in Jordan canonical form. 2

$$\frac{y(s)}{U(s)} = \frac{2s^2 + 6s + 7}{(s+1)^2(s+2)}$$
(12 Marks)

b. Obtain the transfer function for the state model represented by,  $\dot{x} = Ax + Bu$ , y = Cx + DU,

where 
$$A = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix}$$
,  $B = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$ ,  $C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $D = \begin{bmatrix} 0 \end{bmatrix}$  (08 Marks)

- Prove that the modal matrix M diagonalizes the system matrix A. (04 Marks)
  - For the matrix,  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix}$ , find i) Eigen values ii) Eigen vectors iii) Modal matrix (08 Marks)

Compute the state transition matrix for,  $A = \begin{bmatrix} 0 & -3 \\ 1 & -4 \end{bmatrix}$  using i) Laplace – transformation (08 Marks) method. ii) Cayley-Hamilton method.

- (04 Marks) Define state transition matrix and list its properties.
  - A linear time invariant system is characterized by,

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} [\mathbf{u}]; \quad \mathbf{y} = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix}$$

Compute the response y(t) to a unit step input assuming  $X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ . (12 Marks)

c. Evaluate the controllability of the system with 
$$\dot{x} = Ax + BU$$
 where  $A = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$ ,

$$\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$
 (04 Marks)

## PART - B

5 a. A system is described by following state model:

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$$

Compute the state feedback gain matrix "K" so that the control law to u = -Kx places the closed loop poles at  $-2 \pm j4$ , -5, using direct substitution method. (10 Marks)

b. Consider the system,  $\dot{X} = AX + Bu$  and y = CX

where 
$$A = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix}$$
,  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ,  $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ 

Design a full order state observer using Ackermann's formula.

(10 Marks)

(07 Marks)

- 6 a. What is PI and PD controller? What are its effect on system performance? (06 Marks)
  - b. Discuss pole placement by state feedback. What is the necessary condition for design using state feedback? (06 Marks)
  - c. Explain Backlash and Jump resonance with respect to non-linear systems. (08 Marks)
- 7 a. What are singular points? Explain different singular points based on the location of Q point.
  (08 Marks)
  - b. A linear second order servo system is described by the state equation,

$$e + 2\xi\omega_n e + \omega_n^2 e = 0$$

where  $\xi = 0.15$  and  $\omega_n = 1$  rad/sec, e(0) = 1.5 and e(0) = 0. Construct the phase trajectory using the method of isocline. (12 Marks)

- a. Define: i) Positive definiteness ii) Negative definiteness iii) Positive semidefiniteness iv) Negative semidefiniteness v) Indefiniteness. (05 Marks)
  - b. Explain Kravoski's theorem with example.

c. Examine the stability of a non-linear system governed by the equations,

$$\dot{x}_1 = -x_1 + 2x_1^2 x_2$$
;  $\dot{x}_2 = -x_2$ . Assume  $2x_1 x_2 < 1$ . (08 Marks)

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