

18MCA23

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Second Semester MCA Degree Examination, June/July 2019

Discrete Mathematical Structures and Statistics

Time: 3 hrs.

USN

Max. Marks: 100

Note: Answer FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Prove the following conditional is a tautology.

$$[(p \leftrightarrow q) \land (q \leftrightarrow r) \land (r \leftrightarrow p)] \leftrightarrow [(p \rightarrow q) \land (q \rightarrow r) \land (r \rightarrow p)]$$

(07 Marks)

b. Given the following proposition, write

i) Direct proof ii) Indirect-proof

"If n-is an odd integer, then (n+11) – is an even integer."

(07 Marks)

c. Using the laws of logic prove the following conditional expression: $[(p \lor q) \land (p \lor \sim q)] \lor q \Leftrightarrow p \lor q$ (06 Marks)

OR

2 a. Prove the following argument is valid:

$$p \rightarrow q$$

$$r \rightarrow s$$

$$p \lor s$$

$$\therefore q \lor s$$

(07 Marks)

b. Negate and simplify the following:

i)
$$\exists x, [p(x) \lor g(x)]$$
 ii) $[\exists x, [p(x) \lor q(x)]] \rightarrow r(x)$

(07 Marks)

c. Summarize the laws of logic.

(06 Marks)

Module-2

3 a. Determine sets A and B, given that:

$$A - B = \{1, 2, 4\}, B - A = \{7, 8\} \text{ and } A \cup B = \{1, 2, 4, 5, 7, 8, 9\}.$$

(07 Marks)

b. For any three sets A, B, C prove that

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii)
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(07 Marks)

c. State and prove the addition theory in probability.

(06 Marks)

OR

a. A problem is given to four students A, B, C, D whose chances of solving it are 1/2, 1/3, 1/4, 1/5 respectively. Find the probability that the problem is solved. (07 Marks)

b. The probabilities that three persons x, y, z hit a target in one attempt are 1/6, 1/4 and 1/3 respectively. If each of these shoots once at a target-find:

i) The probability that the target is hit

ii) The probability that the target is hit by exactly one person.

(07 Marks)

c. Prove the Demorgan laws, for any two sets:

i)
$$A \cup B = A \cap B$$
 iii

ii)
$$\overline{A \cap B} = \overline{A \cup B}$$

(06 Marks)

Module-3

- 5 Find the number of permutations of the letters of the word 'INSTITUTION'
 - i) How many of these begin with I?
 - ii) How many of these begin with I and end with N?
 - In how many the 3 T's are together?

(07 Marks)

Prove the following by using Mathematical induction for every positive integer n:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1) (2n+1)$$

(07 Marks)

Given the recurrence relation $a_n = a_{n-1} + 4$ with $a_1 = 2$ obtain an explicit formula for the given sequence. (06 Marks)

- Find the coefficient of x^4 in the expansion of $\left(2x^2 \frac{3}{x}\right)^6$ (07 Marks)
 - A man has 7-relatives, 4-of them are ladies and 3-gentlemen. His wife has also 7-relatives 3-of them are ladies and 4-gentlemen. In how many ways can they invite a dinner party of 3-ladies and 3-gentlemen so that there are 3 - of the man's relative and 3 - of the wife's relatives?
 - The Fibonacci numbers are defined by $F_0 = 1$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \ge 2$. Evaluate F₂ to F₁₀. (06 Marks)

Module-4

7 The probability distribution of a finite random variable -X – is given by

X:	-2	-1	0 @	1	2	3
P(X):	0.1	K	0.2	2K	0.3	K

Find: i) The value of K

ii) Mean

iii) Variance and standard deviation.

(07 Marks)

- In a certain town the duration of the shower is exponentially distributed with a mean 5-min. What is the probability that a shower will last for
- i) 10-min or more ii) less than 10-min iii) Between 10 and 12 min.
- (07 Marks)
- The weekly wages of workers in a company are normally distributed with mean of Rs.700 and standard deviation of Rs.50. Find the probability that the weekly wage of a randomly chosen worker is
 - Between Rs.650/- and Rs.750
 - More than Rs.750/-

(06 Marks)

a. Obtain the mean and standard deviation of the Poisson distribution. 8

(07 Marks)

The probability density function of a variate X – is given by the following table:

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X:	0	1	2	3	4	5	6
P(X):	K	3K	5K	7K	9K	11K	13K

Find: i) The value of K

ii) $P(X < 4), P(X \ge 5)$.

(07 Marks)

- The number of telephone lines busy at an instant of time is a binomial variate with P = 0.2. If at an instant 10 lines are choosen at random what is the probability that

 - i) 5-lines are busy ii) At most 2-lines are busy.

(06 Marks)

Module-5

9 a. By the method of least squares, find the straight line that fits the following data: (y = ax + b)

x:	1	2	3	4	5
y:	14	27	40	55	68

(07 Marks)

b. Find the correlation coefficient for the two groups,

x :	92	89	87	86	83	77	71	63	53	50
y:	86	83	91	77	68	85	52	82	37	57

(07 Marks)

c. Define the terms

- i) Coefficient of correlation
- ii) Regression
- iii) Principle of least squares.

(06 Marks)

OR

10 a. Find the correlation coefficient 'r' and the equations of the lines of regression for the following values of x and y

x :	1	2	3	4	5
y:	2	5	3	8	7

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(10 Marks)

b. Fit a curve of the form $y = ae^{bx}$, to the following data:

X:	5	15	20	30	35	40
y:	10	14	25	40	50	62

(10 Marks)