CBCS SCHEME

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16/17MCA15

First Semester MCA Degree Examination, Dec.2018/Jan.2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. How many rows appear in a truth table for the compound proposition $(p \rightarrow r) \vee (\sim s \rightarrow \sim t)(\sim u \rightarrow v)$. **(02 Marks)**

b. Define Tautology. Show that the compound proposition $[(p \vee q) \wedge (\sim p \vee r)] \rightarrow (q \vee r)$ is a Tautology. **(04 Marks)**

c. Let p(x) be the statement "x+1 > 2x". Determine the truth values of each of the following statements if the domain consists of all integers?
i) $\exists x\, p(x)$ ii) $\forall x\, p(x)$ iii) $\exists x \sim p(x)$ iv) $\forall \sim p(x)$ **(04 Marks)**

d. Use rules of inference to show that the hypothesis "Randy works hard", "If Randy work hard, then he is a dull boy", and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job". **(06 Marks)**

OR

2 a. Define the dual of a logical statement. Write the dual of the compound proposition $(p \wedge \sim) \vee (q \wedge F)$. **(03 Marks)**

b. Show that $\sim[p \vee (\sim p \wedge q)]$ and $\sim p \wedge \sim q$ are logically equivalent by using laws of logic. **(04 Marks)**

c. Write the negation of the following quantified statement:
i) "There exists a pig that can swim and catch fish"
ii) "No monkey can speak French". **(04 Marks)**

d. Give a proof by contradiction of the statement "If 3n + 2 is odd, then n is odd". **(05 Marks)**

Module-2

3 a. Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$. **(02 Marks)**

b. Show that the congruence modulo 'm' is an equivalence relation on the set of integers. **(04 Marks)**

c. Let $A = \{a, b, c\}$ and $B = P(A)$ is the power set of A and the relation R on B defined by xRy if $x \subseteq y$. Show that (B, R) is a POSET, and draw its Hasse diagram. Is it a Lattice? **(06 Marks)**

d. Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$. Find fog and gof. **(04 Marks)**

OR

4 a. Define power set of a set, write the power set of $A = \{0, \phi, \{\phi\}\}$. **(02 Marks)**

b. Let R be an equivalence relation on Set A. Then show that the following statements are equivalent $\forall$ a, b $\in$ A. **(05 Marks)**
i) aRb ii) $[a] = [b]$ iii) $[a] \cap [b] \neq \phi$.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

c. Draw the Hasse diagram for divisibility relation on the set A = {3, 5, 9, 15, 24, 45}. Find the (i) least and greatest elements, (ii) minimal and maximal elements, (iii) LUB of {3, 5} and (iv) GLB of {15, 45}. **(05 Marks)**

d. Verify the function $f : R \rightarrow R$ defined by $f(x) = 2x + 1$ is bijection or not. **(04 Marks)**

Module-3

5 a. How many license plates can be made using either two letters followed by four digits or two digits followed by four letters? **(04 Marks)**

b. State Pigeonhole principle. Show that, if there are 30 students in a class, then at least two have last names that begin with the same letter. **(04 Marks)**

c. Find the coefficient of $x^{12} y^{13}$ in the expansion of $(2x - 3y)^{25}$. **(04 Marks)**

d. A person deposits Rs. 10,000 in an account at a bank that yields 11% per year with interest compounded annually. How much will be in the account after 30 years? **(04 Marks)**

OR

6 a. If a department contains 10 men and 15 women then in how many ways are there to form a committee with six members if it must have same number of men and women? **(05 Marks)**

b. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? **(05 Marks)**

c. Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$. **(06 Marks)**

Module-4

7 a. Let S be the sample space for a random experiment E and let A and B be events from S such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$. Determine :

(i) $P(\overline{A \cup B})$ (ii) $P(A \cap \overline{B})$ (iii) $P(\overline{A} \cap \overline{B})$

(iv) $P(\overline{A} \cap B)$ (v) $P(\overline{A} \cup B)$ (vi) $P(A \cup \overline{B})$. **(06 Marks)**

b. Define independent events. If A and B are independent events then shown that (i) A and $\overline{B}$ are independent (ii) $\overline{A}$ and B are independent (iii) $\overline{A}$ and $\overline{B}$ are independent. **(05 Marks)**

c. One bag contains 15 identical (in shape) coins, in which nine are silver and six are Gold. A second bag contains 16 more of these coins in which six are silver and 10 are gold. If one coin from the first bag randomly selected and then places it in the second bag, and then a coin selected at random from the second bag. What is the probability that it is a gold coin? **(05 Marks)**

OR

8 a. The freshman class of a private Engineering college has 300 students. It is known that 180 can program in JAVA, 120 in VISUAL BASIC, 30 in C++, 12 in JAVA and C++, 18 in VISUAL BASIC and C++, 12 in JAVA and VISUAL BASIC and 6 in all 3 languages,

(i) A student is selected at a random, what is the probability that she can program in exactly two languages?

(ii) Two students are selected at random, what is the probability that they can both program in JAVA and both program only in JAVA? **(05 Marks)**

b. Let A, B and C be independent events taken from a sample space S. If $P(A) = \frac{1}{8}$, $P(B) = \frac{1}{4}$ and $P(A \cup B \cup C) = \frac{1}{2}$, then find $P(C) = ?$ **(05 Marks)**

Module-5

c. A company involved in the integration of personal computers gets its graphics cards from three sources. The first source provides 20% of the cards, the second source 35% and the third source 45%. Past experience has show that 5% of the cards from the first source are found to be defective, while those from the second and third sources are found to be defective 3% and 2%, respectively, of the time. (i) What percentage of the company's graphics cards are defective? (ii) If a graphics card is selected and found to be defective, what is the probability it was provided by the third source? **(06 Marks)**

9 a. Define the following with an example each,
(i) Regular graphs (ii) Complete graphs (iii) Bipartite graphs. **(06 Marks)**

b. Explain Konigsberg bridge problem. **(05 Marks)**

c. Use Dijkstra's algorithm to find the length of a shortest path between the vertices a and z in the following weighted graph [Refer Figure Q9(c)] **(05 Marks)**

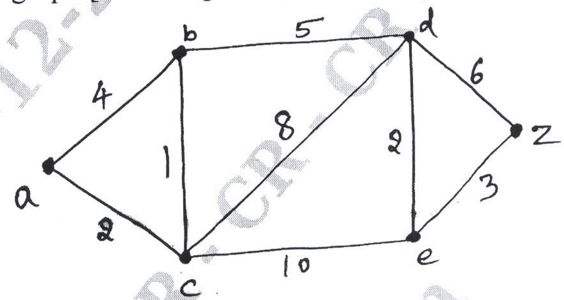

Fig Q9(c)

OR

10 a. How many edges are there in a graph with 10 vertices each of degree six? **(02 Marks)**

b. Define Isomorphism of Graphs, verify the following graphs are isomorphic or not?

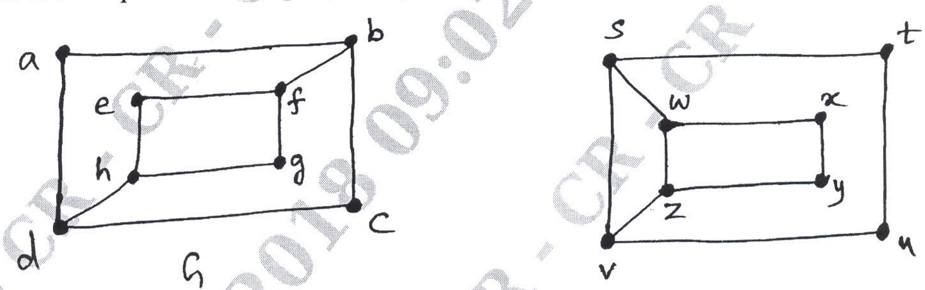

Fig Q10(b) **(05 Marks)**

c. Define planar graphs. Show that the complete graph K_5 is nonplanar. **(05 Marks)**

d. Define chromatic number of a graph. Find the chromatic number of the following graph :

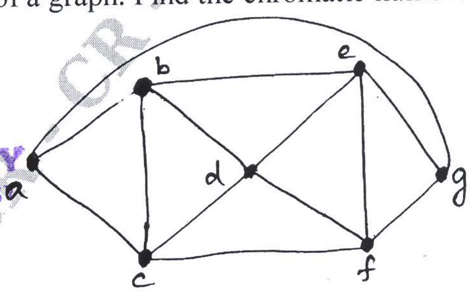

Fig Q10(d) **(04 Marks)**

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