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CMRIT
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Internal Assessment Test 1 – April. 2019

Sub:	Engineering Physics Theory					Sub Code:	18PHY22	Branch:	EC/MECH/ELE
Date:	15-04-2019	Duration:	90 min's	Max Marks:	50	Sem / Sec:	II / I,J,K,L,M,N,O		OBE

Answer any FIVE FULL Questions

Given: $c = 3 \times 10^8$ m/s; $h = 6.625 \times 10^{-34}$ Js; $k = 1.38 \times 10^{-23}$ J/K; $N_A = 6.02 \times 10^{26}$ /K mole;

$m_e = 9.1 \times 10^{-31}$ kg; $e = 1.6 \times 10^{-19}$ C

		MARKS	CO	RBT
1 (a)	Prove that electrons cannot exist in the nucleus using Heisenberg's uncertainty principle.	[07]	CO3	L3
(b)	Calculate the de-Broglie wavelength of the electron accelerated through a potential difference of 1000V.	[03]	CO3	L3
2 (a)	Find the Eigen function and energy Eigen values for a particle in a one dimensional potential well of infinite height.	[07]	CO3	L3
(b)	The ground state energy of an electron in an infinite potential well is 25 eV. Calculate the width of the potential well.	[03]	CO3	L3
3 (a)	Explain the construction and working of CO_2 laser, with the help of suitable diagrams.	[07]	CO4	L4
(b)	Discuss the application of lasers in storing data in CD.	[03]	CO4	L2
4 (a)	Obtain an expression for energy density of radiation under thermal equilibrium in terms of Einstein's coefficents.	[07]	CO4	L4
(b)	If the ratio of population of two energy states is 1.06 x 10^{-30}, find the wavelength of the light emitted by the spontaneous emissions at room temperature (300 K).	[03]	CO4	L3

PTO

5 (a)	What are free vibrations?. Discuss the differential equation for SHM and its solution.	[7]	CO1	L3
(b)	If the effective spring constant of a series combination of two identical springs is 50 N/m, what will be the elongation produced in each spring, when this series combination is loaded with 100 gm.	[3]	CO1	L3
6 (a)	Obtain the general solution for the displacement of a body undergoing damped oscillations and discuss the case of over damping.	[7]	CO1	L3
(b)	Calculate the natural frequency of the oscillator of mass 20 gm, if it is set for critical damping in the medium of damping coefficient r = 0.17 kg/s.	[3]	CO1	L3
7 (a)	Discuss the theory of forced vibrations and hence obtain the expression for amplitude and the phase.	[7]	CO1	L3
(b)	Mention three applications of shock waves.	[3]	CO1	L2
8 (a)	Explain the construction and working of a Reddy shock tube.	[7]	CO1	L3
(b)	Calculate the Mach-number of the shock front generated in a shock tube experiment, if the time delay recorded by the sensors separated by a distance of 10 cm is 0.15 ms. (Sound velocity = 340 m/s)	[3]	CO1	L3

IAT-1 EVEN SEM 2018-19

SCHEME

1.a   {7}

TO SHOW THAT ELECTRON DOES NOT EXIST INSIDE THE NUCLEUS:

We know that the diameter of the nucleus is of the order of 10^{-14}m. If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

$$\Delta x \leq 10^{-14} m$$

Now the uncertainty in momentum is

$$\Delta p \geq \frac{h}{4\pi.\Delta x}$$

$$\Delta p \geq \frac{6.62 \times 10^{-34}}{4\pi \times 10^{-14}}$$

$$\Delta p \geq 0.5 \times 10^{-20} Ns$$

Then the momentum of the electron can atleast be equal to the uncertainty in momentum.

$$p \geq 0.5 \times 10^{-20} Ns$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by (for small velocities -non-relativistic-case)

$$E = \frac{p^2}{2m} = \frac{(0.5 \times 10^{-19})^2}{2 \times 9.1 \times 10^{-31}} = 1.37 \times 10^{-11} J = 85 MeV$$

. The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of decay of nucleus ($n \rightarrow p + e + \overline{\nu}$ / $p \rightarrow n + e + \nu$).

1.b. {3}

$$\lambda_{Electron} = \frac{h}{\sqrt{2meV}} = 0.38 \times 10^{-10} \text{ m}$$

2.a {7}

Particle in an infinite potential well problem:

Consider a particle of mass m moving along X-axis in the region from X= 0 to X = a in a one dimensional potential well as shown in the diagram. The potential energy is assumed to be zero inside the region and infinite outside the region.
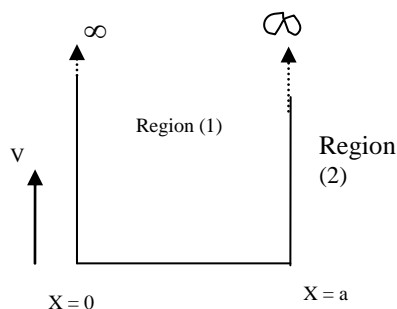
Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \qquad \because V = 0 \text{ for region (1)}$$

But $k^2 = \dfrac{8\Pi^2 mE}{h^2}$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

Auxiliary equation is $\left(D^2 + k^2\right)x = 0$

Roots are $D = +ik$ and $D = -ik$

The general solution is

$$x = Ae^{ikx} + Be^{-ikx}$$

$$= A(\cos kx + i\sin kx) + B(\cos kx - i\sin kx)$$

$$= (A+B)\cos kx + i(A-B)\sin kx$$

$$= C\cos kx + D\sin kx$$

The boundary conditions are

1. At x=0, $\Psi = 0 \qquad \therefore C = 0$

2. At x=a, $\Psi = 0$

D sin ka = 0 $\implies$ ka = n Π ....(2)

where n = 1, 2 3…

$$\therefore \ \Psi = D\sin\left(n\frac{\Pi}{a}\right)x$$

From (1) and (2) $E = \dfrac{n^2 h^2}{8ma^2}$

To evaluate the constant D:

Normalisation : For one dimension

$$\int_0^a \Psi^2 dx = 1$$

$$\int_0^a D^2 \sin^2\left(\frac{n\Pi}{a}\right)x\, dx = 1$$

But $\cos 2\theta = 1 - 2\sin^2\theta$

$$\int_0^a D^2 \frac{1}{2}\left(1 - \cos 2\left(\frac{n\Pi}{a}\right)x\right) dx = 1$$

$$\int_0^a \frac{D^2}{2} dx - \int_0^a \frac{1}{2}\cos 2\left(\frac{n\Pi}{a}\right)x\, dx = 1$$

$$\frac{D^2 a}{2} - \ \left[\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}\right]_0^a = 1$$

$$D^2 \frac{a}{2} - 0 = 1$$

$$D = \sqrt{\frac{2}{a}}$$

$$\therefore \Psi_n = \sqrt{\frac{2}{a}} \; \sin\left(n\frac{\Pi}{a}\right)x$$

2.b. **{3}**

$$E_1 = \frac{1^2 h^2}{8mL^2} = 25eV$$

$$L = 1.22x10^{-10} m$$

3.a. **{7}**

Carbon dioxide laser

Construction
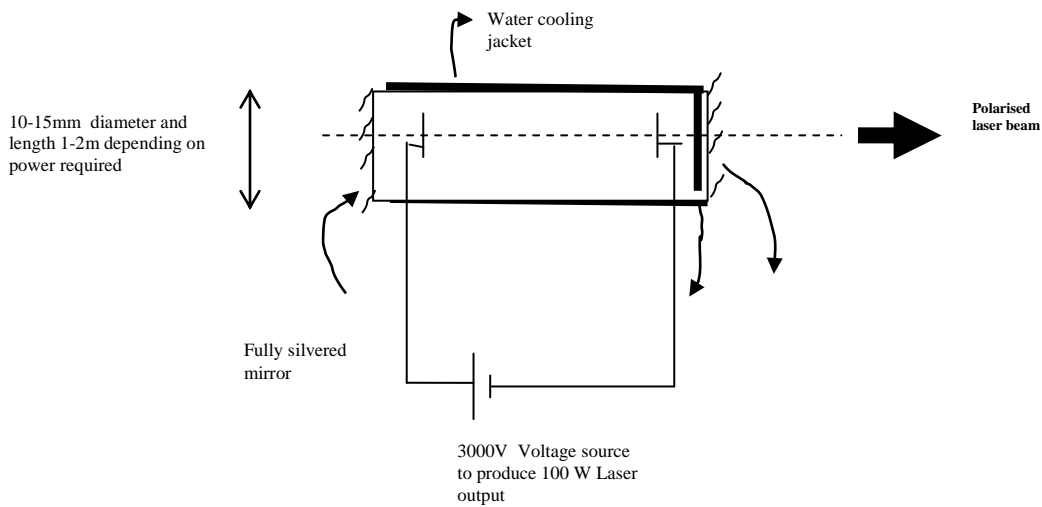
1. Active medium – Mixture of CO_2, N_2 and He in the ratio 1:2:8. Nitrogen absorbs energy from the pumping source efficiently. Helium gas conducts away the heat and also catalyses collisional deexcitation of CO_2 molecules.

2. The discharge tube consists of a glass tube of 10-15mm diameter with a coaxial water cooling jacket.

3. Partially reflecting and fully reflecting mirrors are mounted at the ends of the tube.

4. Optical pumping is achieved by electric discharge caused by applying potential difference of over 1000V.
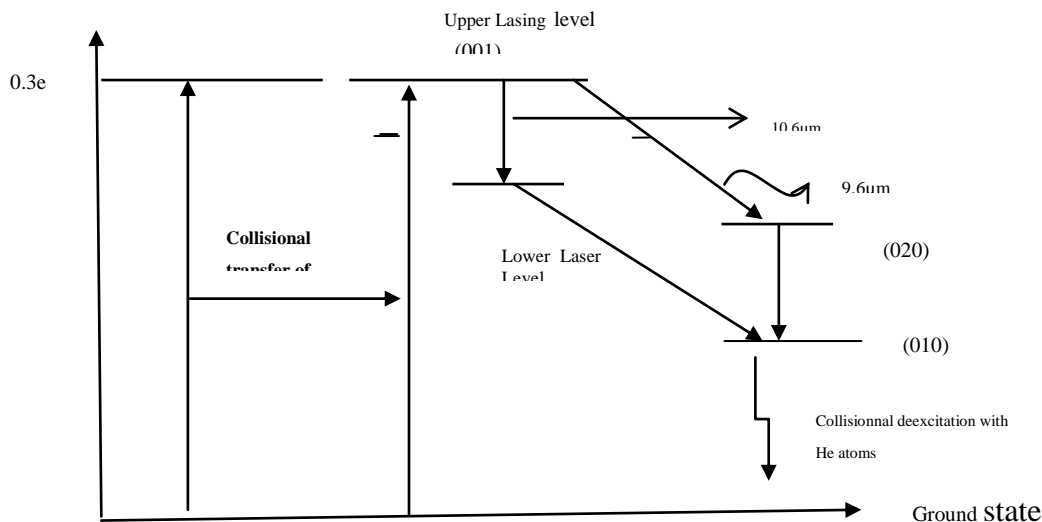
Working:

1.CO_2 is a linear molecule and has three modes of vibration –Symmetric stretching (100), Asymmetric stretching (001) and bending (010).

2. Asymmetric stretching (001) is the upper laser level which is a metastable state. (100) and (020) are the lower lasing states

3.During electric discharge, the electrons released due to ionisation excite N_2 molecules to its first vibrational level which is close to upper lasing level of CO_2.

4.N_2 molecules undergo collisions with CO_2 molecules and excite them to (001). This results in population inversion.

5.Lasing transition occurs between (001) and (100) emitting at 10.6μm and (001) to (020) emitting at 9.6μm

6. CO_2 molecules deexcite to ground state through collisions with Helium atom.

3 b. **{3}**

LASERS IN DATA STORAGE

In a compact disc, series of microscopic holes known as pits are formed by burning. Laser light is reflected from the disc surface and is detected by photodiodes. The amount of light received by the diodes varies according to the presence or absence of pits.
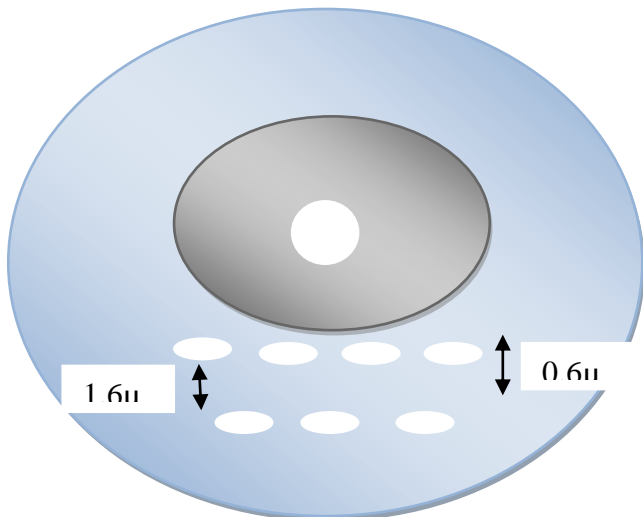
In a CD, 1s and 0s are recorded in the form of pits along a spiral track on a plastic material with a metal coating. The total length of the track would be around 6km.Any transition from pit to land or land to pit is read as 1 while the region completely in the land or pit is read as 0.Seperation between tracks is 1.6μm in CD and 1.1μm in a DVD (**DVD DIGITAL VIDEO DISK).**The laser beam is focused on the surface of CD. The reflected beam reaches photodetector and processed. The laser spot should have minimum size. Holographic storage uses entire volume of the recording medium rather than the surface and hence stores large data.
 Ex: AlGaInP (640nm) is used to read the data in DVD as it can be focused on a very small region.

4.a. {7}

Expression for energy density:

Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E_1 and E_2 be the energy levels in an atom and N1 and N_2 be the number density in these levels respectively. Let U_γ be the energy density of the radiation incident..

$$\gamma = \frac{E_2 - E_1}{h}$$

Rate of absorption is proportional to the number of atoms in lower state and also on the energy density U_γ.

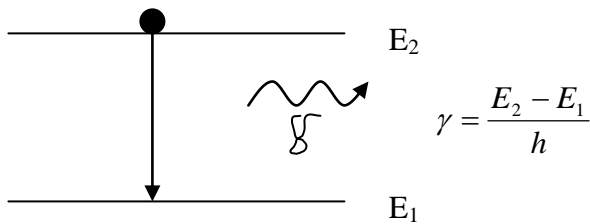$$\text{Rate of absorption} = B_{12} \, N_1 \, U_\gamma$$

Here B_{12} is a constant known as Einsteins coefficient of spontaneous absorption.

Spontaneous emission:

It is a process in which ,atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.
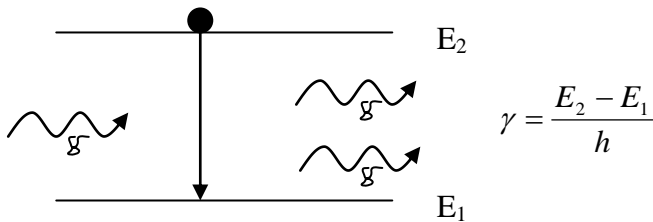
$$\text{Rate of spontaneous absorption} = A_{21} \, N_2$$

Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.

$$\gamma = \frac{E_2 - E_1}{h}$$

Stimulated emission:

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.

$$\gamma = \frac{E_2 - E_1}{h}$$

The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U_γ.

Rate of stimulated emission $= B_{21} \, N_2 \, U_\gamma$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12} \, N_1 \, U_\gamma \quad = \quad A_{21} \, N_2 \quad + \quad B_{21} \, N_2 \, U_\gamma$$

$$U_\gamma = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Rearranging this, we get

$$U_\gamma = \frac{A_{21}}{B_{21}} \left[\frac{1}{\dfrac{B_{12} N_1}{B_{21} N_2} - 1} \right]$$

From Boltzmans law , $\qquad \dfrac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$

Hence

$$U_\gamma = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\gamma}{kT}} - 1} \right]$$

From Planck's radiation law,

$$U_\gamma = \frac{8\pi h\gamma^3}{c^3} \left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]} - 1} \right]$$

Comparing these expressions, we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\gamma^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1$$

4.b. {3}

$$\frac{N_1}{N_2} = e^{\frac{hc}{\lambda kT}}$$

From Boltzmans law , $\quad \dfrac{N_2}{N_1} = 1.06x10^{-30}$

$$\lambda = 695nm = 695x10^{-9}m$$

5.a. {1+6}

Free Oscillations
The oscillations are said to be free oscillations when there are no external forces . The object oscillates with natural frequency.

$$\text{Restoring force } \alpha - \text{displacement}$$

$$F = -k\ x$$

Here k is the proportionality constant known as spring constant. It represents the amount of restoring force produced per unit elongation and is a relative measure of stiffness of the material.

$$F_{\text{Re}storing} = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$Let\ \omega_o^2 = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} + \omega_o^2 x = 0$$

Here ω_o is angular velocity = $2.\pi.f$

$\quad$ f is the natural frequency $\quad f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

The Solution is of the form $\quad$ x(t) = A cosω_ot + B sinω_ot.

$\quad$ This can also be expressed as x(t) = C cos(ω_ot-ɵ) where $C = \sqrt{A^2 + B^2}$ $\quad$ tanɵ = B/A

5.b. {3}

$$k_e = \frac{k.k}{k+k} = 50; k = 100N/m$$

$$\Delta x_1 = \frac{F}{k} = \frac{mg}{100} = 9.8x10^{-3}m$$

6.a. {7}

Damped Oscillations
Mechanical Case:

In a damped harmonic oscillator, the amplitude decreases gradually due to losses such as friction, impedance etc. The oscillations of a mass kept in water, charge oscillations in a LCR circuit are examples of damped oscillations. Let us assume that in addition to the elastic force F = -kx, there is a force that is opposed to the velocity, F = b v where b is a constant known as resistive coefficient and it depends on the medium, shape of the body.
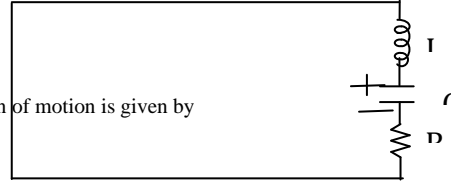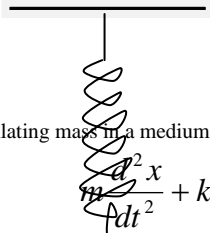
For the oscillating mass in a medium with resistive coefficient b, the equation of motion is given by

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

This is a homogeneous, linear differential e...   Oscillating mass in a liquid

The auxilia... $$...D + \frac{\ddot{}}{m} = 0$$

The roots a...$b^2 - 4mk$ and $D_2 = -\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}$

The solution can be derived as $x(t) = Ce^{-\left(\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2-4mk}\right)t} + De^{-\left(\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2-4mk}\right)t}$

Note: This can be expressed as x(t) = $Ae^{-\frac{b}{2m}t}\cos(\omega t - \phi)$ where $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$$A = \sqrt{C^2 + D^2} \qquad \phi = \tan^{-1}(D/C)$$

Here, the term $Ae^{-\frac{b}{2m}t}$ represents the decreasing amplitude and (ωt-φ) represents phase

Apply Boundary conditions: 1. At t = 0   x = x₀   2. At t = 0 $\frac{dx}{dt} = 0$

$$C = \frac{x_0}{2}\left(1 - \frac{b}{\sqrt{b^2 - 4mk}}\right)$$

$$D = \frac{x_0}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4mk}}\right)$$

Case 1: $b^2 > 4mk$ OVER DAMPING

$$x(t) = e^{-\left(\frac{b}{2m}\right)t}[Ce^{\left(\frac{1}{2m}\sqrt{b^2-4mk}\right)t} + De^{-\left(\frac{1}{2m}\sqrt{b^2-4mk}\right)t}]$$

When the retarding force is greater than k.A, $\frac{b}{2m} > \omega_o$

Over damping takes away the energy of the system and oscillations stop.

6.b. **{3}**

F For critical damping case (if b = r)

$$b^2 = 4mk = r2$$

$$0.17^2 = 4x0.020xk$$

$$k = 0.36$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 0.67 Hz$$

$$Natural frequency = 4.25 rad/s$$

OR If b = r/2m, for critical damping $b^2 = \omega^2$

Then ω = r/2m = 4.25 rad/s

7 .a {7}
Theory of Forced Oscillations

Forced oscillations are produced when an external oscillating force is applied to the particle subject to SHM.

Let F = F₀ Sinω₁t be the oscillating applied force

The equation of motion is given by

$$F = ma = -kx - bv + F_o \sin \omega_f t$$

$$m\frac{d^2 x}{dt^2} + b.\frac{dx}{dt} + kx = F_o \sin \omega_f t$$

$$\frac{d^2 x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_o}{m}\sin \omega_f t$$

$$Let \ \ \frac{b}{m} = 2k; \frac{k}{m} = \omega^2; \frac{F_o}{m} = F$$

$$\frac{d^2 x}{dt^2} + 2k\frac{dx}{dt} + \omega_o^2 x = F \sin \omega_f t \qquad \text{.....(1)}$$

Let one particular solution be $x = a.\sin(\omega_f t - \phi)$

$$\frac{dx}{dt} = \omega_f a.\cos(\omega_f t - \phi)$$

$$\frac{d^2 x}{dt^2} = -\omega_f^2 a.\sin(\omega_f t - \phi)$$

Also

$$F \sin \omega_f t = F.\sin(\omega_f t - \phi + \phi)$$

$$= F \sin(\omega_f t - \phi)\cos\phi + F \cos(\omega_f t - \phi)\sin\phi$$

Substituting in (1)

$$-\omega_f^2 a.\sin(\omega_f t - \phi) + 2ka\omega_f \cos(\omega_f t - \phi) + \omega_o^2 a \sin(\omega_f t - \phi) = F \sin(\omega_f t - \phi)\cos\phi + F \cos(\omega_f t - \phi)\sin\phi$$

Comparing coefficients of $\sin(\omega_f t - \phi) \ \ and \ \ \cos(\omega_f t - \phi)$ on both sides

$$a(.\omega_o^2 - \omega_f^2) = F \cos\phi$$

$$2ka\omega_f = F \sin\phi$$

$$\therefore F^2 = a^2(\omega_o^2 - \omega_f^2)^2 + 4k^2 a^2 \omega_f^2$$

$$a = \frac{F}{\sqrt{(\omega_o^2 - \omega_f^2)^2 + 4k^2\omega_f^2}} \qquad\qquad \tan\phi = \frac{2k\omega_f}{\omega_o^2 - \omega_f^2}$$

7 .b {3}

Uses:

- Aerodynamics – hypersonic shock tunnels, scramjet engines.
- High temperature chemical kinetics – ignition delay
- Rejuvenating depleted bore wells
- Material studies – effect of sudden impact pressure, blast protection materials
- Investigation of traumatic brain injuries
- Needle-less drug delivery
- Wood preservation

8. a {7}

Reddy shock tube:

A shock tube is a device used to study the changes in pressure & temperature which occur due to the propagation of a shock wave. A shock wave may be generated by an explosion caused by the buildup of high pressure which causes diaphragm to burst.

It is a hand driven open ended shock tube. It was conceived with a medical syringe. A plastic sheet placed between the plastic syringe part and the needle part constitutes the diaphragm.
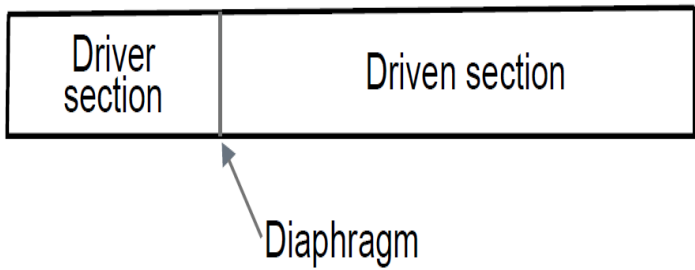
Diaphragm

- A high pressure (driver) and a low pressure (driven) side separated by a diaphragm.
- When diaphragm ruptures, a shock wave is formed that propagates along the driven section.
- Shock strength is decided by driver to driven pressure ratio, and type of gases used.
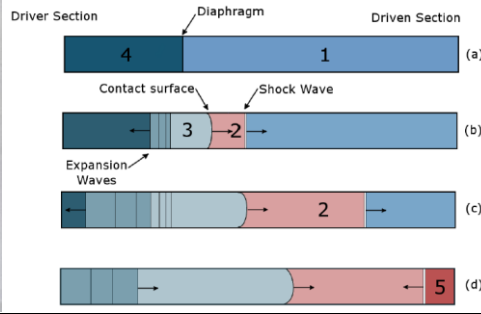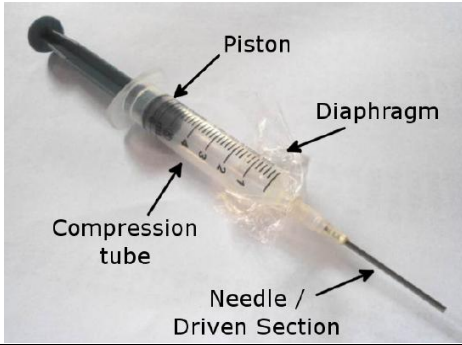
Working:

- The piston is initially at rest and accelerated to final velocity V in a short time t.
- The piston compresses the air in the compression tube. At high pressure, the diaphragm ruptures and the shock wave is set up. For a shock wave to form, $V_{piston} > V_{sound}$.

Formation of shock wave:

As the piston gains speed, compression waves are set up. Such compression waves increase in number. As the piston travels a distance, all the compression waves coalesce and a single shock wave is formed. This wave ruptures the diaphragm.
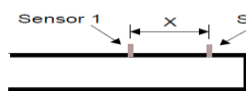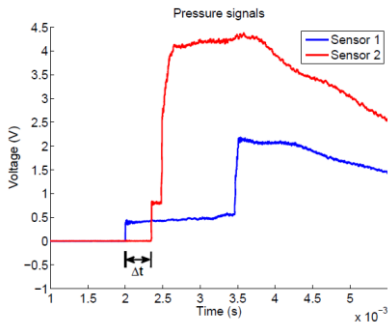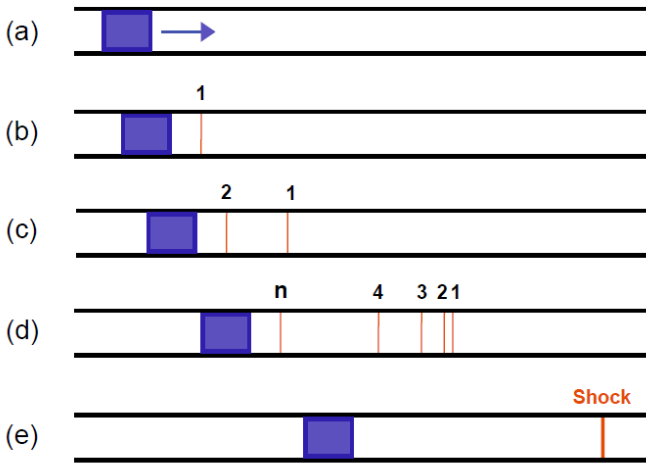
$$U_s = \frac{X}{\Delta t}$$

8.b. {3}

$$V_{shock} = \frac{0.10}{0.00015} = 666.66m/s$$

$$M = \frac{V_{shock}}{V_{Sound}} = 1.9$$