13MCA12

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First Semester MCA Degree Examination, June/July 2016
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. For any statements p, q prove the following:
i) $\sim(p \downarrow q) \Leftrightarrow (\sim p \uparrow \sim q)$ ii) $\sim(p \uparrow q) \Leftrightarrow (\sim p \downarrow \sim q)$ **(04 Marks)**

b. Define Tautology. Verify the compound propostition $\{(p \vee q) \wedge [(p \rightarrow r) \wedge (q \rightarrow r)]\} \rightarrow r$ is a tautology or not. **(05 Marks)**

c. Define dual of logical statement. Verify the principle of duality for the following logical equivalence, $[\sim(p \wedge q) \rightarrow \sim p \vee (\sim p \vee q)] \Leftrightarrow (\sim p \vee q)$. **(06 Marks)**

d. Simplify the following switching network without using truth tables: **(05 Marks)**

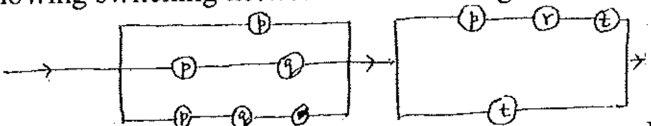

Fig. Q1 (d)

2 a. Verify the validity of the following argument : "If Rochelle gets the supervisor's position and works hard, then she will get a raise. If she gets a raise, then she will buy a car. She has not purchased a car. Therefore, either Rochelle did not get the supervisor's position or she did not work hard." **(06 Marks)**

b. Define open sentence. Write the negation of "All integers are rational numbers and some rational numbers are integers". **(02 Marks)**

c. Define converse, inverse and contrapositive of an implication. Hence find converse, inverse and contrapositive for "$\forall x, (x > 3) \rightarrow (x^2 > 9)$" where universal set is the set of real numbers R. **(06 Marks)**

d. Give : (i) a direct proof, (ii) an indirect proof and (iii) proof by contradiction, for the following statement: "If n is an odd integer then n + 9 is an even integer". **(06 Marks)**

3 a. Find the sets A and B if $A \cap B = \{2,4,7\}$, $A \cup B = \{1,2,3,4,5,7,8,9,10\}$ and $A - B = \{1,8\}$ **(02 Marks)**

b. Among the integers 1 to 200, find the number of integers that are, (i) divisible by 2 or 5 or 9, (ii) not divisible by 5, (iii) not divisible by 2 or 5 or 9, (iv) divisible by 5 and not by 2 and 9. **(08 Marks)**

c. Using laws of set theory simplify the following : (i) $A \cap (B - A)$ (ii) $\overline{\overline{(A \cup B \cap C)} \cup \overline{B}}$ **(05 Marks)**

d. A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations:
i) There is no restriction in the choice.
ii) Two particular persons will not attend separately.
iii) Two particular persons will not attend together. **(05 Marks)**

4 a. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers, prove that $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$. **(05 Marks)**

b. Give a recursive definition for each of the following integer sequence:
i) $a_n = 7n$ ii) $a_n = 3n + 7$ iii) $a_n = 2 - (-1)^n$ **(05 Marks)**

c. Define GCD of two positive integers. Find the GCD of 231 and 1820 and express it as a linear combination. **(05 Marks)**

d. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(05 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

5 a. Define Cartesian product of two sets. For any three non-empty sets A, B and C. Prove that $A \times (B - C) = (A \times B) - (A \times C)$ **(05 Marks)**

b. Define stirling's number of second kind. If $|A| = 7$, $|B| = 4$, find the number of onto functions from A to B. Hence find S(7, 4). **(05 Marks)**

c. Prove that a function $f : A \to B$ is invertible if and only if it is one-one and onto. **(05 Marks)**

d. State the pigeonhole principle. Let ABC be an equilateral triangle with AB = 1 cm. show that if we select five points in the interior of the triangle, there must be at least two points whose distance is less than $\frac{1}{2}$ cm. **(05 Marks)**

6 a. Let A = {1, 2, 3, 4} and R be a relation on A defined by xRy iff x divides y or y divides x. Write down R as a relation of set of ordered pairs, relation matrix M_R and digraph of R. Also find in-degree and out-degree of each vertex. **(05 Marks)**

b. Define partition of a set. If R is a relation defined on $A = \{1, 2, 3, 4\}$ by R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3), (3, 3), (4, 4)}, determine the partition induced by the equivalence relation. **(05 Marks)**

c. Let R be an equivalence relation on A and $a, b \in A$, then prove the following:
i) $a \in [a]$ ii) aRb iff $[a] = [b]$ and
iii) if $[a] \cap [b] \neq \phi$ then $[a] = [b]$. **(05 Marks)**

d. Let $A = \{a, b, c\}$, $B = P(A)$ where P(A) is the power set of A. Let R be a subset relation on A. Prove that (B, R) is a POSET and draw its Hasse diagram. Is it a lattice? **(05 Marks)**

7 a. Define the following with an example:
i) Regular graph ii) Complete graph iii) Bipartite graph. **(06 Marks)**

b. Define Isomorphism of two graphs. Verify the following graphs are isomorphic or not. **(05 Marks)**

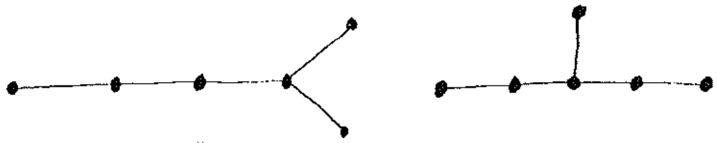

Fig. Q7 (b)

c. Explain Konigsberg bridge problem. **(05 Marks)**

d. Define Hamilton graph. Verify the following graph is Hamilton graph or not. **(04 Marks)**

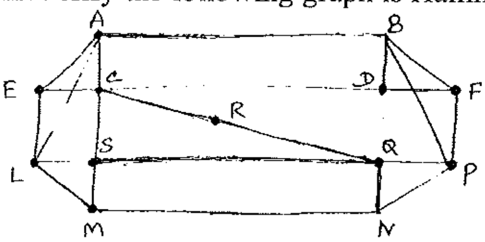

Fig. Q7 (d)

8 a. Define chromatic number. Find the chromatic number of the graph shown below: **(05 Marks)**

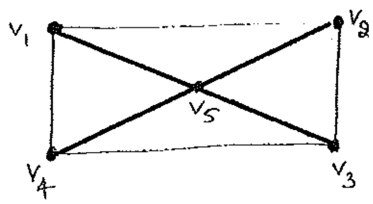

Fig. Q8 (a)

b. Define a Tree. Show that a tree with 'n' vertices has $n - 1$ edges. **(05 Marks)**

c. Define planar graph. Show that the bipartite graphs $K_{2,2}$ and $K_{2,3}$ are planar graphs. **(05 Marks)**

d. A class room contains 25 microcomputers that must be connected to a wall socket that has 4 outlets. Connections are made by using extension cords that have 4 outlets each. Find the least number of cords needed to get this computer set up for the class. **(05 Marks)**

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