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**First Semester MCA Degree Examination, Dec.2016/Jan.2017**  
**Discrete Mathematical Structures**

Time: 3 hrs.

Max. Marks:100

*Note: Answer any FIVE full questions.*

- 1 a. Indicate how many rows are needed in the truth table for the compound proposition  $(p \vee \neg q) \leftrightarrow \{(\neg r \wedge s) \rightarrow t\}$ . Find the truth table of this proposition if p and r are true and q, s, t are false. (05 Marks)
- b. Prove that, for any propositions p, q, r the compound proposition  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$  is tautology. (05 Marks)
- c. Write down the negation of the proposition : "If x is not a real number, then it is not a rational number and not an irrational number". (05 Marks)
- d. Prove the result using laws of logic,  $\neg[(p \vee q) \wedge r] \rightarrow \neg q \Leftrightarrow q \wedge r$ . (05 Marks)
- 2 a. Let  $p(x) : x^2 - 7x + 10 = 0$ ,  $q(x) : x^2 - 2x - 3 = 0$ ,  $r(x) : x < 0$ . Determine the truth or falsity of the following statements when the universe contains only the integers 2 and 5. If a statement is false, provide a counter example.
- i)  $\forall x, p(x) \rightarrow \neg r(x)$
- ii)  $\forall x, q(x) \rightarrow r(x)$
- iii)  $\exists x, p(x) \rightarrow r(x)$ . (06 Marks)
- b. Test the validity of the argument. "If I study, I will not fail in the examination. If I do not watch TV in the evenings, I will study. I failed in the examination. Therefore I must have watched TV in the evening. (07 Marks)
- c. Verify the argument is valid.
- $$\frac{\begin{array}{l} \forall x, [p(x) \vee q(x)] \\ \exists x, \neg p(x) \\ \forall x, [\neg q(x) \vee r(x)] \\ \forall x, [s(x) \rightarrow \neg r(x)] \end{array}}{\therefore \exists x, \neg s(x)}$$
- (07 Marks)
- 3 a. Define symmetric difference of two sets. Determine the sets A and B, given that  $A - B = \{1, 2, 4\}$ ,  $B - A = \{7, 8\}$  and  $A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$ . (06 Marks)
- b. In a set of 50 students, 15 study mathematics, 8 study Physics, 6 study Chemistry and 3 study all these three subjects. Prove that 27 or more students study none of these subjects. (07 Marks)
- c. A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in part C. If is required to answer seven questions selecting at least two questions from each part. In how many different ways can a student select his seven questions for answering? (07 Marks)
- 4 a. Prove that, for each  $n \in \mathbb{Z}^+$ ,  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1)$  using mathematical induction. (06 Marks)
- b. Give a direct proof of the statement "The square of an odd integer is an odd integer". (07 Marks)
- c. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30 ..... Hence find general term. (07 Marks)

- 5 a. Let  $A = \{1, 2, 3, 4\}$  and let  $R$  be the relation on  $A$  defined by  $xRy$  iff  $x > y$ .  
 i) Write down  $R$  as a set of ordered pairs  
 ii) Draw the digraph of  $R$ . (06 Marks)
- b. If  $A = \{1, 2, 3, 4\}$  and  $R, S$  are relations on  $A$  defined by  $R = \{(1, 2), (1, 3), (2, 4), (4, 4)\}$ ,  
 $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 3), (2, 4)\}$ . Find  $RoS, SoS$ . Draw its digraph and write  
 down their matrices. (07 Marks)
- c. Let  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ . On this set define the relation  $R$  by  $(x, y) \in R$  iff  
 $x - y$  is a multiple of 5. Verify that  $R$  is an equivalence relation. (07 Marks)
- 6 a. Define sterling number of second kind. Prove that if 30 dictionaries in a library contain a  
 total of 61,327 pages, then at least one of the dictionaries must have at least 2045 pages. (06 Marks)
- b. Define one-one and onto function. Consider the relations on the set  $A = \{1, 2, 3\}$   
 $f = \{(1, 3), (2, 3), (3, 1)\}$ ,  $g = \{(1, 3), (2, 2), (3, 1)\}$  Which of these are one-one and onto  
 functions? (07 Marks)
- c. Consider the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 2x + 5$ . Let a function  $g : \mathbb{R} \rightarrow \mathbb{R}$  be  
 defined by  $g(x) = \frac{1}{2}(x - 5)$ . Prove that  $g$  is an inverse of  $f$ . (07 Marks)
- 7 a. Define complete graph, complete bipartite graph and connected graph with one example. (06 Marks)
- b. Find the chromatic polynomial for the cycle  $C_4$ . (07 Marks)
- c. Find all paths of length 4 and all the cycles in the graph Fig. Q7(c). (07 Marks)

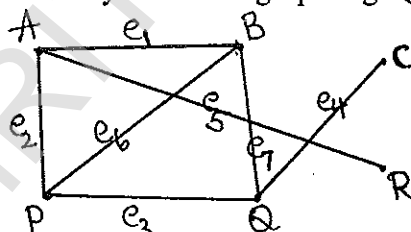


Fig. Q7(c)

- 8 a. Define a tree. Prove that a tree with 'n' vertices has  $n - 1$  edges. (06 Marks)
- b. Apply merge-sort to the list  $-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3$ . (07 Marks)
- c. Obtain an optimal prefix code for the message "MISSION SUCCESSFUL". Indicate the  
 code. (07 Marks)

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