USN

Internal Assessment Test I – September 2019

Sub:	Calculus and Linear Algebra					Sub Code:	18MAT11		
Date:	18/09/2019	Duration:	90 mins	Max Marks:	50	Sem / Sec:	I / I-O (CHEM CYCLE)		OBE

Question 1 is compulsory and answer any SIX questions from the rest.

		MARKS	CO	RBT
1 .	(a) Solve the following system of equations by using Guass Jorden method: $x+2y+z=3, \quad 2x+3y+3z=10, \quad 3x-y+2z=13.$	[04]	CO6	L3
	(b) Find the Radius of curvature of $x^3+y^3=3axy$ at the point $(\frac{3a}{2},\frac{3a}{2})$ on it.	[04]	CO1	L3
2 .	Show that the curves $r^n=a^n\cos n\theta$ and $r^n=b^n\sin n\theta$ intersect Orthogonally.	[07]	CO1	L3
3 .	Investigate the values of λ and μ such that the system of equations $x+y+z=6, \quad x+2y+3z=10, \quad x+2y+\lambda z=\mu$, may have (i) Unique Solution (ii) Infinite solution (iii) No solution.	[07]	CO6	L3
4.	Find the pedel equation of the curve $r=a(1+\cos\theta)$.	[07]	CO1	L3

5.	Find the largest eigen value and the corresponding eigen vector of the matrix A, by using the power method by taking initial vector as$[1 \quad 1 \quad 1]^T$, $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.	[07]	CO6	L3
6.	Solve the following system of equations by using Guass-Seidel method $20\,x + y - 2z = 17, \quad 3x + 20y - z = -18, \quad 2x - 3y + 20z = 25$	[07]	CO6	L3
7.	Reduce the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ to diagonal form.	[07]	CO6	L3
8.	With usual notation ,prove that $cot\varphi = \frac{1}{r}\frac{dr}{d\theta}$	[07]	CO1	L3

IAT-1 (September-19)
Chemistry Cycle
Calculus and Linear Algebra
Solutions and Scheme

Q1. (a) $C = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right]$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$C \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{array}\right]$

$R_1 \to R_1 - R_2$, $R_2 \to 7R_2 - R_3$

$C \sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & -1 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & 8 & 24 \end{array}\right]$

~~$R_2 \to R_2 + 8R$~~ ~~$R_2 \to 8R_2 - 8$~~

$R_2 \to 8R_2 - R_3$, $R_3 \to R_3/8$

$C \sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & -1 \\ 0 & -8 & 0 & 8 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$R_2 \to -R_2/8$, $\Rightarrow C \sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$R_1 \to R_1 - 3R_2$

$C \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$ $\therefore$ The solution is $(2, -1, 3)$

(b) $x^3+y^3=3axy,$

$\Rightarrow 3x^2+3y^2y_1=3a(xy_1+y) \Rightarrow (y^2-xa)y_1=ay-x^2$

$y_1=\frac{ay-x^2}{y^2-ax}, \quad (y_1)_{(\frac{3a}{2},\frac{3a}{2})}=-1$

$y_2=\frac{(y^2-ax)(ay_1-2x)-(ay-x^2)(2yy_1-a)}{(y^2-ax)^2}$

$(y_2)_{(\frac{3a}{2},\frac{3a}{2})}=\frac{\{\frac{9a^2}{4}-\frac{3a^2}{2}\}\{-a-3a\}-\{\frac{3a^2}{2}-\frac{9a^2}{4}\}(-3a-a)}{(\frac{9a^2}{4}-\frac{3a^2}{2})^2}$

$=\frac{(\frac{3a^2}{4})(-4a)-(-\frac{3a^2}{4})(-4a)}{(\frac{3a^2}{4})^2}=\frac{-3a^3-3a^3}{9a^4/16}=\frac{-6a^3}{9a^4}\times 16$

$=-\frac{32}{3a}$

$\rho=\frac{[1+y_1^2]^{3/2}}{y_2}=\frac{[1+1]^{3/2}}{(-32/3a)}=2\sqrt{2}\cdot\frac{3a}{32}=\left|\frac{3a}{8\sqrt{2}}\right|$

Q2. $r^n=a^n\cos n\theta \quad ; \quad r^n=b^n\sin n\theta$

$n\log r=n\log a+\log\cos n\theta \quad ; \quad n\log r=n\log b+\log\sin n\theta$

$\frac{n}{r}\frac{dr}{d\theta}=0+\frac{-n\sin n\theta}{\cos n\theta} \quad ; \quad \frac{n}{r}\frac{dr}{d\theta}=0+\frac{n\cos n\theta}{\sin n\theta}$

~~$\cot\phi_1=-n\cot$~~

$\cot\phi_1=-\tan n\theta \quad\quad \cot\phi_2=\cot n\theta \Rightarrow$

$=\cot(\pi/2+n\theta) \quad\quad \phi_2=n\theta$

$\phi_1=\pi/2+n\theta$

$\therefore$ Angle between two curves $=|\phi_1-\phi_2|=|\pi/2+n\theta-n\theta|=\pi/2$

$\therefore$ the given curves are orthogonal

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(2)

Q3. $C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_2$

$C \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$

i) for unique solution

$\rho(A) = \rho(C) =$ no. of variables (3)

for this $\lambda - 3 \neq 0, \cancel{4} \Rightarrow \lambda \neq 3,$

ii) for infinite solution.

$\rho(A) = \rho(C) \cancel{=} < 3$ (no. of variables)

$\lambda - 3 = 0, \quad \mu - 10 = 0 \Rightarrow \lambda = 3, \ \mu = 10$

iii) for no solution,

$\cancel{\lambda(A) \neq \lambda(C)}$ $\rho(A) \neq \rho(C)$

$\therefore \lambda - 3 = 0, \quad \mu - 10 \neq 0$

$\lambda = 3, \quad \mu \neq 10.$

Q4. $r = a(1+\cos\theta)$

$\log r = \log a + \log(1+\cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{-\sin\theta}{1+\cos\theta} = \frac{-2\sin\theta/2\,\cos\theta/2}{2\cos^2\theta/2}$

$\cot\phi_1 = -\tan\theta/2 = \cot(\pi/2 + \theta/2) \Rightarrow \phi_1 = \pi/2 + \theta/2$

$\therefore$ Pedal equation is given by

$p = r\sin\phi$

$$p = r\sin(\pi/2 + \theta/2) = r\cos\theta/2 \quad \text{—— (1)}$$

By given equation, $r = a(1+\cos\theta) \Rightarrow r = 2a\cos^2\theta/2$ —— (2)

from (1) $p^2 = r^2\cos^2\theta/2$

$$= r^2 \cdot \frac{r}{2a} = \frac{r^3}{2a}$$

$\therefore$ Pedal equation is $2ap^2 = r^3$

Answer.

Q5. $x^{(0)} = \begin{bmatrix}1\\1\\1\end{bmatrix}$,

$$AX^{(0)} = \begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 1\begin{bmatrix}1\\0\\1\end{bmatrix} = \lambda^{(1)}X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}2\\-2\\2\end{bmatrix} = 2\begin{bmatrix}1\\-1\\1\end{bmatrix} = \lambda^{(2)}X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}\begin{bmatrix}1\\-1\\1\end{bmatrix} = \begin{bmatrix}3\\-4\\3\end{bmatrix} = 4\begin{bmatrix}0.75\\-1\\0.75\end{bmatrix} = \lambda^{(3)}X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}\begin{bmatrix}0.75\\-1\\0.75\end{bmatrix} = \begin{bmatrix}2.5\\-3.5\\2.5\end{bmatrix} = 3.5\begin{bmatrix}0.71\\-1\\0.71\end{bmatrix} = \lambda^{(4)}X^{(4)}$$

$$AX^{(4)} = \begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}\begin{bmatrix}0.71\\-1\\0.71\end{bmatrix} = \begin{bmatrix}2.42\\-3.42\\2.42\end{bmatrix} = 3.42\begin{bmatrix}0.708\\-1\\0.708\end{bmatrix} = \lambda^{(5)}X^{(5)}$$

$$AX^{(5)} = \begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 2\end{bmatrix}\begin{bmatrix}0.708\\-1\\0.708\end{bmatrix} = \begin{bmatrix}2.416\\-3.416\\2.416\end{bmatrix} = 3.416\begin{bmatrix}0.7073\\-1\\0.7073\end{bmatrix}$$

Thus after five iterations the largest eigen value is and corresponding eigen vector is $\lambda = 3.416$, (3)

$[0.7073 \quad -1 \quad 0.7073]^T$

Q6 The given equations are diagonally dominant and hence we first write them in the following form

$$x = \frac{1}{20}[17 - y + 2z]$$

$$y = \frac{1}{20}[-18 - 3x + z]$$

$$z = \frac{1}{20}[25 - 2x + 3y],$$ we start with the trial

Solution $x=0,\ y=0,\ z=0$

First Iteration:

$$x^{(1)} = \frac{17}{20} = 0.85, \quad y^{(1)} = \frac{1}{20}[-18 - 3(0.85)] = -1.0275$$

$$z^{(1)} = \frac{1}{20}[25 - 2(0.85) + 3(-1.0275)] = 1.0109$$

Second Iteration:-

$$x^{(2)} = \frac{1}{20}[17 - (-1.0275) + 2(1.0109)] = 1.0025$$

$$y^{(2)} = \frac{1}{20}[-18 - 3(1.0025) + 1.0109)] = -0.9998$$

$$z^{(2)} = \frac{1}{20}[25 - 2(1.0025) + 3(-0.9998)] = 0.9998$$

Third Iteration :

$$x^{(3)} = \frac{1}{20}[17 - (0.9998) + 2(0.9998)] = 0.9999$$

$$y^{(3)} = \frac{1}{20}[-18 - 3(0.9999) + 0.9998] = -1.0000$$

$$z^{(3)} = \frac{1}{20}[25 - 2(0.9999) + 3(-1.0000)] = 1.0000$$

Thus, $\boxed{x=1, y=-1, z=1}$ is the required solution.

Q7. $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$.

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix} = 0 \Rightarrow (4-\lambda)(3-\lambda) - 2 = 0$$

$$\Rightarrow 12 - 7\lambda + \lambda^2 - 2 = 0 \Rightarrow \lambda^2 - 7\lambda + 10 = 0$$

$$(\lambda - 2)(\lambda - 5) = 0 \Rightarrow \lambda = 2, 5$$

Thus the eigen values are 2 and 5.

Case-(i) $\lambda = 2$, from $\begin{bmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow 2x + y = 0 \quad 2x = -y \Rightarrow \frac{x}{-1} = \frac{y}{2}$$

$$X_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Case-(ii) $\lambda = 5$, $\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$-x + y = 0 \Rightarrow x = 1, \quad X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$P = [X_1\ X_2] = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix},$$

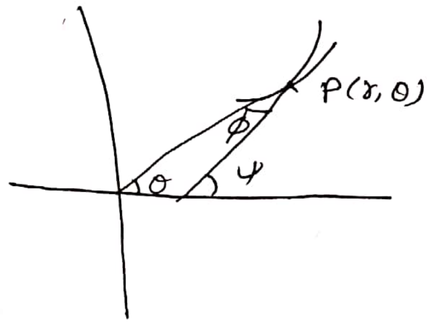
$$P^{-1} = \frac{\operatorname{adj}(P)}{|P|}, \quad |P| = -1-2 = -3$$

$$P^{-1} = -\frac{1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

$$D = P^{-1}AP = -\frac{1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} -2 & 5 \\ 4 & 5 \end{bmatrix}$$

$$= -\frac{1}{3}\begin{bmatrix} -6 & 0 \\ 0 & -15 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Q8. Let $P(r,\theta)$ be the point on curve $r = f(\theta)$.



from figure. $\psi = \theta + \phi$

$$\Rightarrow \tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} \quad \text{—— (1)}$$

Let (x,y) be cartesian co-ordinate of P then $x = r\cos\theta$.
$y = r\sin\theta$

$$\tan\psi = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r\cos\theta + r_1\sin\theta}{r_1\cos\theta - r\sin\theta}, \quad r_1 = \frac{dr}{d\theta}$$

$$= \frac{r\cos\theta\left(1 + \frac{r_1}{r}\tan\theta\right)}{r\cos\theta\left(\frac{r_1}{r} - \tan\theta\right)} = \frac{\left(\frac{r}{r_1} + \tan\theta\right)}{1 - \frac{r}{r_1}\tan\theta} \quad \text{—— (2)}$$

from (1) & (2), $\tan\phi = \frac{r}{r_1} \Rightarrow \cot\phi = \frac{r_1}{r} =$

$$\Rightarrow \boxed{\cot\phi = \frac{1}{r}\frac{dr}{d\theta}}$$