IAT-1 Physics Odd SEM 2019-20

SCHEME

1.a **{7}**

TO SHOW THAT ELECTRON DOES NOT EXIST INSIDE THE NUCLEUS:

We know that the diameter of the nucleus is of the order of 10^{-14}m.If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

$$\Delta x \le 10^{-14}\, m$$

Now the uncertainty in momentum is

$$\Delta p \ge \frac{h}{4\pi.\Delta x}$$

$$\Delta p \ge \frac{6.62\times10^{-34}}{4\pi\times10^{-14}}$$

$$\Delta p \ge 0.5\times10^{-20}\, Ns$$

Then the momentum of the electron can atleast be equal to the uncertainty in momentum.

$$p \ge 0.5\times10^{-20}\, Ns$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by (for small velocities -non-relativistic-case)

$$E = \frac{p^2}{2m} = \frac{(0.5\times10^{-19})^2}{2\times9.1\times10^{-31}} = 1.37\times10^{-11}\, J = 85 MeV$$

. The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of decay of nucleus ($\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e} + \bar{\nu}$ / $\mathrm{p} \rightarrow \mathrm{n} + \overset{+}{\mathrm{e}} + \nu$).

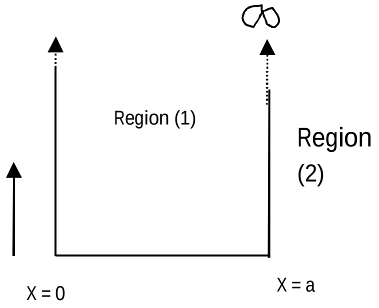
1.b. **{3}**

N = P/ Eg = 2.22×10^{16}

2.a **{7}**

Particle in an infinite potential well problem:

Consider a particle of mass m moving along X-axis in the region from X= 0 to X = a in a one dimensional potential well as shown in the diagram. The potential energy is assumed to be zero inside the region and infinite outside the region.



Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \qquad \because V = 0 \text{ for region (1)}$$

But $k^2 = \dfrac{8\Pi^2 mE}{h^2}$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

Auxiliary equation is $\left(D^2 + k^2\right)x = 0$

Roots are D = +ik and D = -ik

The general solution is

$$\begin{aligned} x &= Ae^{ikx} + Be^{-ikx} \\ &= A(\cos kx + i\sin kx) + B(\cos kx - i\sin kx) \\ &= (A+B)\cos kx + i(A-B)\sin kx \\ &= C\cos kx + D\sin kx \end{aligned}$$

The boundary conditions are

1. At x=0, $\Psi = 0 \qquad \therefore C = 0$

2. At x=a, $\Psi = 0$

D sin ka = 0 $\Rightarrow$ ka = n Π ………(2)

where n = 1, 2 3…

$$\therefore\ \Psi = D\sin\left(n\frac{\Pi}{a}\right)x$$

From (1) and (2) E = $\dfrac{n^2h^2}{8ma^2}$

To evaluate the constant D:

Normalisation : For one dimension

$$\int_0^a \Psi^2 dx = 1$$

$$\int_0^a D^2 \sin^2(\frac{n\Pi}{a})x dx = 1$$

But $\cos 2\theta = 1 - 2\sin^2\theta$

$$\int_0^a D^2 \frac{1}{2}(1 - \cos 2(\frac{n\Pi}{a})x) dx = 1$$

$$\int_0^a \frac{D^2}{2} dx - \int_0^a \frac{1}{2}\cos 2(\frac{n\Pi}{a})x) dx = 1$$

$$\frac{D^2 a}{2} - \quad [\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}]_0^a = 1$$

$$D^2 \frac{a}{2} - 0 = 1$$

$$D = \sqrt{\frac{2}{a}}$$

$$\therefore \Psi_n = \sqrt{\frac{2}{a}} \sin\left(n\frac{\Pi}{a}\right)x$$

2.b. {3}

From Boltzmans law ,

$$\frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}} = e^{\frac{1.54 X 10^{-19}}{1.38 X 10^{-23} X 300}}$$

$$\frac{N_1}{N_2} = 9.36 X 10^{32}$$

3a. {7}

Time independent Schrödinger equation

A matter wave can be represented in complex form as

$$\Psi = A\sin kx(\cos wt + i\sin wt)$$

$$\Psi = A\sin kx e^{iwt}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = kA\cos kx e^{iwt}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A\sin kx e^{iwt} = -k^2\Psi \text{ .... (1)} \quad \textbf{[2]}$$

From debroglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\Pi p}{h}$$

$$k^2 = 4\Pi^2 \frac{p^2}{h^2} \text{ .... (2)} \quad \textbf{[2]}$$

Total energy of a particle

E = Kinetic energy + Potential Energy

$$E = \frac{1}{2} m v^2 + V$$

$$E = \frac{p^2}{2m} + V$$

$$p^2 = (E - V)2m$$

Substituting in (2)

$$k^2 = \frac{4\Pi^2(E - V)2m}{h^2}$$

$\therefore$ From (1) **[2]**

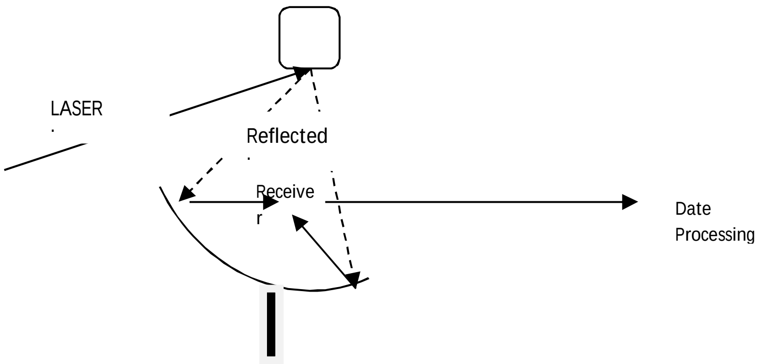
$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m(E - V)\Psi}{h^2}}$$

3b. {3}

LASER RANGE FINDER

Laser rangefinders have numerous applications such as measuring of rooms and buildings in the construction sector, to determine the depth of snow in inaccessible areas, Cloud base height for atmospheric study, air pollutant distribution, attitude characterization of space debris, trajectory of aircraft, satellites. Laser technology is more cost effective.

The laser rangefinder uses a laser signal is transmitted and returned from a target. The time delay between transmission and receipt of the signal is used to determine the distance to the target based on the speed of light. The receiver consists of reflector, photodetector and amplifier.



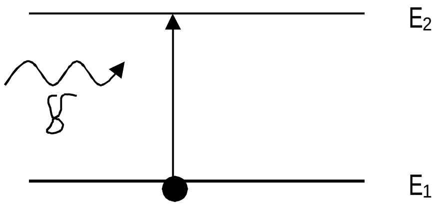
4.a. {7}

Expression for energy density:

Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E_1 and E_2 be the energy levels in an atom and N1 and N_2 be the number density in these levels respectively. Let U_γ be the energy density of the radiation incident..



$$\gamma = \frac{E_2 - E_1}{h}$$

Rate of absorption is proportional to the number of atoms in lower state and also on the energy density U_γ.

Rate of absorption = $B_{12}\ N_1\ U_\gamma$

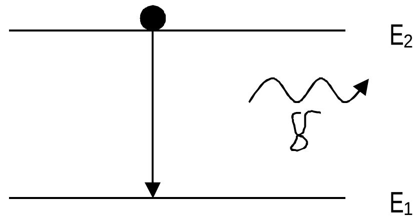
Here B_{12} is a constant known as Einsteins coefficient of spontaneous absorption.

Spontaneous emission:

It is a process in which ,atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

Rate of spontaneous absorption = $A_{21}\ N_2$

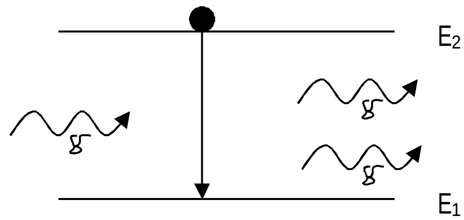
Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.



$$\gamma = \frac{E_2 - E_1}{h}$$

Stimulated emission:

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



$$\gamma = \frac{E_2 - E_1}{h}$$

The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U_γ.

Rate of stimulated emission = $B_{21}\ N_2\ U_\gamma$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12}\ N_1\ U_\gamma \;=\; A_{21}\ N_2 \;+\; B_{21}\ N_2\ U_\gamma$$

$$U_\gamma = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Rearranging this, we get

$$U_\gamma = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right]$$

From Boltzmans law , $\frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$

Hence

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\gamma}{kT}} - 1}\right]$$

From Planck's radiation law,

$$U_\gamma = \frac{8\pi h\gamma^3}{c^3}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]} - 1}\right]$$

Comparing these expressions, we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\gamma^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1$$

4b. **{3}**

Eigen Value $E = \frac{n^2h^2}{8mL^2}$

Eigen Function $\therefore \Psi_n = \sqrt{\frac{2}{a}}\,\sin\left(n\frac{\Pi}{a}\right)x$

$$E = \frac{n^2h^2}{8mL^2}$$

For L= 1A^0 E = 6.03X10^{-18}J= 37 eV

5.a. **{7}**

Carbon dioxide laser

Construction

1.Active medium – Mixture of CO_2, N_2 and He in the ratio 1:2:8. Nitrogen absorbs energy from the pumping source efficiently.Helium gas conducts away the heat and also catalyses collisional deexcitation of CO_2 molecules.

2.The discharge tube consists of a glass tube of 10-15mm diameter with a coaxial water cooling jacket.

3.Partially reflecting and fully reflecting mirrors are mounted at the ends of the tube.

4.Optical pumping is achieved by electric discharge caused by applying potential difference of over 1000V.

Working:

1.CO_2 is a linear molecule and has three modes of vibration –Symmetric stretching (100), Asymmetric stretching (001) and bending (010).

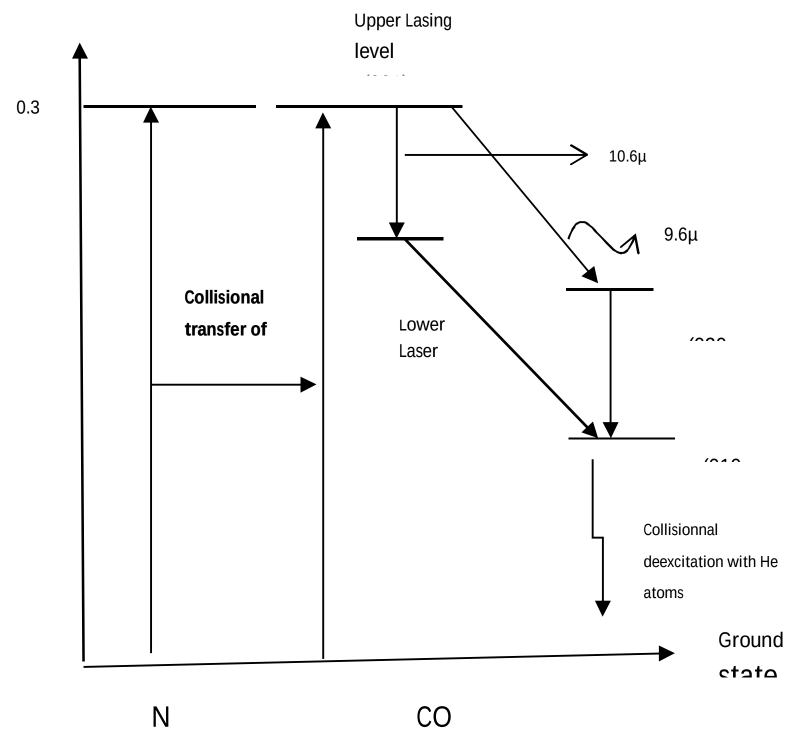
2. Asymmetric stretching (001) is the upper laser level which is a metastable state. (100) and (020) are the lower lasing states

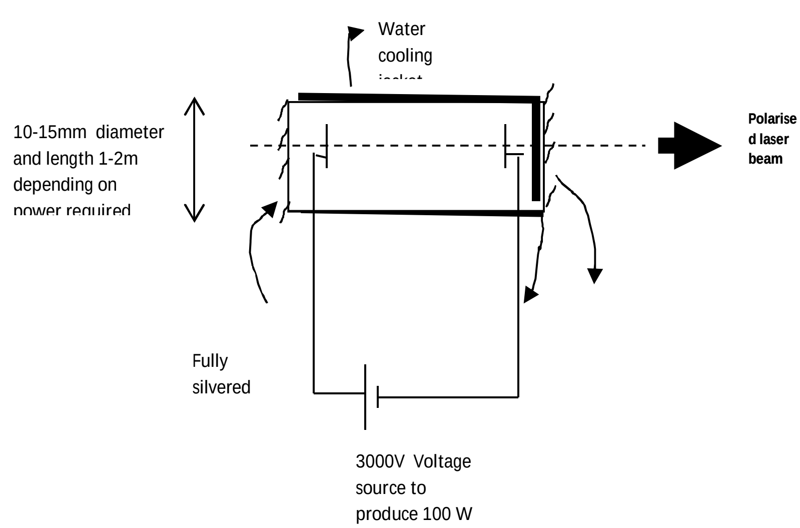
3.During electric discharge, the electrons released due to ionisation excite N_2 molecules to its first vibrational level which is close to upper lasing level of CO_2.

4.N_2 molecules undergo collisions with CO_2 molecules and excite them to (001). This results in population inversion.

5.Lasing transition occurs between (001) and (100) emitting at 10.6µm and (001) to (020) emitting at 9.6µm

6. CO_2 molecules deexcite to ground state through collisions with Helium atom.





5.b. **{3}**

$$\lambda = \frac{h}{mv}$$

$$v = 1.45x10^{-10}m$$

6a **{7}**

SIMPLE HARMONIC MOTION

It is the periodic oscillations of an object caused when the restoring force on the object is proportional to the displacement. The restoring force is directed opposite to displacement.

Ex: 1. Oscillation of mass connected to spring

2. Oscilations of prongs of Tuning fork

3. Simple pendulum (described in APPENDIX)

Restoring force α – displacement

F = -k x

Here k is the proportionality constant known as spring constant. It represents the amount of restoring force produced per unit elongation and is a relative measure of stiffness of the material.

$$F_{\mathrm{Re}\,storing} = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$Let\ \omega_o{}^2 = \frac{k}{m}$$

$$\frac{d^2x}{dt^2} + \omega_o{}^2 x = 0$$

Here ω_o is angular velocity = 2.π.f

f is the natural frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

The Solution is of the form $x(t) = A\cos\omega_o t + B\sin\omega_o t$.

This can also be expressed as $x(t) = C\cos(\omega_o t - \theta)$ where

$C = \sqrt{A^2 + B^2}$ tanθ = B/A

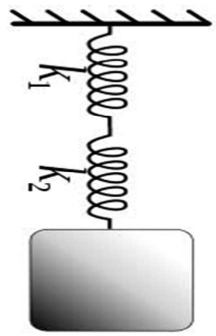
6b. **{3}**

ω = 2pi/T=62.8rad/s

$k = \omega^2 m$ = 788.7 N/m

7a **{6}**

Expression for Spring Constant for Series Combination



Consider a load suspended through two springs with spring constants k_1 and k_2 in series combination. Both the springs experience same stretching force. Let Δx_1 and Δx_2 be their elongation.

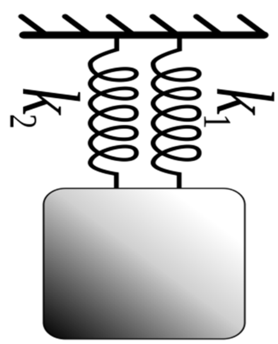
Total elongation is given by

$$\Delta X = \Delta X_1 + \Delta X_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{F}{k_{eqv}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k_{eqv}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Expression for Spring Constant for Parallel Combination



Consider a load suspended through two springs with spring constants k_1 and k_2 in parallel combination. The two individual springs both elongate by x but experience the load nonuniformly.

Total load across the two springs is given by

$$F = F_1 + F_2$$

$$k_{eqv}.\Delta X = k_1.\Delta X + k_2.\Delta X$$

$$k_{eqv} = k_1 + k_2$$

7b. **{4}**

$$A(t) = A_{\max} e^{-\frac{b}{2m}t}$$

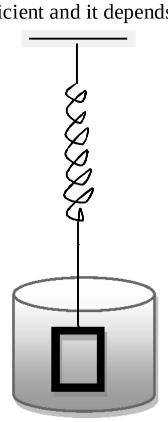
For A(t)=A_{max}/2 and taking natural log

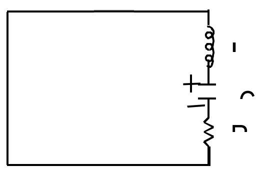
$$t = \frac{\ln 0.5 X\, 2m}{b} = 99s$$

8a **{7}**

Damped Oscillations

Mechanical Case:
In a damped harmonic oscillator, the amplitude decreases gradually due to losses such as friction, impedance etc. The oscillations of a mass kept in water, charge oscillations in a LCR circuit are examples of damped oscillations. Let us assume that in addition to the elastic force F = -kx, there is a force that is opposed to the velocity, F = b v where b is a constant known as resistive coefficient and it depends on the medium, shape of the body.





Oscillating mass in a liquid

For the oscillating mass in a medium with resistive coefficient b, the equation of motion is given by

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

This is a homogeneous, linear differential equation of second order.

The auxiliary equation is $D^2 + \frac{b}{m}D + \frac{k}{m} = 0$

The roots are $D_1 = -\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2 - 4mk}$ and $D_2 = -\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}$

The solution can be derived as

$$x(t) = Ce^{-\left(\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}\right)t} + De^{-\left(\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2 - 4mk}\right)t}$$

Note: This can be expressed as x(t) = $Ae^{-\frac{b}{2m}t}\cos(\omega t - \phi)$ where

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$A = \sqrt{C^2 + D^2} \qquad \phi = \tan^{-1}(D/C)$$

Here, the term $Ae^{-\frac{b}{2m}t}$ represents the decreasing amplitude and (ωt-ϕ) represents phase

Apply Boundary conditions: 1. At t = 0 x = x_o 2. At t = 0 $\frac{dx}{dt} = 0$

$$C = \frac{x_0}{2}\left(1 - \frac{b}{\sqrt{b^2 - 4mk}}\right)$$

$$D = \frac{x_0}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4mk}}\right)$$

8b **{3}**

V_{max} = ω A = (2π/ T) A

A = V_{max} T/2π = 0.31 m