ANALYSIS OF INDETERMINATE STRUCTURES.

SOLUTION OF IAT - I

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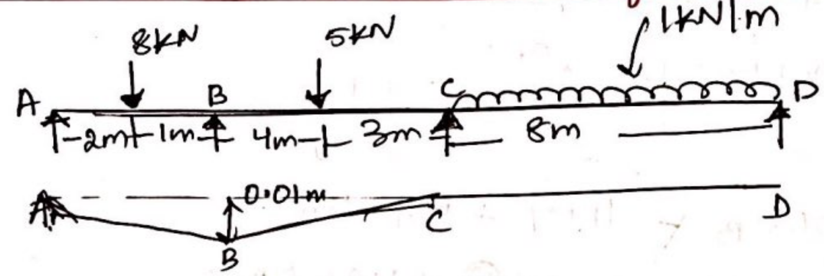
$EI = 2.1\times10^5\times10^3 \times 85\times10^5\times10^{\ldots}$

$= 1785\ kNm^2$



step 1: Fixed End Moments.

$$M_{FAB} = -\frac{Wab^2}{l^2} = \frac{-8\times2\times1^2}{3^2} = \underline{\underline{-1.78\ kNm}}$$

$$M_{FBA} = \frac{Wa^2b}{l^2} = \frac{8\times2^2\times1}{3^2} = \underline{\underline{+3.56\ kNm}}$$

$$M_{FBC} = -\frac{Wab^2}{l^2} = \frac{-5\times4\times3^2}{7^2} = \underline{\underline{-3.67\ kNm}}$$

$$M_{FCB} = +\frac{Wa^2b}{l^2} = \frac{+5\times4^2\times3}{7^2} = \underline{\underline{+4.90\ kNm}}$$

$$M_{FCD} = -\frac{Wl^2}{12} = \frac{-1\times8^2}{12} = \underline{\underline{-5.33\ kNm}}$$

$$M_{FDC} = \frac{Wl^2}{12} = \frac{1\times8^2}{12} = \underline{\underline{5.33\ kNm}}$$

step 2: Slope deflection method:

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= -1.78 + \frac{2\times1785}{3}\left[\theta_B - \frac{3\times0.01}{3}\right]$$

$$= -1.78 + 1190\theta_B - 11.9$$

$$= -13.68 + 1190\theta_B + 2380\theta_A \rightarrow (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$= +3.56 + \frac{2\times1785}{3}\left[2\theta_B + \theta_A - \frac{3\times0.01}{3}\right]$$

$$= 3.56 + 2380\theta_B - 11.9 + 1190\theta_A.$$

$$= 2380\theta_B - 8.34 \rightarrow (2) + 1190\theta_A.$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

$$= 3.67 + \frac{2\times1785}{7}\left[2\theta_B + \theta_C - \frac{3\times0.01}{7}\right]$$

$$= 3.67 + 1020\theta_B + 510\theta_C - 2.18$$

$$= 1.49 + 1020\theta_B + 510\theta_C \longrightarrow (3).$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$$

$$= 4.90 + \frac{2\times1785}{7}\left[2\theta_C + \theta_B - \frac{3\times0.01}{7}\right]$$

$$= 4.90 + 510\theta_B + 1020\theta_C - 2.18$$

$$= +2.71 + 510\theta_B + 1020\theta_C \longrightarrow (4)$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D - \frac{3\delta}{l}\right]$$

$$= -5.33 + \frac{2\times1785}{8}\left[2\theta_C + \theta_D - \frac{3\times0.01}{8}\right]$$

$$= -5.33 + 892.5\theta_C - 1.67 + 446.25\theta_D.$$

$$= -7 + 892.5\theta_C + 446.25\theta_D \rightarrow (5).$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[2\theta_D + \theta_C - \frac{3\delta}{l}\right]$$

$$= 5.33 + \frac{2\times1785}{8}\left[\theta_C - \frac{3\times0.01}{8}\right]$$

$\rightarrow (6).$

Step 3: Equilibrium condition:

at joint B,

$$M_{BA} + M_{BC} = 0.$$

$$1190\theta_A + 2380\,\theta_B - 8.34 + 1.49 + 1020\theta_B + 510\theta_C = 0.$$

$$1190\theta_A - 6.85 + 3400\theta_B + 510\theta_C = 0. \longrightarrow (7).$$

at joint C,

$$M_{CB} + M_{CD} = 0.$$

$$2.71 + 510\,\theta_B + 1020\,\theta_C - 7 + 892.5\theta_C + 446.25\,\theta_D = 0.$$

$$-4.29 + 510\theta_B + 1912.5\theta_C + 446.25\theta_D = 0. \longrightarrow (8)$$

at D.

$$M_{DC} = 0.$$

$$3.65 + 446.25\theta_C + 892.5\,\theta_D = 0. \longrightarrow (9).$$

at A $M_{AB} = 0.$

$$-13.68 + 1190\,\theta_B + 2380\theta_A = 0. \longrightarrow (10).$$

$\theta_A = 6.09\times10^{-3}$ $\theta_B = -6.92\times10^{-4}$

$\theta_C = 3.82\times10^{-3}$ $\theta_D = -6.004\times10^{-3}$

Step 4: Final moments.

sub $\theta_A, \theta_B, \theta_C, \theta_D$ values in eqn (1), (2), (3), (4), (5) & (6), we get.

$M_{AB} = \cancel{2.73\,kNm}\ -9.28\times10^{-3} \simeq 0$ kNm.

$M_{BA} = -2.73$ kNm.

$M_{BC} = 2.73$ kNm.

$M_{CB} = 6.25$ kNm.

$M_{CD} = -6.26$ kNm

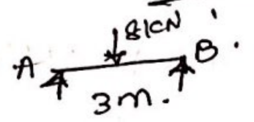
$M_{DC} = -3.895\times10^{-3} \simeq 0$ kNm.

Step 5: S.F.D & B.M.D.

$V_A + V_B + V_C + V_D = 8 + 5 + 8$

$= 21$ kN.

At span AB.

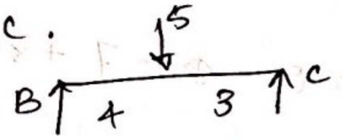


$\Sigma M_B = 0.$

$V_A \times 3 - 8 \times 1 + 0 - 2.73 = 0.$

$V_A = 3.58$ kN.

At span BC.

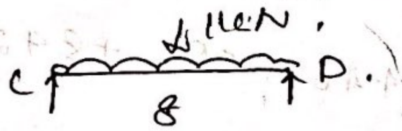


$\Sigma M_B = 0.$

$-V_C \times 7 + 5 \times 4 + 2.73 + 6.25 = 0.$

$V_C = 4.14$ kN.

span CD,

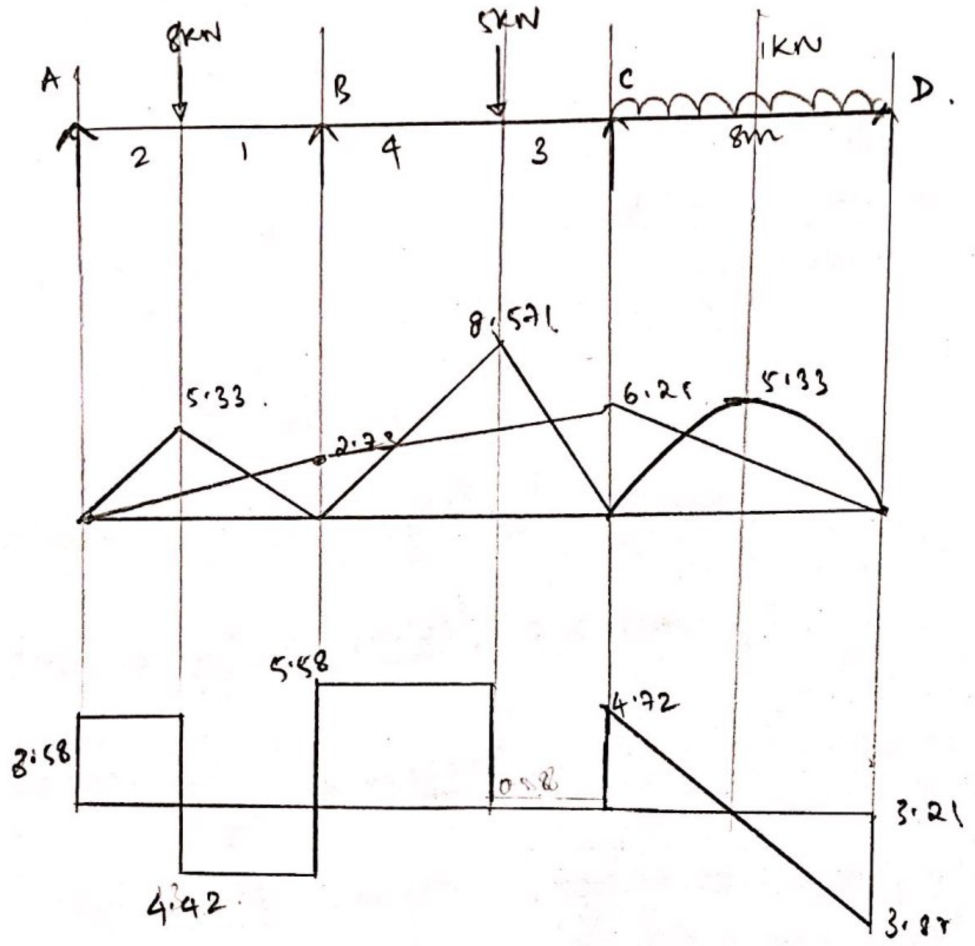


$\Sigma M_C = 0.$

$-V_D \times 8 + 1 \times 8 \times \frac{8}{2} - 6.26 = 0.$

$V_D = 3.21$ kN.

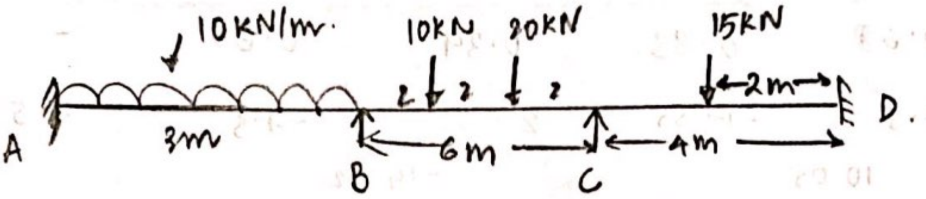
$V_B = 10.07$ kN.



BMD

SFD

2) Analyse the continuous beam using moment distribution method Draw SFD & BMD.



→ Fixed end Moments

$$M_{FAB} = -\frac{WL^2}{12} \Rightarrow -\frac{10(3)^2}{12} \Rightarrow -7.5\,KNm$$

$$M_{FBA} \Rightarrow \frac{WL^2}{12} \Rightarrow \frac{10(3)^2}{12} = 7.5\,KNm$$

$$M_{FBC} \Rightarrow -\frac{Wab^2}{l^2} - \frac{Wab^2}{l^2} \Rightarrow \frac{-10(2)(4)^2}{6^2} - \frac{20(4)(2)^2}{6^2} = -17.77\,KNm$$

$$M_{FCB} = \frac{Wa^2b}{l^2} + \frac{Wa^2b}{l^2} \Rightarrow \frac{10(2)^2(4)}{6^2} + \frac{20(4)^2(2)}{6^2} = \underline{22.22\,KNm}$$

$$M_{FCD} = -\frac{WL}{8} \Rightarrow -\frac{15(4)}{8} \Rightarrow \underline{-7.5\,KNm}$$

$$M_{FDC} = \frac{WL}{8} \Rightarrow \frac{15(4)}{8} \Rightarrow \underline{7.5\,KNm}$$

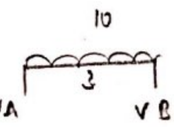
→ Distribution factor.

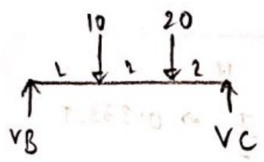
Joint	Member	K	ΣK	K/ΣK
B	BA	$\frac{I}{L} \Rightarrow 0.333\,I$	$0.499\,I$	0.66
	BC	$\frac{I}{L} \Rightarrow 0.1666\,I$		0.33
C	CB	$\frac{I}{L} \Rightarrow 0.1666\,I$	$0.416\,I$	0.39
	CD	$\frac{I}{L} \Rightarrow 0.25\,I$		0.60

Joint	A	B		C		D.
Member	AB	BA	BC	CB	CD	DC
DF	~~[illegible]~~	0.68	0.33.	0.39	0.60	—
FEM	-7.5	7.5	-17.55	22.22	-7.5	7.5.
		10.05		-14.72		
Balance	—	6.83	3.31	-5.74	-8.83.	—
carry	3.415 ~~3.365~~	—	-2.87	1.65		-4.415.
		(2.87)		(1.65)		
Balance	—	1.952	0.9471	-0.643	-0.99.	—
carry	0.991	—	-0.321	0.473		-0.495
		0.321		-0.473		
Balance	—	0.218	0.105	-0.184	-0.28	—
carry	0.1075	—	-0.092	0.0525	—	-0.14
		0.092		-0.052.		
Balance		0.062	0.030	-0.020	0.0312	—
Final FEM	-3.07	16.42	-20.80.	17.80	-17.56	2.45.

$\frac{67}{33}{1000}$

$V_A + V_B + V_C + V_D = 10 \times 3 + 10 + 20 + 15 = 75$





$M_B = 0$

$V_A(3) - 10(3)\left(\frac{3}{2}\right) + M_{BA} + M_{AB} = 0.$

$3V_A - 10(3)\frac{3}{2} - 3.07 + 16.42 = 0$

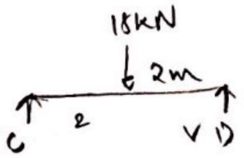
$V_A \Rightarrow 10.55\ kN$

$M_B = 0.$

$-V_C(6). + 10(2) + 20(4) + M_{BC} + M_{CB} = 0$

$-6V_C + 20 + 80 - 20.80 + 17.80 = 0$

$V_C = +16.3\ kN.$



$V_A + V_B + V_C + V_D = 75$

$V_B \Rightarrow 44.43\ kN$

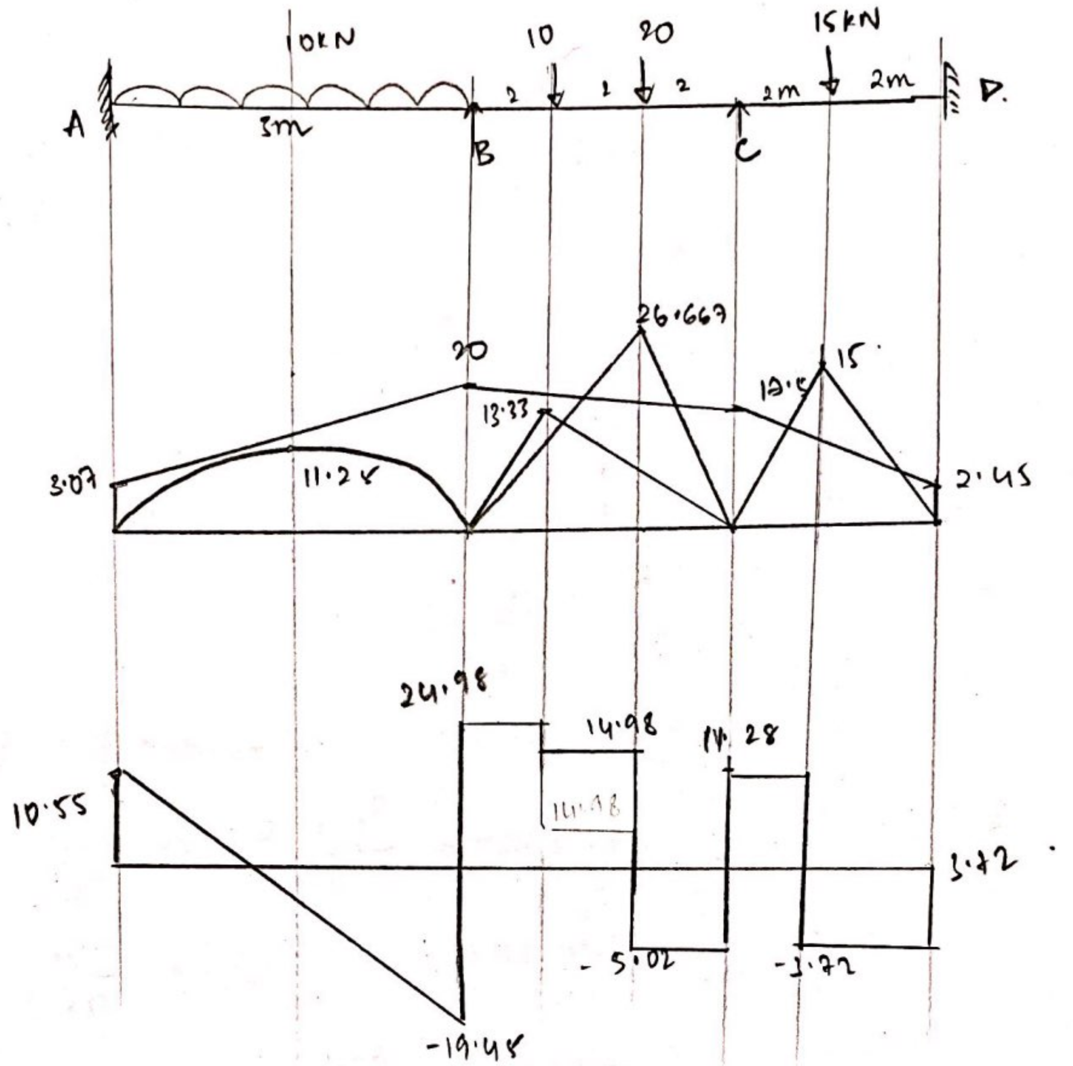
$M_C = 0$

$-V_D(4) + 15(2) + M_{CD} + M_{DC} = 0$

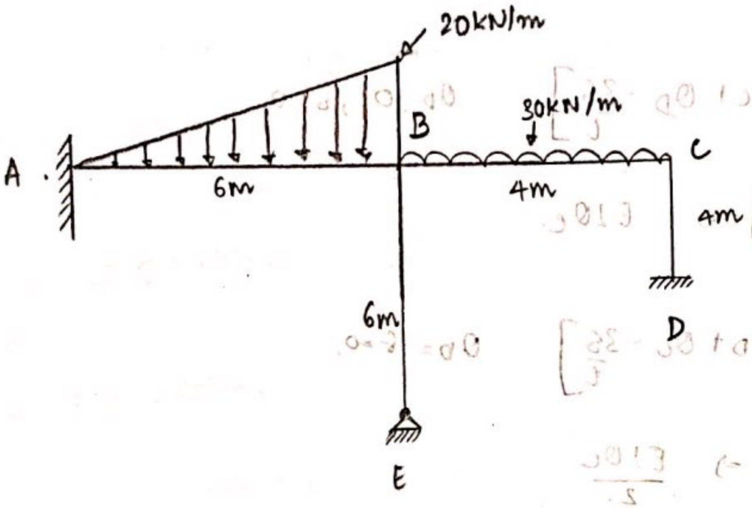
$-4V_D = -30 + 17.56 - 2.45$

$V_D = +3.72\ kN$

Scanned with CamScanner


10KN
10
20
15KN
2
1
2
2m
2m
A
3m
B
C
D.
26.667
20
15
13.33
12.5
3.07
11.25
2.45
24.98
14.98
14.28
10.55
3.72
-5.02
-3.72
-19.48

Analyse the portal frame shown using slope deflection method. Draw BMD & SFD.



⇒ Fixed end Moments

$$M_{FAB} = \frac{-WL^2}{30} \Rightarrow \frac{-20 \times 6^2}{30} \Rightarrow -24\,kNm$$

$$M_{FBA} = \frac{WL^2}{20} \Rightarrow \frac{20 \times 6^2}{20} \Rightarrow 36\,kNm.$$

$$M_{FBC} = \frac{-wL^2}{12} \Rightarrow \frac{-30(4)^2}{12} \Rightarrow -40\,kNm$$

$$M_{FCB} = \frac{wL^2}{12} \Rightarrow \frac{30(4)^2}{12} \Rightarrow 40\,kNm.$$

$$M_{FBE} = M_{FEB} = M_{FCD} = M_{FDC} = 0$$

⇒ Slope Deflection Equation.

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right] \qquad \theta_A = 0,\ \delta = 0.$$

$$\Rightarrow -24 + \frac{2EI}{6}\left[\theta_B\right]. \Rightarrow \frac{2}{6}EI\theta_B - 24.$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right] \qquad \theta_A = \delta = 0.$$

$$\Rightarrow 36 + \frac{2EI}{6}\left[2\theta_B\right] \Rightarrow \frac{2}{3}EI\theta_B + 36.$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right] \quad \delta = 0$$

$$\Rightarrow -40 + \frac{2EI}{4}\left[2\theta_B + \theta_C\right] \Rightarrow EI\theta_B + \frac{EI\theta_C}{2} - 40$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right] \quad \delta = 0.$$

$$40 + \frac{2EI}{4}\left[2\theta_C + \theta_B\right] \Rightarrow \frac{EI\theta_B}{2} + EI\theta_C + 40$$

$$M_{CD} \Rightarrow M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D - \frac{3\delta}{l}\right] \quad \theta_D = 0\ , \delta = 0$$

$$0 + \frac{2EI}{4}\left[2\theta_C\right] \Rightarrow EI\theta_C.$$

$$M_{DC} \Rightarrow M_{FDC} + \frac{2EI}{l}\left[2\theta_D + \theta_C - \frac{3\delta}{l}\right] \quad \theta_D = \delta = 0.$$

$$\frac{2EI}{4}\left[\theta_C\right] \Rightarrow \frac{EI\theta_C}{2.}$$

~~M_{BE}~~ $M_{EB} = 0$ support at E is hinged & hence $\theta_E = 0$

$$M_{BE} = M_{FBE} + \frac{3EI}{l}\left[\theta_B\right] - \frac{M_{FEB}}{2.} \quad \ldots \quad \Big| \; M_{FBE} - \frac{M_{FEB}}{2} + \frac{3EI}{l}(\theta_B)$$

$$M_{BE} \Rightarrow \frac{3EI}{6}\theta_B \Rightarrow \frac{EI\theta_B}{2.}$$

⇒ Equilibrium condition

@ joint B

$$M_{BA} + M_{BC} + M_{BE} = 0.$$

$$\frac{2}{3}EI\theta_B + 36 + EI\theta_B + \frac{EI\theta_C}{2} - 40 + \frac{EI\theta_B}{2} \Rightarrow 0$$

$$2.16EI\theta_B + 0.5EI\theta_C \Rightarrow 4. \longrightarrow (1)$$

at joint C

$$\theta_B = \frac{6.87}{EI}$$

$$M_{CB} + M_{CD} = 0$$

$$\theta_C \Rightarrow \frac{-21.71}{EI}$$

$$\frac{EI\theta_B}{2} + EI\theta_C + 40 + EI\theta_C = 0.$$

$$0.5EI\theta_B + 2EI\theta_C = -40\ .$$

~~at joint A~~ $\theta_B = 6.87/EI$ rad

~~$M_{AB} = 0$~~. $\theta_C \Rightarrow -21.71/EI$ rad.

Final moments

$M_{AB} = \cancel{28.76 KNm} = -21.71 KNm$

$M_{BA} \Rightarrow \cancel{21.52 KNm} = 40.58 KNm.$

$M_{BC} \Rightarrow -43.98 KNm$

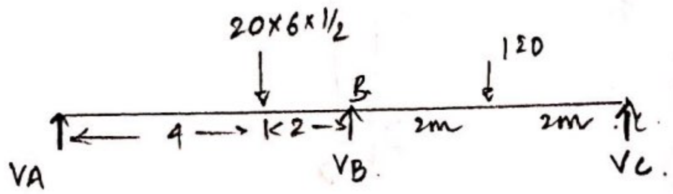
$M_{CB} = 21.725 KNm$

$M_{CD} \Rightarrow -21.71. KNm$

$M_{DC} \Rightarrow -10.85 KNm$

$M_{EB} = 0$

$M_{BE} \Rightarrow 3.435 KNm.$



$M_B = 0$. (LHS).

$VA(6) - 20 \times 6 \times \frac{1}{2} \times \cancel{2} + M_{AB} + M_{BA} = 0.$

$6V_A - 120 - 21.71 + 40 = 0$

$V_A = 16.95 KN.$

M_B (RHS). = 0.

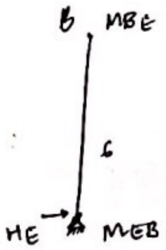
$-V_C(4) + 120(2) + M_{BC} + M_{CB}. = 0.$

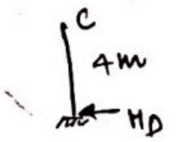
$-4V_C + 240 + (-43.98) + 21.725$

$V_C \Rightarrow 54.4325$

$V_A + V_B + V_C \Rightarrow 20 \times 6 \times \frac{1}{2} \times 30 \times 4 = 240$

$V_B \Rightarrow 168.62$





$4(H_D) + M_{CD} + M_{DC} = 0.$

$4H_D - 21.71 - 10.85 = 0.$

$4H_D \Rightarrow 32.56, 0$

$H_D = 8.14$ KN