1. Weight of characters:

F → 2

A → 2

L → 4

O → 1

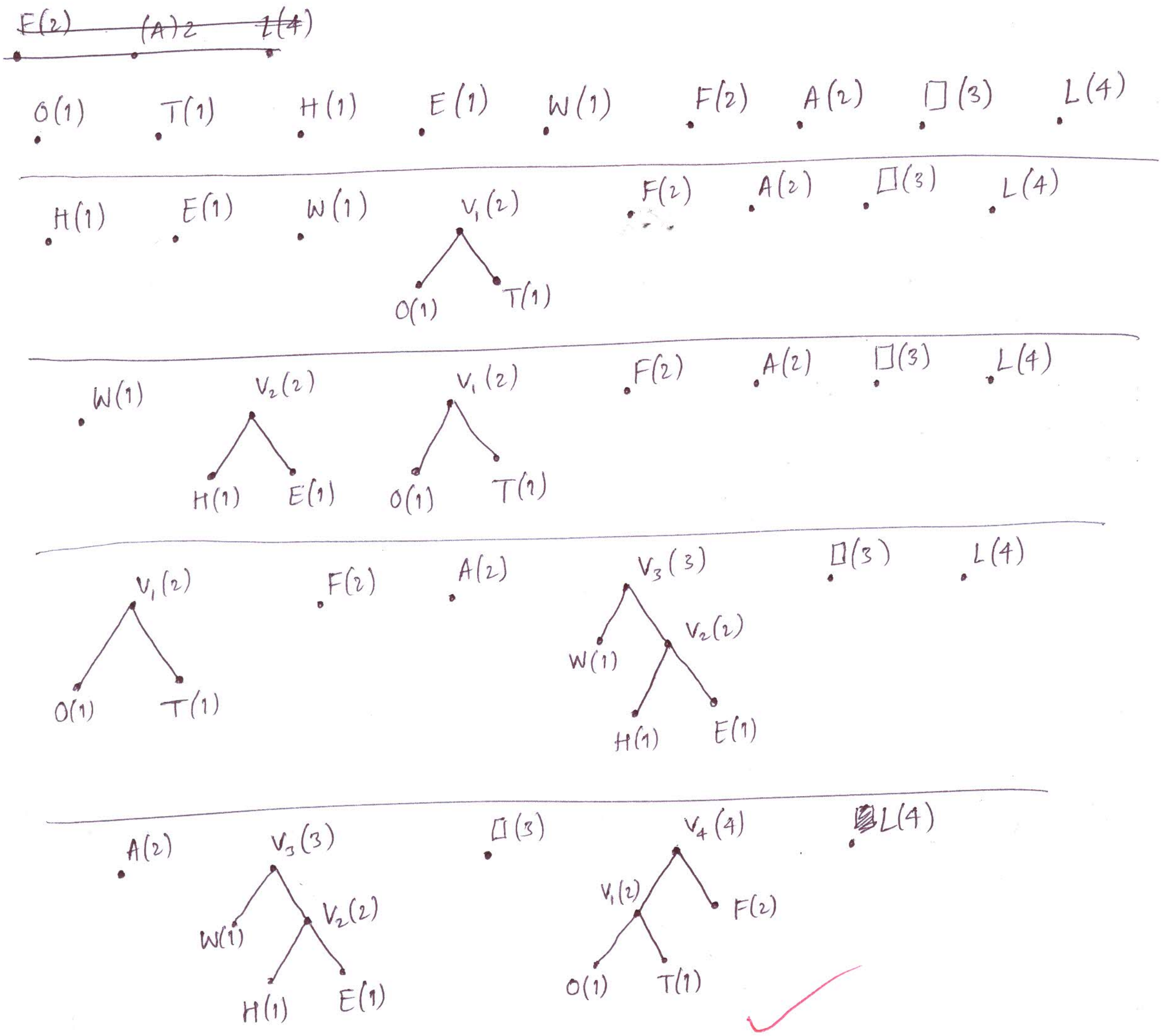
T → 1

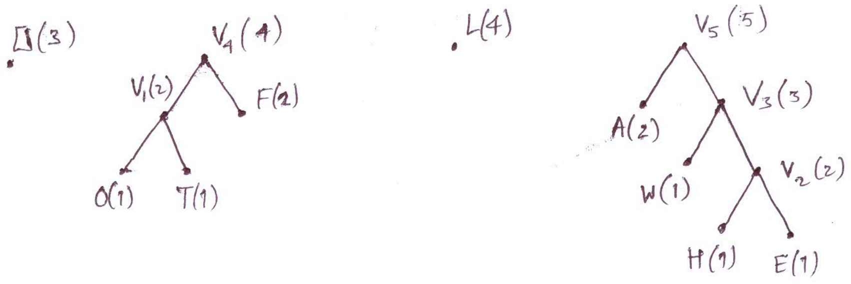
H → 1

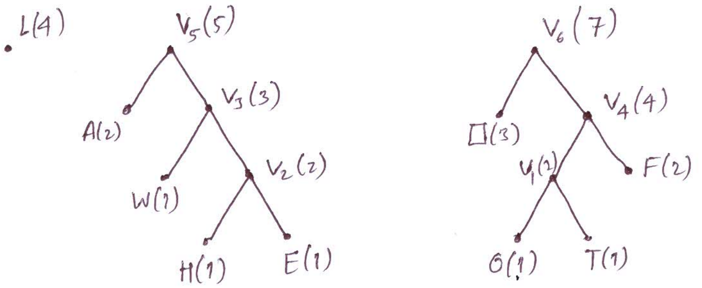
E → 1

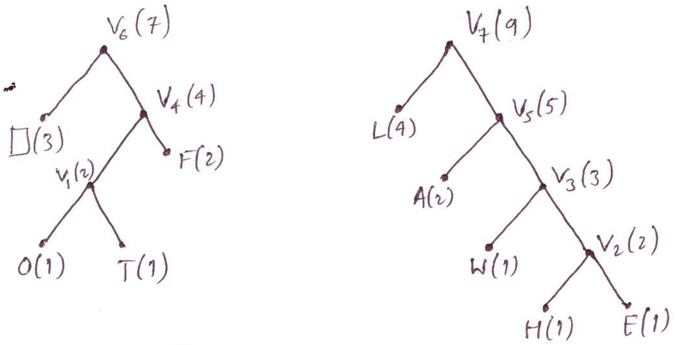
W → 1

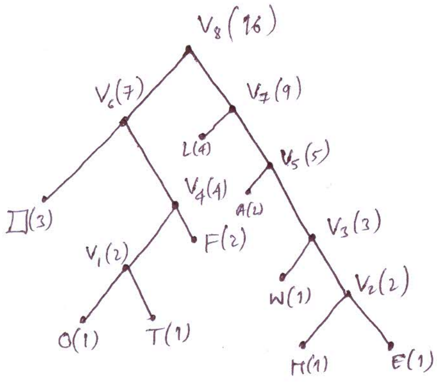
□ → 3











Prefix codes:

□ : 00

O : 0100

T : 0101

F : 011

L : 10

A : 110

W : 1110

H : 11110

E : 11111

∴ ~~Opti.~~ Code for the message:

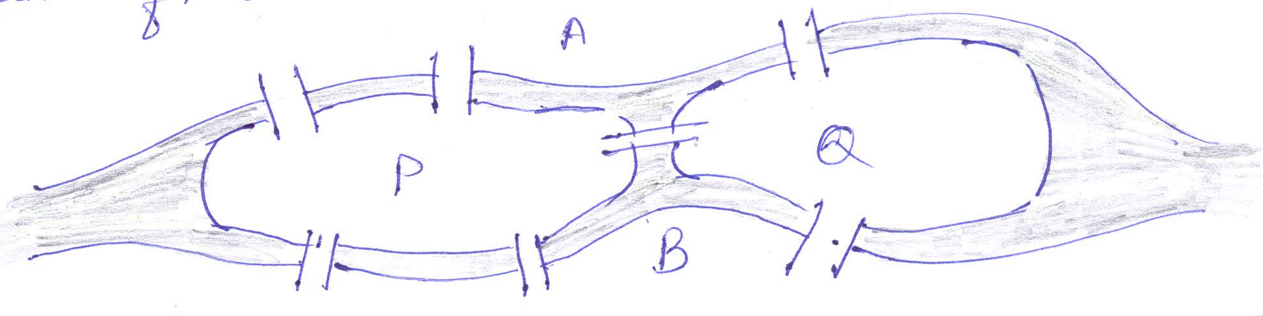
011110 10100000100011000 10111110 111110011101101010

2.

The Köonigsberg Bridge Problem

In 18th century, the city of Konigsberg in Prussia was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. The problem was, by starting at any of the four land areas, can we return to that area after crossing each of the seven bridges exactly once?

[four land areas - two of these parts are the banks of the river and two are islands]

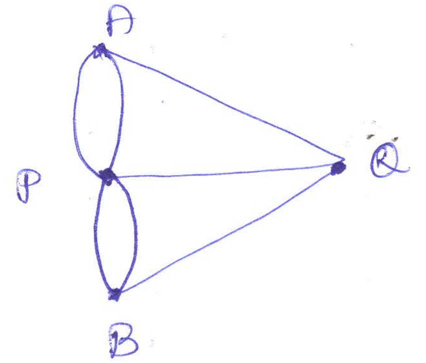


This is the starting point for the development of graph theory. In 1736, Euler analyzed this problem with the help of a graph and gave the sol? This problem is known as Konigsberg Bridge Problem.

Let the land areas be denoted by A, B, P, Q.

A, B are banks of the river & P, Q are islands.

Treat four land areas as four vertices and 7 Bridges as 7 edges. So, we get the graph -

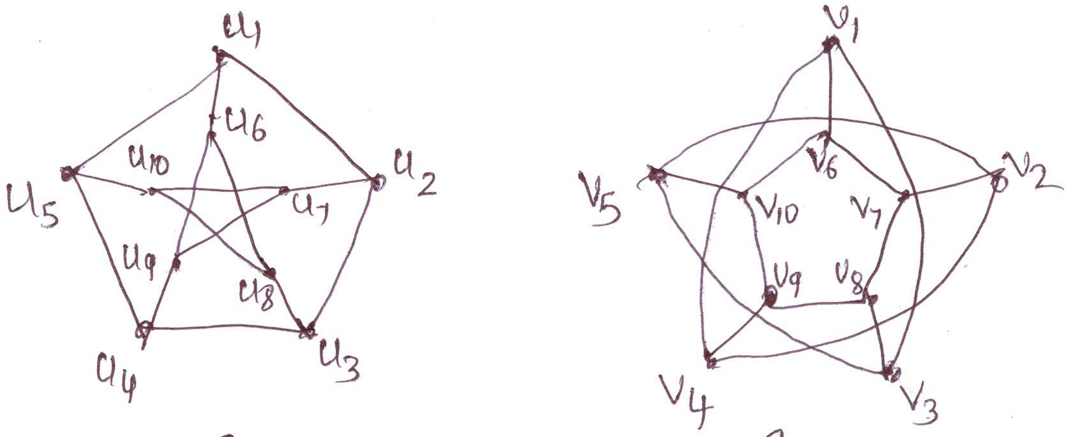


We note that, in this graph deg (A) = 3

deg (B) = 3, deg (P) = 5, deg (Q) = 3

which are not even, ∴ graph doesn't have an Euler circuit. ∴ It is not possible to walk over each of the seven bridges exactly once and return to the starting point.

3.



G_1 G_2

Both the graphs have 10 vertices.

——— " ——— 15 edges.

——— " ——— all the vertices of degree 3 (3-regular graph).

Mapping b/w vertices:

$u_1 \leftrightarrow v_6$

$u_2 \leftrightarrow v_7$

$u_3 \leftrightarrow v_8$

$u_4 \leftrightarrow v_9$

$u_5 \leftrightarrow v_{10}$

$u_6 \leftrightarrow v_1$

$u_7 \leftrightarrow v_2$

$u_8 \leftrightarrow v_3$

$u_9 \leftrightarrow v_4$

$u_{10} \leftrightarrow v_5$

Mapping b/w the edges:

$\{u_1, u_2\} \leftrightarrow \{v_6, v_7\}$

$\{u_2, u_3\} \leftrightarrow \{v_7, v_8\}$

$\{u_3, u_4\} \leftrightarrow \{v_8, v_9\}$

$\{u_4, u_5\} \leftrightarrow \{v_9, v_{10}\}$

$\{u_5, u_1\} \leftrightarrow \{v_{10}, v_6\}$

$\{u_1, u_6\} \leftrightarrow \{v_6, v_1\}$

$\{u_2, u_7\} \leftrightarrow \{v_7, v_2\}$

$\{u_3, u_8\} \leftrightarrow \{v_8, v_3\}$

$\{u_4, u_9\} \leftrightarrow \{v_9, v_4\}$

$\{u_5, u_{10}\} \leftrightarrow \{v_{10}, v_5\}$

$\{u_{10}, u_7\} \leftrightarrow \{v_5, v_2\}$

$\{u_7, u_9\} \leftrightarrow \{v_2, v_4\}$

$\{u_9, u_6\} \leftrightarrow \{v_4, v_1\}$

$\{u_6, u_8\} \leftrightarrow \{v_1, v_3\}$

$\{u_8, u_{10}\} \leftrightarrow \{v_3, v_5\}$

There exists a 1-1 correspondence b/w the two graphs.

$\therefore$ Both the graphs are isomorphic.

②

4. <u>Thm:-</u> "A tree with n vertices has n-1 edges."

$n=1$

$n=2$

$n=3$ 

Assume that the theorem holds for all trees with n vertices where $n \leq k$, for a specified +ve int k.

Consider a tree with k+1 vertices and let 'e' be an edge in this with end vertices ~~e~~ u & v. Deletion of e will disconnect the graph and T-e consists of exactly two components, say T_1 and T_2. Now T_1 and T_2 are trees.

No of vertices in $T_1 = k_1 \leq k+1$

No of vertices in $T_2 = k_2 < k+1$

∴ No of edges in $T_1 = k_1 - 1$

— " — $T_2 = k_2 - 1$

∴ Total edges $= (k_1 + k_2) - 2$
$= (k+1) - 2$
$= k-1$

Total no of vertices in T_1 and T_2 ~~is~~ $= k+1$.

∴ Total no of edges ~~taken~~ in T_1 and $T_2 = (k+1) - 2$
$= k-1$

But T_1 and T_2 together is T-e. ~~Th~~

∴ T-e contains k-1 edges.

∴ T has exactly k edges.

Thus, the theorem is true for $n = k+1$

Hence by M.I, the result is true for all positive integers n.

5) A compound proposition which is always true regardless of truth values of its components is called a tautology.

A compound proposition which is always false regardless of truth values of its components is called a contradiction.

p	q	r	$(p \vee q)$	$(p \to r)$	$(q \to r)$	① $(p \vee q) \wedge (p \to r) \wedge (q \to r)$	① $\to r$
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
0	1	0	1	1	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Since all the entries of the last colum are 1's, given compound proposition is a tautology.

5) Determine the order $|V|$ of the graph $G = (V, E)$ in the following cases:

(1) G is a cubic graph with 9 edges

(2) G is regular graph with 15 edges

(3) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.

(1) Let the order of G be n.

Since G is a cubic graph, all vertices of G have degree 3. Sum of the degrees of vertices $= 3n$

Since G has 9 edges, we should have

$$3n = 2 \times 9$$

$$\Rightarrow n = 6$$

$\therefore$ Thus, the order of $G = 6$.

(2) Since G is regular, all the vertices must have same degree say k. Let the ~~degree~~ order of G be n.

$\therefore$ ~~Total no. of~~ Sum of ~~all~~ the deg of all vertices $= kn$.

$\therefore \quad kn = 15 \times 2$ since G has 15 edges.

$$\Rightarrow k = 30/n.$$

Since k has to be a +ve int, n must be a divisor of 30

$n = 30, 15, 10, 6, 5, 3, 2, 1$

6

③ G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.

Let the order of G be n.

~~Since two vertices of G are of~~

Sum of the degrees of vertices $= 4 + 4 + 3(n-2)$

$8 + 3(n-2) = 10 \cdot 2$ from hand-shaking prop.

$\Rightarrow 3(n-2) = 20 - 8 = 12$

$\Rightarrow n - 2 = 4$

$\Rightarrow n = 6$

Thus, order of G is 6.

(7) $p \to q$

$q \to (r \wedge s)$

$\neg r \vee (\neg t \vee u)$

$p \wedge t$

$\Rightarrow$ $p \to (r \wedge s)$ (Syllogism in 1 & 2)

$r \to (t \to u)$

p

t

$\Rightarrow$ $r \wedge s$ (modus Pones in 2 & 3)

$r \to (t \to u)$

t

$\Rightarrow$ r (conj. simplification)

$r \to (t \to u)$

t

$\Rightarrow$ $t \to u$ (modus Pones in 1 & 2)

t

$\therefore u$

$\therefore$ valid argument

(8) (i) $p \to (q \to r)$

$\equiv \neg p \vee (\neg q \vee r)$ $(p \to q \equiv \neg p \vee q)$

$\equiv (\neg p \vee \neg q) \vee r$ (Assoc.)

$\equiv \neg (p \wedge q) \vee r$ (De-morgans)

$\equiv (p \wedge q) \to r$ (")

(ii) $[p \vee q \vee (\neg p \wedge \neg q \wedge r)]$

$\equiv [(p \vee q) \vee \{\neg (p \vee q) \wedge r\}]$ (De-morgan's)

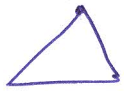
$\equiv [(p \vee q) \vee \neg (p \vee q)] \wedge (p \vee q \vee r)$ (Distributive)

$\equiv T_0 \wedge (p \vee q \vee r)$ (Inverse law)

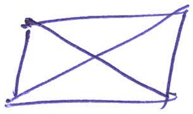
$\equiv p \vee q \vee r$ (Identity)

(9) (i) <u>Complete graph</u> – A simple graph of order $n \geq 2$ in which there is an edge between every pair of vertices. A complete graph with $n (\geq 2)$ vertices is denoted by K_n

K_2

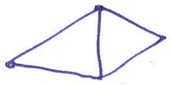


K_3



K_4

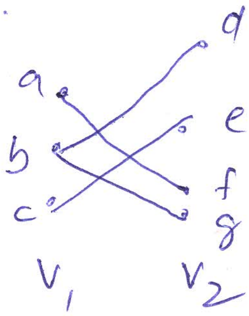
(ii) <u>Simple graph</u> – A graph without any loop and multiple edges.



(iii) <u>Bipartite graph</u> – Suppose a simple graph G is s.t. its vertex set V is the union of two of its mutually disjoint non-empty subsets V_1 & V_2 which are such that each edge in G joins a vertex in V_1 and a vertex in V_2. Then G is called a bipartite graph. If E is the edge set of this graph, the graph is denoted by $G = (V_1, V_2; E)$. The set V_1 & V_2 are bipartite of the vertex set V.

$V_1 = \{a, b, c\}$

$V_2 = \{d, e, f, g\}$



(iv) <u>Spanning graph</u> – A graph G_1 is the spanning subgraph of G, if it contains all the vertices of G,