

1.

A Signal is a function of a set of independent variables, with time perhaps the most prevalent single variable. A signal itself carries some kind of information available for observation. In general, a signal is a function or sequence of values that represents information.

A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals.

Classification of a Signals.

1.2.1 Continuous-Time and Discrete-Time Signals

1.2.2 Even and Odd Signals.

1.2.3 Periodic and Non-periodic Signals.

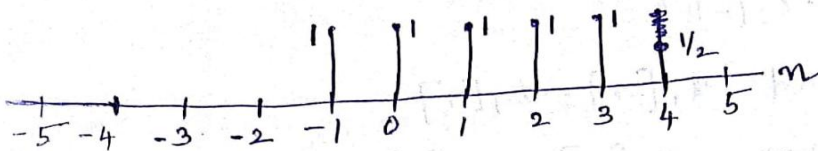
1.2.4 Deterministic and Random Signals.

1.2.5 Energy and Power Signals.

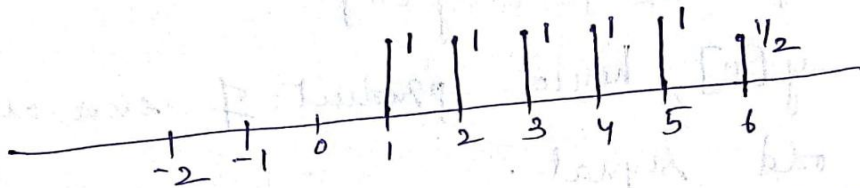
2.

(3)

$x[n]$



(i) $x[n-2]$ shifted 2 places to the right (delay)



(ii) $x[4-n] = x[-n+4]$ advancing and reflect'n $x[-n+4]$



(iii) $x[2n]$ time scaling - compression



3.

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$E = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1 - 1/4} = \frac{1}{3/4} = \frac{4}{3}$$

Energy signal

$$x[n] = u[n]$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N 1^2 = \lim_{N \rightarrow \infty} \frac{1}{2N+1} (N+1) = \frac{1}{2} < \infty$$

[L'Hopital's rule]
[L'Hopital's]

$$x[n] = 2^n u[-n]$$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^0 4^n$$

sub $n = -m$

$$E = \sum_{m=\infty}^0 4^{-m} = \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m = \frac{1}{1 - 1/4} = \frac{4}{3}$$

4.

(5) Consider $y[n] = x_1[n] \cdot x_2[n]$

$$y[-n] = x_1[-n] \cdot x_2[-n]$$

Given $x_1[n]$ is odd $x_1[-n] = -x_1[n]$

$x_2[n]$ is even $x_2[-n] = x_2[n]$

$$y[-n] = -x_1[n] \cdot x_2[n] = -y[n]$$

$y[-n] = -y[n]$, hence product of even and odd signal is odd signal.

(1) $\sum_{n=-\infty}^{\infty} x[n] = 0$

L.H.S $\sum_{n=-\infty}^{\infty} x[n] = 0$

$$= \sum_{n=-\infty}^{-1} x[n] + x[0] + \sum_{n=1}^{\infty} x[n]$$

$$= \sum_{n=1}^{\infty} x[-n] + x[0] + \sum_{n=1}^{\infty} x[n]$$

$$= x[0] + \sum_{n=1}^{\infty} \{x[n] + x[-n]\}$$

for odd $x[0] = 0$ $x[-n] = -x[n]$

Sub in equation

$$= 0 + 0 = 0$$

5.

ion: Let $y(t) = T\{x(t)\} = \frac{dx(t)}{dt}$

(i) Linearity: $T\{ax_1(t) + bx_2(t)\} = \frac{d}{dt}\{ax_1(t) + bx_2(t)\}$

$$= a \frac{dx_1(t)}{dt} + b \frac{dx_2(t)}{dt}$$

$$= a T\{x_1(t)\} + b T\{x_2(t)\}$$

\therefore System is linear

(ii) Time-invariance: $T\{x(t-t_0)\} = \frac{d}{dt}x(t-t_0)$

$$y(t-t_0) = \frac{d}{dt}x(t-t_0)$$

$$\therefore y(t-t_0) = T\{x(t-t_0)\}$$

\therefore System is time-invariant

(iii) Memory : Differentiator has memory.

(iv) Causal : The output does not depend on the future values of the input. So causal.

(v) Stability : If $|x(t)| \leq B_x$,

$$\text{then } |y(t)| = \left| \frac{dx(t)}{dt} \right| \notin B_y$$

\therefore system is unstable.

② Linearity

$$H\{x_1(t)\} = [x_1(t)]^2$$

$$H\{x_2(t)\} = [x_2(t)]^2$$

$$aH\{x_1(t)\} + bH\{x_2(t)\} = a[x_1(t)]^2 + b[x_2(t)]^2 \quad \text{--- ①}$$

$$H\{ax_1(t) + bx_2(t)\} = [ax_1(t) + bx_2(t)]^2 \quad \text{--- ②}$$

$$\text{①} \neq \text{②}$$

Not linear.

③ Time-invariance

$$y(t-t_0) = (x(t-t_0))^2$$

$$T\{x(t-t_0)\} = (x(t-t_0))^2$$

Time-invariant

④ Memory

$$y(t) = (x(t))^2 \quad \text{depends only on the present value, Memoryless}$$

⑤ Causality

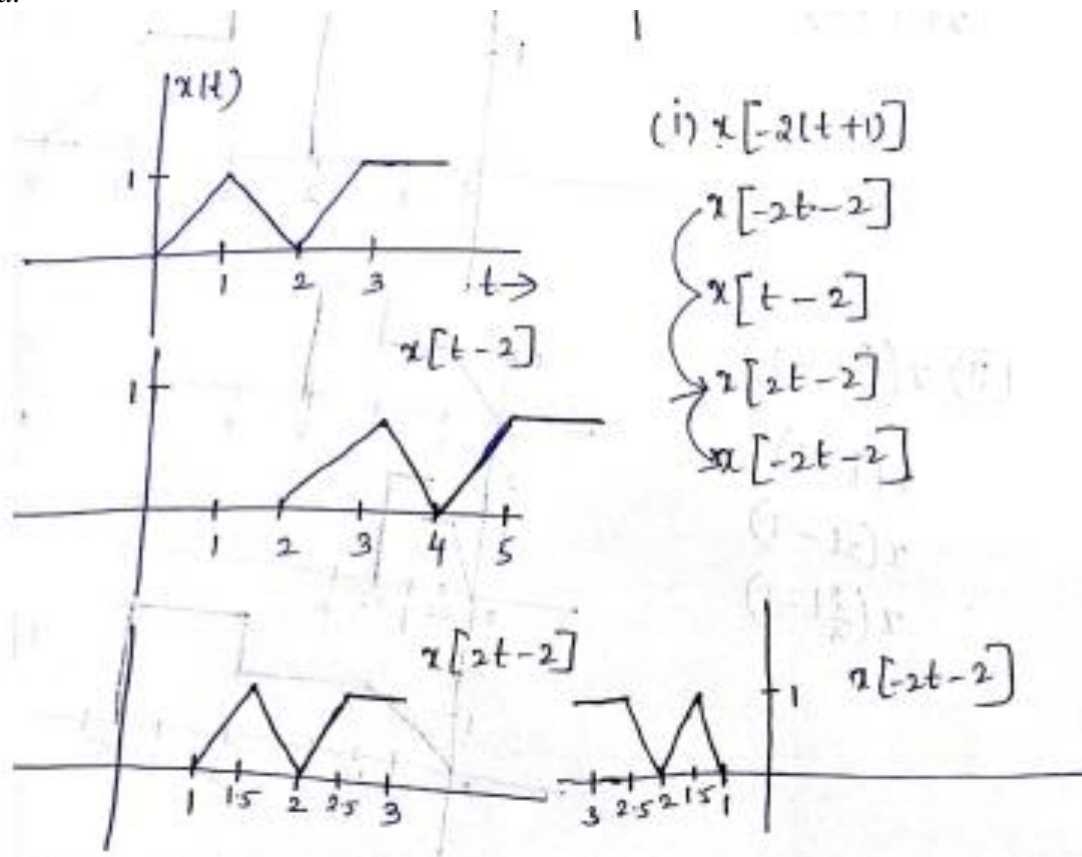
Does not depend on future value, so Causal.

⑥ Stability $0 < |x(t)| < \infty$

$$0 < |(x(t))^2| < \infty$$

Stable.

6.
a.



(i) $x[-2(t+1)]$

$x[-2t-2]$

$x[t-2]$

$x[2t-2]$

$x[-2t-2]$

b.

(ii) $x\left[\frac{t}{2} + 1\right]$

first perform $x[t+1]$

one place to left.

then $x\left[\frac{t}{2} + 1\right]$ expand by 2.

then $x[t+1]$