

Internal Test1 –September 2019

Sub:	High Voltage Engineering						Code:	15EE73/ 10EE73	
Date:	24/09 /2019	Duration:	90 mins	Max Marks:	50	Sem:	VII	Branch:	EEE
Note: Answer any FIVE full questions with neat diagram wherever necessary.									

		Marks	OBE	
			CO	RBT
1.	Define Townsend’s first and second ionization coefficients. Explain the Townsends criterion for a spark. Discuss the limitations of Townsend’s theory.	[3+4 +3]	CO1	L2
2.	Explain the following methods of breakdown of liquid dielectrics, (i) suspended particle theory; (ii) stressed oil volume theory; (iii) Cavitation Theory.	[4+4 +3]	CO1	L2
3.	Discuss the working principle and application of high frequency a.c high voltage generation with a neat diagram.	[5+5]	CO2	L2
4.	State and explain Paschen’s law. Derive the expression of minimum breakdown potential and corresponding <b>pd</b> (min).	[5+5]	CO1	L2
5.	Explain the different testing methods involved with respect to overhead line insulators.	[10]	CO6	L2
6	Describe with a neat diagram the working of a three stage cascaded transformer. Label the power ratings of various stages of the transformer and explain why cascading more than three stages not practically possible	[3+5 +2]	CO3	L2
7a.	In an experiment in a certain gas it was found that the steady state current is $5.5 \times 10^{-8}$ A at 8kV at a distance of 0.4cm between the plane electrodes. Keeping the field constant and reducing the distance to 0.1 cm results in a current of $5.5 \times 10^{-9}$ A. Calculate Townsend’s primary ionization coefficient $\alpha$ .	[5]	CO1	L3
7b	With respect to Paschen’s Law, the given constants are: $A=12, B=365 \quad \gamma=0.02$ or air. Determine $(pd)_{\min}$ and $V_{b_{\min}}$ .	[5]	CO1	
1.				

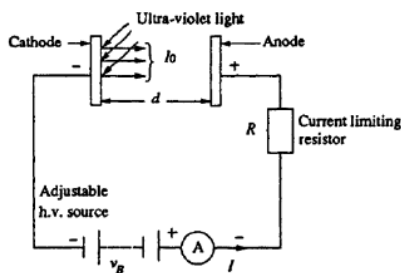


Fig. 2.1 Arrangement for study of a Townsend discharge

$\alpha$ , the average number of ionizing collisions made by an electron per centimetre travel in the direction of the field ( $\alpha$  depends on gas pressure  $p$  and  $E/p$ , and is called the Townsend's first ionization coefficient).

The secondary ionization coefficient  $\gamma$  is defined in the same way as  $\alpha$ , as the net number of secondary electrons produced per incident positive ion, photon, excited particle, or metastable particle, and the total value of  $\gamma$  is the sum of the individual coefficients due to the three different processes, i.e.,  $\gamma = \gamma_1 + \gamma_2 + \gamma_3 = \gamma$  is called the Townsend's secondary ionization coefficient and is a function of the gas pressure  $p$  and  $E/p$ .

The single avalanche process described in the previous section becomes complete when the initial set of electrons reaches the anode. However, since the amplification of electrons [ $\exp(\alpha d)$ ] is occurring in the field, the probability of additional new electrons being liberated in the gap by other mechanisms increases, and these new electrons create further avalanches. The other mechanisms are

(i) The positive ions liberated may have sufficient energy to cause liberation of electrons from the cathode when they impinge on it.

(ii) The excited atoms or molecules in avalanches may emit photons, and this will lead to the emission of electrons due to photo-emission.

(iii) The metastable particles may diffuse back causing electron emission.

The electrons produced by these processes are called secondary electrons. The secondary ionization coefficient  $\gamma$  is defined in the same way as  $\alpha$ , as the net number of secondary electrons produced per incident positive ion, photon, excited particle, or metastable particle, and the total value of  $\gamma$  is the sum of the individual coefficients due to the three different processes, i.e.,  $\gamma = \gamma_1 + \gamma_2 + \gamma_3 = \gamma$  is called the Townsend's secondary ionization coefficient and is a function of the gas pressure  $p$  and  $E/p$ .

Following Townsend's procedure for current growth, let us assume

FIGURE 10.15.

At  $x = 0, n_x = n_0$  (2.6)

Also,  $\frac{dn_x}{dx} = \alpha n_x$ ; or  $n_x = n_0 \exp(\alpha x)$  (2.7)

Then, the number of electrons reaching the anode ( $x = d$ ) will be

$$n_d = n_0 \exp(\alpha d) \quad (2.8)$$

The number of new electrons created, on the average, by each electron is

$$\exp(\alpha d) - 1 = \frac{n_d - n_0}{n_0} \quad (2.9)$$

Therefore, the average current in the gap, which is equal to the number of electrons travelling per second will be

$$I = I_0 \exp(\alpha d) \quad (2.10)$$

where  $I_0$  is the initial current at the cathode.

Following Townsend's procedure for current growth, let us assume

$n_0'$  = number of secondary electrons produced due to secondary ( $\gamma$ ) processes.

Let  $n_0''$  = total number of electrons leaving the cathode.

Then 
$$n_0'' = n_0 + n_0' \quad (2.11)$$

The total number of electrons  $n$  reaching the anode becomes,

$$n = n_0'' \exp(\alpha d) = (n_0 + n_0') \exp(\alpha d);$$

and 
$$n_0' = \gamma [n - (n_0 + n_0')]$$

Eliminating  $n_0'$ , 
$$n = \frac{n_0 \exp(\alpha d)}{1 - \gamma [\exp(\alpha d) - 1]}$$

or 
$$I = \frac{I_0 \exp(\alpha d)}{1 - \gamma [\exp(\alpha d) - 1]} \quad (2.12)$$

Townsend's Criterion For Breakdown:

Equation (2.12) gives the total average current in a gap before the occurrence of breakdown. As the distance between the electrodes  $d$  is increased, the denominator of the equation tends to zero, and at some critical distance  $d = d_s$ .

$$1 - \gamma [\exp(\alpha d) - 1] = 0 \quad (2.13)$$

For values of  $d < d_s$ ,  $I$  is approximately equal to  $I_0$ , and if the external source for the supply of  $I_0$  is removed,  $I$  becomes zero. If  $d = d_s$ ,  $I \rightarrow \infty$  and the current will be limited only by the resistance of the power supply and the external circuit. This condition is called Townsend's breakdown criterion and can be written as

$$\gamma [\exp(\alpha d) - 1] = 1$$

Normally,  $\exp(\alpha d)$  is very large, and hence the above equation reduces to

$$\gamma \exp(\alpha d) = 1 \quad (2.14)$$

For a given gap spacing and at a give pressure the value of the voltage  $V$  which gives the values of  $\alpha$  and  $\gamma$  satisfying the breakdown criterion is called the spark breakdown voltage  $V_s$  and the corresponding distance  $d_s$  is called the sparking distance.

#### Limitation In Townsend's Mechanism

Townsend mechanism when applied to breakdown at atmospheric pressure was found to have certain drawbacks.

A) according to the Townsend theory, current growth occurs as a result of ionization processes only.

But in practice, breakdown voltages were found to depend on the gas pressure and the geometry of the gap.

B) Mechanism predicts **time lags** of the order of **10-5S**, while in actual practice breakdown was observed to occur at very short times of the order of **10-8S**.

C) Townsend mechanism predicts a very diffused form of discharge, in actual practice, discharges were found to be filamentary and irregular.

The Townsend mechanism failed to explain all these observed phenomena and as a result, around 1940, Raether and, Meek and Loeb independently proposed the **Streamer theory**.

2i)

### i)Suspended Particle Theory

- In commercial liquids, the presence of solid impurities cannot be avoided.
- These impurities will be present as fibrous or as dispersed solid particles.
- The permittivity of these particles ( $\epsilon_1$ ) will be different from the permittivity of the liquid ( $\epsilon_2$ ).
- If we consider these impurities to be spherical particles of radius r, and if the applied field is E, then the particles experience a force F, where

$$F = r^3 \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + 2\epsilon_2} E \cdot \frac{dE}{dx}$$

- this force is directed towards a place of higher stress if  $\epsilon_1 > \epsilon_2$  and towards a place of lower stress if  $\epsilon_1 < \epsilon_2$  when  $\epsilon_1$  is the permittivity of gas bubbles.
- The force given above increases as the permittivity of the suspended particles ( $\epsilon_1$ ) increases. If  $\epsilon_1 \rightarrow \infty$

$$F = r^3 \frac{1 - \epsilon_2/\epsilon_1}{1 + 2\epsilon_2/\epsilon_1} E \frac{dE}{dx}$$

Let  $\epsilon_1 \rightarrow \infty$

$$F = r^3 E \cdot \frac{dE}{dx}$$

- Force will tend the particle to move towards the strongest region of the field.
- In a uniform electric field which usually can be developed by a small sphere gap, the field is the strongest in the uniform field region. **Here  $dE/dx \rightarrow 0$  so that the force on the particle is zero and the particle remains in equilibrium.**
- Particles will be dragged into the uniform field region.
- Permittivity of the particles is higher than that of the liquid, the presence of particle in the uniform field region will cause flux concentration at its surface.
- Other particles if present will be attracted towards the higher flux concentration.
- The movement of the particle under the influence of electric field is opposed by the viscous force posed by the liquid and since the particles are moving into the region of high stress, diffusion must also be taken into account.
- We know that the viscous force is given by (Stoke's relation)
- **$F_v = 6\pi\eta r v$**
- where  $\eta$  is the viscosity of liquid, r the radius of the particle and v the velocity of the particle.
- Equating the electrical force with the viscous force we have

$$6\pi\eta r v = r^3 E \frac{dE}{dx} \quad \text{or} \quad v = \frac{r^2 E}{6\pi\eta} \frac{dE}{dx}$$

- However, if the diffusion process is included, the drift velocity due to diffusion will be given by

$$v_d = -\frac{D}{N} \frac{dN}{dx} = -\frac{KT}{6\pi\eta r} \frac{dN}{N dx}$$

$$E_0 = \frac{1}{(\epsilon_1 - \epsilon_2)} \left[ \frac{2\pi\sigma(2\epsilon_1 + \epsilon_2)}{r} \left\{ \frac{\pi}{4} \sqrt{\left( \frac{V_b}{2rE_0} \right) - 1} \right\} \right]^{\frac{1}{2}}$$

- Where  $\sigma$  is the surface tension of the liquid,
- $\epsilon_1$  is the permittivity of the liquid,
- $\epsilon_2$  is the permittivity of the gas bubble,
- $r$  is the initial radius of the bubble assumed as a sphere
- $V_b$  is the voltage drop in the bubble (corresponding to minimum on the Paschen's curve).
- From this equation, it can be seen that the breakdown strength depends on the initial size of the bubble which in turn is influenced by the hydrostatic pressure and temperature of the liquid.

### iii) Stressed Oil Volume Theory

- In commercial liquids where minute traces of impurities are present, breakdown strength is determined by the "**largest possible impurity**" or "**weak link**".
- **On a statistical basis** it was proposed that electrical breakdown strength of oil is defined by the weakest region in the oil, the region which is **stressed to maximum** and by the **volume of oil included** in that region.
- In non-uniform fields, the stressed oil volume is taken as the volume which is contained between the **maximum stress ( $E_{max}$ ) contour and 0.9  $E_{max}$  contour.**

According to this theory the breakdown strength is inversely proportional to the stressed oil volume.

- The **breakdown voltage** is highly **influenced** by the **gas content** in the oil, the **viscosity** of the oil, and the **presence of other impurities**.
- These being uniformly distributed, increase in the stressed oil volume consequently results in a reduction in the breakdown voltage.
- The variation of the breakdown voltage stress with the stressed oil volume is shown in Fig.

3

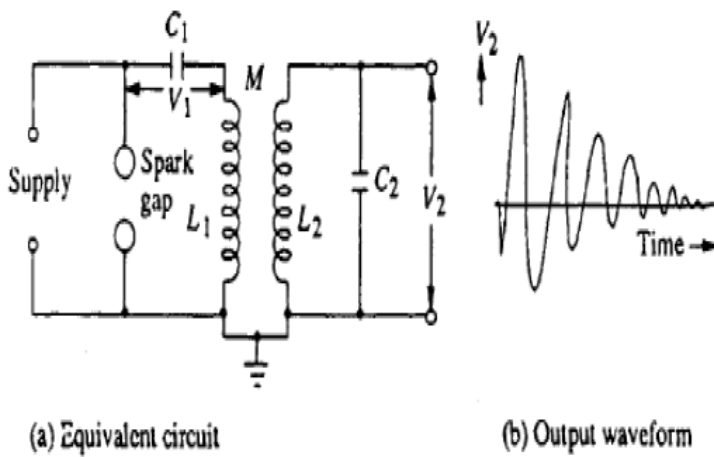
## Generation of High Frequency a.c. High Voltages

### • High frequency high voltages are required for rectifier d.c. power supplies

- testing electrical apparatus for switching surges, high frequency high voltage damped oscillations are needed which need high voltage high frequency transformers.

The advantages of these high frequency transformers are:

- the absence of iron core in transformers and hence saving in cost and size,
  - pure sine wave output,
  - slow build-up of voltage over a few cycles and hence no damage due to switching surges, and
  - uniform distribution of voltage across the winding coils due to subdivision of coil stack into a number of units.
  - The commonly used high frequency resonant transformer is the Tesla coil
  - Doubly tuned resonant circuit
  - The primary voltage rating is 10 kV
  - The secondary may be rated to as high as 500 to 1000 kV.
  - The primary is fed from a d.c. or a.c. supply through the condenser  $C_1$
  - A spark gap  $G$  connected across the primary is triggered at the desired voltage  $V_1$  which induces a high self-excitation in the secondary.
  - The primary and the secondary windings ( $L_1$ ) and ( $L_2$ ) are wound on an insulated former with no core (air-cored) and are immersed in oil.
  - The windings are tuned to a frequency of 10 to 100 kHz by means of the condensers  $C_1$  and  $C_2$
-



- output voltage  $V_2$  is a function of  $L_1$ ,  $L_2$ ,  $C_1$ ,  $C_2$ ,  $M$ .
- winding resistances will be small, contribute only for damping of the oscillations.
- neglecting the winding resistances.
- Let the condenser  $C_1$  be charged to a voltage  $V_1$  when the spark gap is triggered.
- Let a current  $i_1$  flow through the primary winding  $L_1$  and produce a current  $i_2$  through  $L_2$  and  $C_2$

Laplace Transform

$$V_1 = \frac{1}{C_1} \int_0^t i_1 dt + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\frac{V_1}{s} = \left[ L_1 s + \frac{1}{C_1 s} \right] I_1 + M s I_2$$

$$0 = \frac{1}{C_2} \int_0^t i_2 dt + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$0 = [M s] I_1 + \left[ L_2 s + \frac{1}{C_2 s} \right] I_2$$

$$V_2 = \frac{1}{C_2} \int_0^t i_2 dt; \text{ or its transformed equation is}$$

$$V_2(s) = \frac{I_2}{C_2 s}$$

$$V_2 = \frac{M V_1}{\sigma L_1 L_2 C_1} \frac{1}{\gamma_2^2 - \gamma_1^2} [\cos \gamma_1 t - \cos \gamma_2 t]$$

where,

$$\sigma^2 = 1 - \frac{M^2}{L_1 L_2} = 1 - K^2$$

→ Townsend's Criteria.

$\gamma [\exp(\alpha d) - 1]$  enables the evaluation of breakdown voltage of the gap by the use of appropriate values of  $\alpha/p$  and  $\gamma$  so corresponding to  $E/p$  when the current is too low to damage the cathode and also space charge distortions are minimum.

→ Calculated and experimental values matches when gaps are short or long pressure is relatively high. low.

⇒ Breakdown voltage for uniform field gaps as a function of gap length and gap pressure can be derived from threshold equation by expressing the ionization coefficient  $\alpha/p$  as a function of field strength  $E$  and gas pressure  $p$ .

$$\frac{\alpha}{p} = f\left(\frac{E}{p}\right).$$

$$\therefore e^{f(E/p)pd} = \frac{1}{\gamma} + 1.$$

taking  $\ln$  both sides;

$$f\left(\frac{E}{p}\right) pd = \ln\left[\frac{1}{\gamma} + 1\right] = k.$$

for uniform field  $E = \frac{V_b}{d}$ .

$$\therefore f\left(\frac{V_b}{pd}\right) \times (pd) = k.$$

$$\therefore f\left(\frac{V_b}{pd}\right) = \frac{k}{pd}.$$

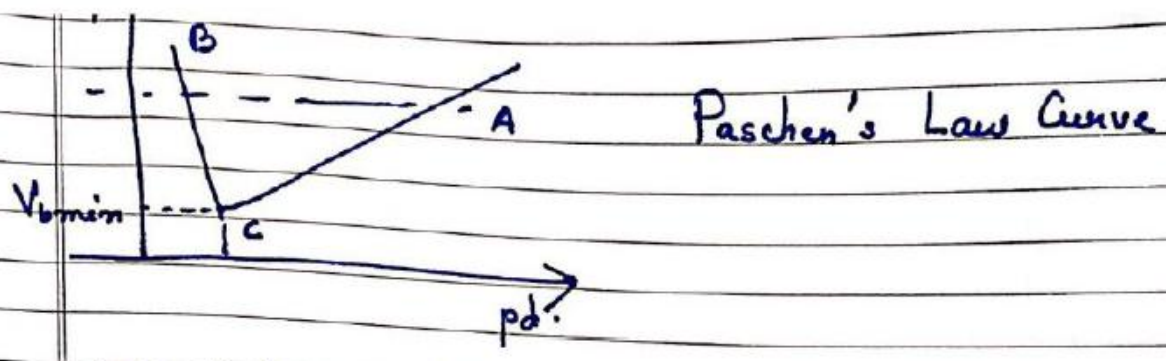
$$V_b = F(pd).$$

This shows the breakdown voltage of a uniform field gap is unique function of product of gap pressure and electrode material.

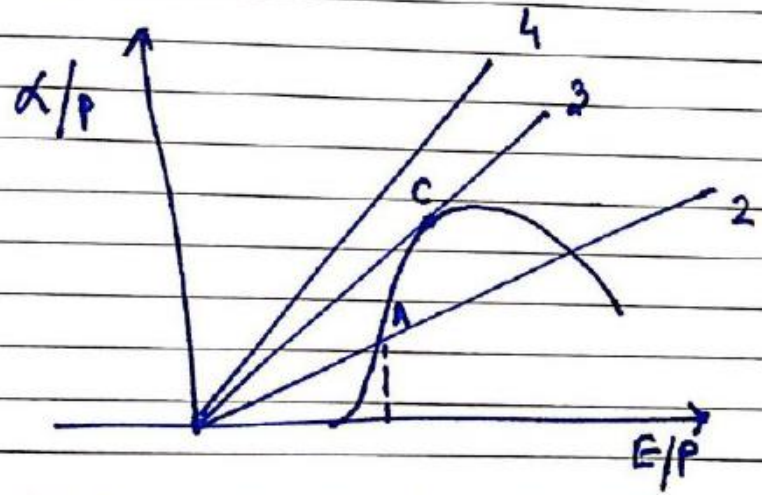
→ Paschen's Law.

→ This relation does not mean that breakdown voltage is proportional to product  $pd$ , some region of the product  $pd$  the relation is linear.





Let Experimentally obtained relation between ionization coeff  $\alpha/p$  and field strength  $(E/p)$ .  
 $(E_b/p)_c \Rightarrow$  represents an onset of ionization



The Townsend's criteria

$$\alpha d = k.$$

$$\frac{\alpha}{p} \times \frac{V}{E} = \frac{k}{p} \quad \text{or} \quad \frac{\alpha}{p} = \frac{k}{V} \cdot \frac{E}{p}$$

This is equation to a straight line, with slope equal to  $K/V$ , depending upon the value of  $K$ .

Higher the voltage smaller slope  
 $\Rightarrow$  this line will intersect ionization curve at two points. A and B.

$\rightarrow$  there must be two breakdown voltages at a constant pressure  $p$ , one corresponding to small value of gap length, i.e. higher  $E$  ( $E = V/d$ ) i.e. point B

$\Rightarrow$  other to the longer gap length i.e. smaller  $E$  or  $E/p$  i.e. point A.

$\Rightarrow$  At low values of voltage  $V$ , the slope of the straight line is large.

$\therefore$  no intersection between line and curve.

$\Rightarrow$  This means no breakdown occurs with small voltages below Paschen's minimum irrespective of the value of  $pd$ .

Point C on the curve indicates the lowest breakdown voltage or minimum sparking potential.

$\Rightarrow$  Spark over voltage corresponding to points A, B, C are shown in previous figure.

⇒ Analytical expression for minimum sparking potential can be obtained using general expression for  $\alpha/p$ .

$$\frac{\alpha}{p} = A e^{-Bp/E}$$

$$\therefore \alpha = p A e^{-Bp/E}$$

$$\alpha = p A e^{-Bpd/V_b}$$

$$e^{Bpd/V_b} = \frac{pA}{\alpha}$$

$$\therefore \frac{1}{\alpha} = \frac{e^{Bpd/V_b}}{pA}$$

on d.  $\frac{1}{\alpha d} = \frac{e^{Bpd/V_b}}{pA}$

$$\text{as } \alpha d = \ln\left(1 + \frac{1}{\gamma}\right)$$

$$\therefore d = \frac{e^{Bpd/V_b}}{pA} \times \alpha d$$

$$= \frac{e^{Bpd/V_b}}{pA} \times \ln\left(1 + \frac{1}{\gamma}\right)$$

as  $\gamma = \text{constant}$

$$\therefore \ln\left(1 + \frac{1}{\gamma}\right) = K$$

$$\therefore d = \frac{e^{Bpd/V_b}}{pA} \times K \quad \text{--- (A)}$$

⇒ Analytical expression for minimum sparking potential can be obtained using general expression for  $\alpha/p$ .

$$\frac{\alpha'}{p} = A e^{-Bp/E}$$

$$\therefore \alpha = p A e^{-Bp/E}$$

$$\alpha = p A e^{-Bpd/V_b}$$

$$e^{Bpd/V_b} = \frac{pA}{\alpha}$$

$$\therefore \frac{1}{\alpha} = \frac{e^{Bpd/V_b}}{pA}$$

$$\text{on } d, \frac{1}{\alpha d} = \frac{e^{Bpd/V_b}}{pA}$$

$$\text{as } \alpha d = \ln\left(1 + \frac{1}{\gamma}\right)$$

$$\therefore d = \frac{e^{Bpd/V_b}}{pA} \times \alpha d$$

$$= \frac{e^{Bpd/V_b}}{pA} \times \ln\left(1 + \frac{1}{\gamma}\right)$$

as  $\gamma = \text{constant}$

$$\therefore \ln\left(1 + \frac{1}{\gamma}\right) = K$$

$$\therefore d = \frac{e^{Bpd/V_b}}{pA} \times K \quad \text{--- (A)}$$

In order to obtain minimum potential

$$V_b = f(pd).$$

Taking logarithm in both sides.

$$\frac{Bpd}{V_b} = \ln \frac{Apd}{K}$$

$$V_b = \frac{Bpd}{\ln \frac{Apd}{K}}$$

Differentiating  $V_b$  w.r.t  $pd$  and equating the derivative to zero.

$$\frac{dV_b}{d(pd)} = \frac{\ln \frac{Apd}{K} \times B - Bpd \cdot \frac{K}{Apd} \cdot \frac{A}{K}}{\left[ \ln \left( \frac{Apd}{K} \right) \right]^2}$$

$$\ln \frac{1}{\ln \frac{Apd}{K}} = \frac{1}{\left[ \ln \left( \frac{Apd}{K} \right) \right]^2}$$

$$\ln \left( \frac{Apd}{K} \right) = 1.$$

To explain the fact that there exists a minimum sparking potential in the relation between sparking potential and gap length assuming  $p$  to be constant can be explained by considering the  $\eta$  of ionization of electrons traversing the gap with different electron energies.

Assuming that the Townsend's second ionization coefficient  $\gamma$  is small for values  $(pd) > (pd)_{min}$ , electrons crossing the gap make more frequent collision with gas molecules than at  $(pd)_{min}$ , but the energy gained between the successive collision is small than at  $(pd)_{min}$ . Hence probability of ionization is lower unless the  $V \uparrow$

In case of  $(pd) < (pd)_{min}$ , electrons cross the gap without making any collision and thus the sparking potential is higher. point  $(pd)_{min} \Rightarrow$  corresponding to highest ionization efficiency and hence minimum sparking potential.

$$\therefore (Pd)_{\min} = \frac{e}{A} K$$

$$(V_b)_{\min} = \frac{B e K / A}{1} = \frac{B}{A} e K$$

$$V_{b \min} = 2.718 \frac{B}{A} \ln \left( 1 + \frac{1}{\gamma} \right)$$

## 5. Tests on Insulators

The tests are as (i) **type tests**, and (ii) **the routine tests**.

- **Type tests** to prove or check the design features and the quality.

**The routine tests** to check the quality of the individual test piece.

**Type tests** are done on samples when **new designs or design changes** are introduced.

**the routine tests** are done to ensure the **reliability of the individual test objects** and **quality and consistency** of the materials used in their manufacture.

**High voltage tests** include

- the power frequency tests,
- impulse tests.

All the insulators are tested for both categories of test.

### Power Frequency Tests

(a) **Dry and Wet Flashover Tests** : In these tests the a.c. voltage of power frequency is applied across the insulator and increased at a uniform rate of about 2 per cent per second of 75% of the estimated test voltage, to such a value that a breakdown occurs along the surface of the insulator.

- If the test is conducted under **normal conditions without any rain or precipitation**, it is called "**dry flashover test**".

If the test is done under **conditions of rain**, it is called "**wet flashover test**".

### b) Wet and Dry Withstand Tests (One Minute):

- In these tests, the voltage specified in the relevant specification is applied under dry or wet conditions for a period of one minute with an insulator mounted as in service conditions.
- The test piece should withstand the specified voltage.
- **Impulse Tests**
- **Impulse Withstand Voltage Test** : This test is done by applying standard impulse voltage of specified value **under dry conditions with both positive and negative polarities of the wave**.
- If **five consecutive waves** do not cause a **flashover or puncture**, the insulator is deemed to have **passed the test**.
- If **two applications** cause **flashover**, the object is deemed to have failed.
- If there is only **one failure**, additional **ten applications** of the voltage wave are made.
- If the test object has **withstood** the subsequent applications, it is said to have passed the test.
- (b) **Impulse Flashover Test** The test is done as **above with the specified voltage**.
- Usually, the probability of failure is determined for 40% and 60% failure values or 20% and 80% failure values, since it is difficult to adjust the test voltage for the exact 50% flashover values.
- The average value of the upper and the lower limits is taken.
- The insulator surface should not be damaged by these tests, but slight marking on its surface or chipping off of the cement is allowed.

- **Pollution Testing** Because of the problem of pollution of outdoor electrical insulation and consequent problems of the maintenance of electrical power systems, pollution testing is gaining importance.
- The **normal types of pollution** are
  - dust, micro-organisms, bird secretions, flies, etc.,
  - industrial pollution like smoke, petroleum vapours, dust, and other deposits,
  - coastal pollution in which corrosive and hygroscopic salt layers are deposited on the insulator surfaces,
  - Desert pollution in which sand storms cause deposition of sand and dust layers,
  - ice and fog deposits at high altitudes and in polar countries.
- **Effect of Pollution:**
  - Pollutions cause corrosion, non-uniform gradients along the insulator strings and surface of insulators and also cause deterioration of the material.
  - pollution causes partial discharges and radio interference
  - **pollution testing** is important for **extra high voltage** systems.
  - there is no standard pollution test available.
  - The popular test that is normally done is the **salt fog test**.
  - **the maximum normal withstand voltage** is applied on the **insulator** and then **artificial salt fog** is created around the insulator by **jets of salt water** and **compressed air**.
  - If the **flashover** occurs within **one hour**, the test is repeated **with fog of lower salinity**, otherwise, **with a fog of higher salinity**.
  - The maximum salinity at which the insulator withstands three out of four tests without flashover is taken as the representative figure.
  - Much work is yet to be done to standardize the test procedures

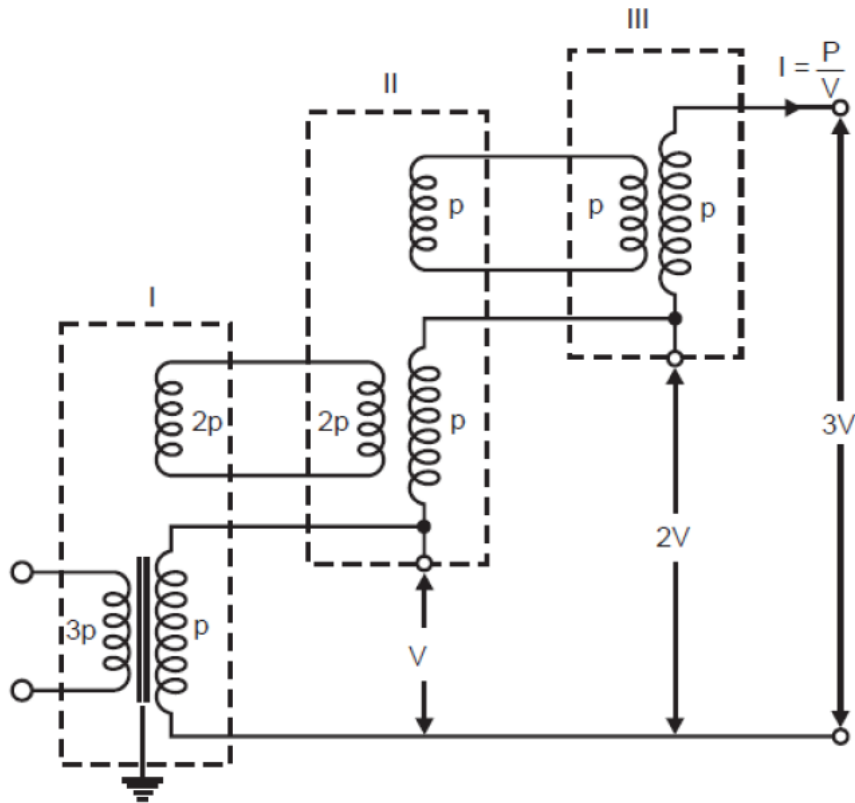
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### **Cascaded Transformer**

Figure shows a basic scheme for cascading three transformers. The primary of the first stage transformer is connected to a low voltage supply. A voltage is available across the secondary of this transformer. The tertiary winding (excitation winding) of first stage has the same number of turns as the primary winding, and feeds the primary of the second stage transformer. The potential of the tertiary is fixed to the potential  $V$  of the secondary winding as shown in Fig. 2.9. The secondary winding of the second stage transformer is connected in series with the secondary winding of the first stage transformer, so that a voltage of  $2V$  is available between the ground and the terminal of secondary of the second stage transformer. Similarly, the stage-III transformer is connected in series with the second stage transformer. With this the output voltage between ground and the third stage transformer, secondary is  $3V$ . it is to be noted that the individual stages except the upper most must have three-winding transformers. The upper most, however, will be a two winding transformer.

shows metal tank construction of transformers and the secondary winding is not divided. Here the low voltage terminal of the secondary winding is connected to the tank. The tank of stage-I transformer is earthed. The tanks of stage-II and stage-III transformers have potentials of  $V$  and  $2V$ , respectively above earth and, therefore, these must be insulated from the earth with suitable solid insulation. Through h. t. bushings, the leads from the tertiary winding and the h. v. winding are brought out to be connected to the next stage transformer.





The main disadvantage of cascading the transformers is that the lower stages of the primaries of the transformers are loaded more as compared with the upper stages. The loading of various windings is indicated by  $P$  in Fig. 2.9. For the three-stage transformer, the total output  $VA$  will be  $3VI = 3P$  and, therefore, each of the secondary winding of the transformer would carry a current of  $I = P/V$ . The primary winding of stage-III transformer is loaded with  $P$  and so also the tertiary winding of second stage transformer. Therefore, the primary of the second stage transformer would be loaded with  $2P$ . Extending the same logic, it is found that the first stage primary would be loaded with  $P$ . Therefore, while designing the primaries and tertiaries of these transformers, this factor must be taken into consideration.

7a

**Solution:** The current at the anode  $I$  is given by

$$I = I_0 \exp(\alpha d)$$

where  $I_0$  is the initial current and  $d$  is the gap distance.

Given,

$$d_1 = 0.4 \text{ cm} \quad d_2 = 0.1 \text{ cm}$$

$$I_1 = 5.5 \times 10^{-8} \text{ A} \quad I_2 = 5.5 \times 10^{-9} \text{ A}$$

$$\frac{I_1}{I_2} = \exp \alpha(d_1 - d_2)$$

i.e.,  $10 = \exp(\alpha \times 0.3)$

i.e.,  $0.3\alpha = \ln(10)$

$\therefore \alpha = 7.676/\text{cm} \cdot \text{torr}$

7b

2. With respect to Paschen's Law, the given constants are:  $A=12$ ,  $B=365$ ,  $\gamma=0.02$  for air. Determine  $(pd)_{\min}$  and  $V_{b\min}$ .

**Solution:** We know that

$$(pd)_{\min} = \frac{eK}{A}$$

where

$$K = \ln(1 + 1/\gamma)$$

Therefore,  $(pd)_{\min} = \frac{e}{A} \ln(1 + 1/\gamma)$

Substituting the values, we have

$$(pd)_{\min} = \frac{2.718}{12} \ln(1 + 1/0.02) = 0.89 \quad \text{Ans.}$$

Now  $V_{b\min} = \frac{B}{A} eK = \frac{365}{12} \times 2.718 \ln 51 = 325 \text{ Volts} \quad \text{Ans.}$