# CBCS SCHEME

USN | | | | | | | | | |

**15MAT31**

## Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
## Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*





### Module-1

1 a. Obtain the Fourier expansion of the function f(x) = x over the interval $(-\pi, \pi)$. Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ **(08 Marks)**

b. The following table gives the variations of a periodic current A over a certain period T:

| t (sec) | 0 | T/6 | T/3 | T/2 | 2T/3 | 5T/6 | T |
|---|---|---|---|---|---|---|---|
| A (amp) | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |

Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the first harmonic. **(08 Marks)**

**OR**

2 a. Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0 \le x \le 2$. **(06 Marks)**

b. Represent the function

$$f(x) = \begin{cases} x, & \text{for } 0 < x < \pi/2 \\ \pi/2 & \text{for } \pi/2 < x < \pi \end{cases}$$

in a half range Fourier sine series. **(05 Marks)**

c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data:

| x° | 0 | 45 | 90 | 135 | 180 | 225 | 270 | 315 |
|---|---|---|---|---|---|---|---|---|
| y | 2 | 3/2 | 1 | 1/2 | 0 | 1/2 | 1 | 3/2 |

**(05 Marks)**

### Module-2

3 a. Find the complex Fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$ Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. If $\bar{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$ show that $u_0 = 0$ $u_1 = 0$ $u_2 = 2$ $u_3 = 11$. **(05 Marks)**

c. Obtain the Fourier cosine transform of the function

$$f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4 - x, & 1 < x < 4 \\ 0 & x > 4 \end{cases}$$ **(05 Marks)**

**OR**

4 a. Obtain the Z-transform of $\cos n\theta$ and $\sin n\theta$. **(06 Marks)**

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx \quad m > 0$. **(05 Marks)**

c. Solve by using Z-transforms $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = 0 = y_1$. **(05 Marks)**

## Module-3

5 a. Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at x = 6. **(06 Marks)**

| | | | | | |
|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 10 | 12 | 13 | 16 | 19 |

b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x | 1 | 3 | 4 | 2 | 5 | 8 | 9 | 10 | 13 | 15 |
| y | 8 | 6 | 10 | 8 | 12 | 16 | 16 | 10 | 32 | 32 |

**(05 Marks)**

c. Use Newton-Raphson method to find a real root of $x\sin x + \cos x = 0$ near $x = \pi$. Carryout the iterations upto four decimal places of accuracy. **(05 Marks)**

**OR**

6 a. Show that a real root of the equation $\tan x + \tanh x = 0$ lies between 2 and 3. Then apply the Regula Falsi method to find third approximation. **(06 Marks)**

b. Compute the coefficient of correlation and the equation of the lines of regression for the data:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

**(05 Marks)**

c. Fit a curve of the form $y = ae^{bx}$ for the data:

| | | | |
|---|---|---|---|
| x | 0 | 2 | 4 |
| y | 8.12 | 10 | 31.82 |

**(05 Marks)**

## Module-4

7 a. From the following table find the number of students who have obtained:
i) Less than 45 marks
ii) Between 40 and 45 marks.

| Marks | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|
| Number of students | 31 | 42 | 51 | 35 | 31 |

**(06 Marks)**

b. Construct the interpolating polgnomial for the data given below using Newton's general interpolation formula for divided differences and hence find y at x = 3.

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 2 | 4 | 5 | 6 | 8 | 10 |
| y | 10 | 96 | 196 | 350 | 868 | 1746 |

**(05 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by Weddle's rule. Taking seven ordinates. Hence find $\log_e 2$. **(05 Marks)**

**OR**

8 a. Use Lagrange's interpolation formula to find f(4) given below. **(06 Marks)**

| x | 0 | 2 | 3 | 6 |
|---|---|---|---|---|
| f(x) | -4 | 2 | 14 | 158 |

b. Use Simpson's 3/8th rule to evaluate $\int_1^4 e^{1/x}dx$. **(05 Marks)**

c. The area of a circle (A) corresponding to diameter (D) is given by

| D | 80 | 85 | 90 | 95 | 100 |
|---|---|---|---|---|---|
| A | 5026 | 5674 | 6362 | 7088 | 7854 |

Find the area corresponding to diameter 105 using an appropriate interpolation formula. **(05 Marks)**

## Module-5

9 a. Evaluate Green's theorem for $\phi_c(xy + y^2)\,dx + x^2dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(06 Marks)**

b. Find the extremal of the functional $\int_a^b \left(x^2 + y^2 + 2y^2 + 2xy\right)dx$. **(05 Marks)**

c. Varity Stoke's theorem for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ C is its boundary. **(05 Marks)**

**OR**

10 a. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y1}\right) = 0$. **(06 Marks)**

b. If $\vec{F} = 2xy\hat{i} + y^2z\hat{j} + xz\hat{k}$ and S is the rectangular parallelepiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3. Evaluate $\iint_S \vec{F}.\hat{n}\,ds$. **(05 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them or prove that the geodesics on a plane are straight lines. **(05 Marks)**

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