CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find modulus and amplitude of $1-\cos\theta+i\sin\theta$. **(05 Marks)**

b. Express $\dfrac{3+4i}{3-4i}$ in $a+ib$ form. **(05 Marks)**

c. Find the value of 'λ' so that the points A(−1, 4, −3), B(3, 2, −5), C(−3, 8, −5) and D(−3, λ, 1), may lie on one plane. **(06 Marks)**

OR

2 a. Find the angle between the vectors $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$. **(05 Marks)**

b. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$. **(05 Marks)**

c. Find the real part of $\dfrac{1}{1+\cos\theta+i\sin\theta}$. **(06 Marks)**

Module-2

3 a. Obtain the n^{th} derivative of $\sin(ax+b)$. **(05 Marks)**

b. Find the pedal equation of $r^n=a^n\cos n\theta$. **(05 Marks)**

c. If $u=\dfrac{yz}{x}$, $v=\dfrac{zx}{y}$, $w=\dfrac{xy}{z}$, show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}=4$. **(06 Marks)**

OR

4 a. If $u=\log\left(\dfrac{x^4+y^4}{x+y}\right)$ show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3$. **(05 Marks)**

b. If $u=f(x-y,\ y-z,\ z-x)$, show that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **(05 Marks)**

c. If $y=a\cos(\log x)+b\sin(\log x)$, show that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$ **(06 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi} x\sin^8 x\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1 x^2(1-x^2)^{3/2}\,dx$. **(05 Marks)**

c. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx$. **(06 Marks)**

OR

6 a. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z)\,dx\,dy\,dz$. **(05 Marks)**

c. Evaluate $\int_0^\infty \frac{x^4}{(1+x^2)^4}\,dx$. **(06 Marks)**

Module-4

7 a. If $\vec{r} = (t^2+1)\hat{i} + (4t-3)\hat{j} + (2t^2-6t)\hat{k}$, find the angle between the tangents at t = 1 and t = 2. **(05 Marks)**

b. If $\vec{r} = e^{-t}\hat{i} + 2\cos 3t\,\hat{j} + 2\sin 3t\,\hat{k}$, find the velocity and acceleration at any time t, and also their magnitudes at t = 0. **(05 Marks)**

c. Show that $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ is irrotational. Also find a scalar function 'φ' such that $\vec{F} = \nabla\phi$. **(06 Marks)**

OR

8 a. Find the unit normal vector to the surface $x^2y + 2xz = 4$ at (2, –2, 3). **(05 Marks)**

b. If $\vec{F} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$ find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ at (1, –1, 1). **(05 Marks)**

c. If $\frac{d\vec{a}}{dt} = \vec{w} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{w} \times \vec{b}$, then show that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{w} \times (\vec{a} \times \vec{b})$ **(06 Marks)**

Module-5

9 a. Solve $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$. **(05 Marks)**

b. Solve $(y^3 - 3x^2y)dx + (3xy^2 - x^3)dy = 0$. **(05 Marks)**

c. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$. **(06 Marks)**

OR

10 a. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(05 Marks)**

b. Solve $x^2y\,dx - (x^3 + y^3)dy = 0$ **(05 Marks)**

c. Solve $y(x+y)dx + (x+2y-1)dy = 0$ **(06 Marks)**

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