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10MAT11

First Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least TWO from each part.***

PART – A

1 a. Choose the correct answers for the following :

i) The n^{th} derivative of x^n is

A) 0 B) nx^{n-1} C) n D) n!

ii) n^{th} derivative of log (ax + b) is

A) $\dfrac{(-1)^n n! a^n}{(ax+b)^{n+1}}$ B) $\dfrac{(-1)^{n-1}(n-1)! a^n}{(ax+b)^n}$ C) $\dfrac{(-1)^n (n-1)! a^n}{(ax+b)^n}$ D) None of these

iii) If f(x) is continuous in [a, b], differentiable in (a, b) and f(a) = f(b) then there exist atleast one value $c \in (a, b)$ such that $f'(c)$ = ……

A) 0 B) 1 C) not equal to zero D) none of these

iv) Using Lagrange's mean value theorem for $f(x) = e^x$ in [0, 1], c = ……

A) log e B) 0 C) log (e-1) D) log (e+1)

(04 Marks)

b. If $y = \dfrac{\sinh^{-1}x}{\sqrt{1+x^2}}$ prove that $(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2 y_n = 0$. **(06 Marks)**

c. State and prove Cauchy's mean value theorem. **(05 Marks)**

d. Using Maclaurin's theorem expand log (secx) in ascending powers of x upto the first three non-vanishing terms. **(05 Marks)**

2 a. Choose the correct answers for the following :

i) $\lim\limits_{x \to 0}\left(\dfrac{1-\cos x}{x^2}\right)$ = ...

A) 1 B) 0 C) $\dfrac{1}{2}$ D) 2

ii) The angle between the radius vector and tangent for the curve $r = ae^{\theta\cot\alpha}$ is

A) α B) 0 C) 1 D) $\dfrac{1}{2}$

iii) The radius of curvature of the catenary y = c cosh (x/c) at the point where it crosses the y-axis is

A) $\dfrac{1}{c}$ B) 0 C) 1 D) C

iv) If $x^3 + y^3 - 3axy = 0$ then $\dfrac{dy}{dx}$ at $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ = ...

A) 0 B) -1 C) 1 D) None of these

(04 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Evaluate: $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{1/x}$ **(06 Marks)**

c. Find the pedal equation for the curve $r \sin^2\left(\frac{\theta}{2}\right) = a$ **(05 Marks)**

d. For the curve $y = \frac{ax}{a+x}$ show that $r \sin^2\left(\frac{\theta}{2}\right) = a\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$ **(05 Marks)**

3 a. Choose the correct answers for the following :

i) If u = f(y/x) then

A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ B) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$ D) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

ii) $\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x + \sqrt{y}}}$ is a homogeneous function of degree

A) $\frac{1}{2}$ B) 1 C) $-\frac{1}{2}$ D) 0

iii) Taylor's expansion of $e^x \log(1 + y)$ about the origin is

A) $y + xy - \frac{y^2}{2}$ B) $y - xy + \frac{y^2}{2}$ C) $y + xy + \frac{y^2}{2}$ D) None of these

iv) If an error of 1% is made in measuring its length and breadth the percentage error in the area of a rectangle is

A) 0.2% B) 0.02% C) 2% D) 1% **(04 Marks)**

b. If $u = x \log(xy)$ where $x^3 + y^3 + 3xy = 1$ find $\frac{du}{dx}$. **(06 Marks)**

c. If x = a cos h u cos v, y = a sin h u sin v prove that $\frac{\partial(x,y)}{\partial(u,v)} = \frac{a^2}{2}[\cosh 2u - \cos 2v]$. **(05 Marks)**

d. Find the extreme values of $x^3y^2(1-x-y)$. **(05 Marks)**

4 a. Choose the correct answers for the following :

i) If $\vec{F}$ is solenoidal then $\nabla \cdot F$ = ...

A) -1 B) 0 C) 1 D) 2

ii) Curl (grad ϕ) = ...

A) $\nabla\phi$ B) $\nabla x \phi$ C) $\nabla.\phi$ D) 0

iii) If $\vec{r} = xi + yj + zk$ then ∇ log r = ...

A) $\frac{1}{r}$ B) $-\frac{\vec{r}}{r^2}$ C) $\frac{1}{r^2}$ D) $\frac{\vec{r}}{r^2}$

iv) Physical interpretation of $\nabla\phi$ is that

A) it gives the maximum rate of change of ϕ

B) it gives the minimum rate of change of ϕ

C) 0

D) None of these **(04 Marks)**

b. Prove that $\nabla r^n = nr^{n-2}\,\vec{r}$ where $\vec{r} = xi + yj + 2k$ **(06 Marks)**

c. Find constants a, b, c such that $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational. **(05 Marks)**

d. Express Curl $\vec{A}$ in orthogonal curvilinear coordinates. **(05 Marks)**

PART – B

5 a. Choose the correct answers for the following :

i) $\int_0^{\pi/2} \cos^6 x \, dx = ...$

A) 0 B) $\frac{5K}{16}$ C) $\frac{5}{16}$ D) $\frac{3}{16}$

ii) Asymptote to the curve $y^2(a - x) = x^2(a + x)$ $(a > 0)$ is

A) $x = a$ B) $x = -a$ C) $x = \pm a$ D) No asymptote

iii) The curve $r = a \sin 3\theta$ is symmetrical about

A) initial line B) pole C) $\theta = \frac{\pi}{2}$ D) None of these

iv) The volume of the solid generated by the revolution of the curve $y = f(x)$ between $x = a$ and $x = b$ about the x-axis is given by

A) $\int_a^b \pi x^2 dy$ B) $\int_a^b \pi x^2 dx$ C) $\int_a^b \pi y^2 dy$ D) $\int_a^b \pi y^2 dx$

(04 Marks)

b. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$ $(\alpha \geq 0)$ **(06 Marks)**

c. Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$. **(05 Marks)**

d. The cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, $0 \leq \theta \leq 2\pi$ rotates about its base. Find the volume of the solid generated. **(05 Marks)**

6 a. Choose correct answers for the following :

i) The order and degree of the differential equation $[1 + (y')^2]^{3/2} = c\, y''$

A) 1 and 3 B) 3/2 and 2 C) 3/2 and 1 D) 2 and 2

ii) The integrating factor of $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$ is

A) $e^{\tan^{-1} x}$ B) $e^{\tan^{-1} y}$ C) $\tan^{-1} x$ D) $\tan^{-1} y$

iii) The integrating factor of $M\,dx + N\,dy = 0$ if $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$ is

A) $\int f(x)dx$ B) $\int f(y)dy$ C) $e^{\int f(x)dy}$ D) $e^{\int f(x)dx}$

iv) Orthogonal trajectory of the hyperbola $xy = c^2$ is

A) $x^2 - y^2 = c$ B) $x^2 + y^2 = c$ C) $x^2 = cy^2$ D) $x = cy^2$

(04 Marks)

b. Solve: $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$ **(06 Marks)**

c. Solve: $x\,dx = y\,(x^2 + y^2 - 1)\,dy$ **(05 Marks)**

d. Find the orthogonal trajectories of the family of curves $r = 4a \sec\theta \tan\theta$. **(05 Marks)**

7 a. Choose the correct answers for the following :

i) If $3x + 2y + z = 0$, $x + 4y + z = 0$, $2x + y + 4z = 0$ be a system of equations then
  A) it is consistent B) it has only the trivial solution $x = 0, y = 0, z = 0$
  C) it can be reduced to a single equation and so a solution does not exist
  D) the determinant of the matrix of coefficient is zero.

ii) Matrix has a value. This statement
  A) is always true B) depends upon the matrices
  C) false D) None of these

iii) The rank of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ is
  A) 2 B) 1 C) 3 D) 0

iv) A is a square matrix such that $AA' = I$ then value of $A'A$ is
  A) A^2 B) A^{-1} C) O D) I **(04 Marks)**

b. For the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ find non-singular matrices P and Q such that PAQ in the normal form. Hence find the rank of A. **(06 Marks)**

c. Test for consistency the following system of equations and if consistent solve them
$x_1 + 2x_2 - x_3 = 3$, $3x_1 - x_2 + 2x_3 = 1$, $2x_1 - 2x_2 + 3x_3 = 2$, $x_1 - x_2 + x_3 = -1$. **(05 Marks)**

d. Solve by Gauss elimination method
$5x_1 + x_2 + x_3 + x_4 = 4$, $x_1 + 7x_2 + x_3 + x_4 = 12$
$x_1 + x_2 + 6x_3 + x_4 = -5$, $x_1 + x_2 + x_3 + 4x_4 = -6$ **(05 Marks)**

8 a. Choose the correct answers for the following :

i) A square matrix A is called orthogonal if
  A) $A = A^2$ B) $AA^{-1} = I$ C) $A^1 = A^{-1}$ D) None of these

ii) The two eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15 then the third eigen value is
  A) 0 B) 3 C) 8 D) 7

iii) A homogeneous expression of the second degree in any number of variables is called
  A) Cubic form B) Linear form C) Quadratic form D) None of these

iv) A square matrix A of order n is similar to a square matrix B of order n if
  A) AB = Null matrix B) $A = P^{-1}BP$ C) AB = unit matrix D) None of these **(04 Marks)**

b. Find all the eigen values and the eigen vector corresponding largest eigen value of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ **(06 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form and hence find A^4. **(05 Marks)**

d. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ to canonical form. **(05 Marks)**

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