USN | | | | | | | | | | 10MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Find the Fourier series of the function $f(x) = x$ over the interval $(-\pi, \pi)$. **(06 Marks)**

b. Expand $f(x) = (x-1)^2$ as a half range cosine series in the interval (0, 1). Hence show that $1+\frac{1}{2^2}+\frac{1}{3^2}+---=\frac{\pi^2}{6}$. **(07 Marks)**

c. For the function specified by the following table, find the Fourier series upto first harmonic.

θ°	0	60	120	180	240	300	360
f(θ)	0.8	0.6	0.4	0.7	0.9	1.1	0.8

(07 Marks)

2 a. Find the Fourier transform of :

$$f(x) = 1-x^2, \quad |x| \le 1$$
$$= 0, \quad |x| > 1$$ **(06 Marks)**

b. Find the inverse Fourier sine transform of $f_s(s) = \frac{e^{-as}}{s}$. **(07 Marks)**

c. Obtain Fourier cosine transform of

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}.$$ **(07 Marks)**

3 a. Obtain D'Alembert's solution of one dimensional wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$ **(06 Marks)**

b. Solve the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < \ell$ under the conditions $u(0, t) = 0$, $u(\ell, t) = 0$, $u(x,0) = \frac{kx}{\ell}$ where k is positive constant. **(07 Marks)**

c. Obtain all possible solutions of Laplace equation : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by the method of separation of variables. **(07 Marks)**

4 a. Fit a curve $y = ax + b$ by the least squares method to the data :

x	0	1	3	6	8
y	1	3	2	5	4

(06 Marks)

b. Solve the following LPP by graphical method :
Min $z = 20x_1 + 10x_2$
Subject to $x_1 + 2x_2 \le 40$
$3x_1 + x_2 \ge 30$
$4x_1 + 3x_2 \ge 60$
$x_1 \ge 0, x_2 \ge 0.$ (07 Marks)

c. Solve by Simplex Method :
Max $z = 5x_1 + 3x_2$
Subject to $x_1 + x_2 \le 2$
$5x_1 + 2x_2 \le 10$
$3x_1 + 8x_2 \le 12$
$x_1, x_2 \ge 0.$ (07 Marks)

PART – B

5 a. Using Regula Falsi Method, find the root of the equation $xe^x = 2$ that lies between 0.8 and 0.9 correct to three decimal places. (06 Marks)

b. Solve by Relaxation Method :
$10x - 2y - 2z = 6$
$-x + 10y - 2z = 7$
$-x - y + 10z = 8.$ (07 Marks)

c. Find the largest eigenvalue and the corresponding eigenvector of the matrix :

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

by power method, taking initial vector $[1 \ 0 \ 0]'$. Perform 4 iterations. (07 Marks)

6 a. Using Newton's backward interpolation formula find f(13.2) given that :

x	10	11	12	13
f(x)	21	23	27	33

(06 Marks)

b. Apply Lagrange's formula to find the value of x corresponding to f(x) = 15 from the data:

x	5	6	9	11
f(x)	12	13	14	16

(07 Marks)

c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by using Simpson's $1/3^{rd}$ rule and taking six equal parts. Hence find the value of $\log_e^2$. (07 Marks)

7 a. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ at the interior points of the square region shown in the Fig.Q7(a) **(06 Marks)**

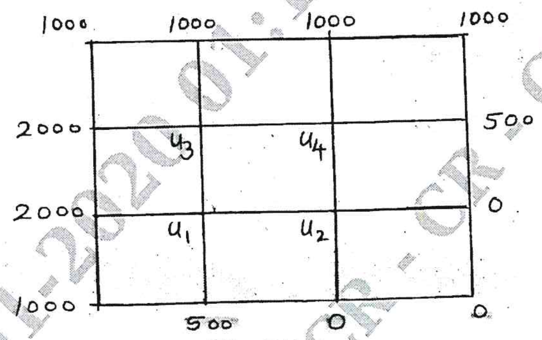

Fig.Q7(a)

b. Obtain the numerical solution of heat equation $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$ under the condition $u(0, t) = u(4, t) = 0, t \geq 0$ and $u(x, 0) = x(4 - x), 0 < x < 4$ with $h = 1$ and $0 \leq t \leq 1, k = 1/4$. **(07 Marks)**

c. By using the explicit three level formula solve the wave equation $\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}$ subject to the boundary conditions $u(0, t) = u(4, t) = 0, t \geq 0$, and initial conditions $u(x, 0) = x(4 - x)$, $\frac{\partial u}{\partial t}(x, 0)=0, 0 \leq x \leq 4$. Take $h = 1, k = ½$ and $0 < t \leq 2$. **(07 Marks)**

8 a. Prove that $Z(n^p) = -z\frac{d}{dz}Z(n^{p-1})$ where p is positive integer. **(06 Marks)**

b. Find : i) $Z(n + 1)$ ii) $Z[\sin(3n + 5)]$. **(07 Marks)**

c. Using z–transform solve the difference equation : $u_{n+2}-3u_{n+1}+2u_n=0$, given $u_0 = 0$, $u_1 = -1$. **(07 Marks)**

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