



**Third Semester B.E. Degree Examination, Dec.2019/Jan.2020**  
**Advanced Mathematics – I**

Time: 3 hrs.

Max. Marks: 100

Note: Answer FIVE full questions.

- 1 a. Express  $\frac{(2+3i)^2}{(1+i)^2}$  in the form of complex number  $a + ib$ . (06 Marks)
- b. Prove that  $(1+i)^4 + (1-i)^4 = -8$ . (07 Marks)
- c. Find the cube root of  $(\sqrt{3} - i)$ . (07 Marks)
- 2 a. Find  $n^{\text{th}}$  derivative of  $\sin(ax + b)$ . (06 Marks)
- b. If  $y = a \cos(\log x) + b \sin(\log x)$ . Show that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ . (07 Marks)
- c. Find  $n^{\text{th}}$  derivative of  $\log \left( \frac{2x+3}{2-3x} \right)^{\frac{1}{10}}$ . (07 Marks)
- 3 a. Find the angle between the curves.  $r = a(\sin \theta + \cos \theta)$  and  $r = 2a \cos \theta$ . (06 Marks)
- b. Find the pedal equation for the curve  $r^2 = a^2 \sec(2\theta)$ . (07 Marks)
- c. Expand  $y = \text{Log}(\cos x)$  using Maclaurin's series upto 4<sup>th</sup> degree term. (07 Marks)
- 4 a. If  $u = \sin^{-1} \left[ \frac{x^3 + y^3 + z^3}{ax + by + cz} \right]$  show that  $xu_x + yu_y + zu_z = 2 \tan u$ . (06 Marks)
- b. If  $u = f(r, s, t)$  where  $r = \frac{x}{y}$ ,  $s = \frac{y}{z}$ ,  $t = \frac{z}{x}$ . Find  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ . (07 Marks)
- c. If  $u = x + y + z$ ,  $v = y + z$ ,  $w = z$  find  $\frac{\partial(uvw)}{\partial(xyz)}$ . (07 Marks)
- 5 a. Obtain reduction formula for  $\int \sin^n x \, dx$  where  $n$  is a positive integer. (06 Marks)
- b. Evaluate:  $\int_0^1 x^9 \sqrt{1-x^2} \, dx$ . (07 Marks)
- c. Evaluate:  $\int_0^1 \int_0^2 (x^2 + y^2) \, dx \, dy$ . (07 Marks)
- 6 a. Evaluate:  $\int_0^1 \int_0^2 \int_0^2 x^2 y z \, dx \, dy \, dz$ . (06 Marks)
- b. Prove that  $\beta(m, n) = \beta(n, m)$ . (07 Marks)
- c. Evaluate:  $\int_0^2 \frac{x^2}{\sqrt{2-x}} \, dx$ . (07 Marks)

- 7 a. Solve  $\frac{dy}{dx} = e^{-y}(e^x + x^2)$ . (06 Marks)
- b. Solve  $(x^2 + y^2)dx = 2xy dy$ . (07 Marks)
- c. Solve  $\frac{dx}{dy} = \frac{x}{y} + 2y^2$ . (07 Marks)
- 8 a. Solve  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^x$ . (06 Marks)
- b. Solve  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \cos x$ . (07 Marks)
- c. Solve  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 12x^2$ . (07 Marks)

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