SCHEME OF EVALUATION
2017-18 EVEN SEMESTER
IAT-1

1.a. {2 + 2.5 + 2.5}

Max Planck developed a structural model for black body radiation that leads to a theoretical equation for the wavelength distribution that is in complete agreement with the experimental results at all wavelengths.

According to his theory

1. a black body is imagined to be consisting of large number of electrical oscillators.
2. an oscillator emits or absorbs energy in discrete units. It can emit or absorb energy by making a transition from one quantum state to another in the form of discrete energy packets known as photons whose energy is an integral multiple of hv where h is the planks constant and v is the frequency.
3. Energy emitted per unit volume per unit energy range is given by the product of number of modes of vibration in the given energy range and the energy per mode.The Energy density per unit wavelength range per unit volume is given by

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]}-1}\right]d\lambda$$

Where h is Planck's constant, c is velocity of light, T is absolute temperature, λ is the wavelength and k is Boltzmann constant

Deduction of Weins law:

It is applicable at smaller wavelengths.

For smaller wavelengths $e^{\frac{h\gamma}{kT}} >> 1$

$$\therefore e^{\frac{h\gamma}{kT}} >> 1 = e^{\frac{h\gamma}{kT}}$$

So Planck's radiation law becomes

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]}}\right]$$

Deduction of Rayleigh Jeans Law:

It is applicable at longer wavelengths.

For longer wavelengths $\frac{h\gamma}{kT} << 1$

$$\therefore e^{\frac{h\gamma}{kT}} = 1 + \frac{h\gamma}{kT} + \left(\frac{h\gamma}{kT}\right)^2 \frac{1}{2|} + \ldots\ldots\ldots\ldots = 1 + \frac{h\gamma}{kT}$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\cdot\frac{1}{1+\frac{h\gamma}{kT}-1}d\lambda = \frac{8\pi kT}{\lambda^4}d\lambda$$

1. b. {1+2}

From Weins displacement law

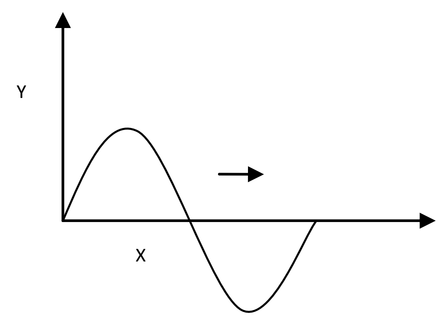
$$\lambda_{max}.T = 2.89x10^{-3}\,mK$$

Given $\lambda_{max}. = 490nm$;

$$.T = 2.89x10^{-3}/490X10^{-9} = 5897K$$

2.a. {1+1+4}

Phase velocity(V_p): It is the speed with which an isolated pulse / constant phase propagates in a medium.



A single pulse is shown in this diagram .It is represented as

Y = A sin [wt – kx]

where Y is the displacement of a particle at a distance 'x' from the origin at a time 't', A is the amplitude , w is the angular velocity and k is the wave number.

Imagine two points A and B at X_1 and X_2 at same phase on the wave. Then

$(wt_1-kX_1) = (wt_2-kX_2)$

$$\frac{w}{k} = \frac{x_1 - x_2}{t_1 - t_2}$$

$$\therefore V_p = \frac{w}{k}$$

Group Velocity(V_g): It is the velocity with which the resultant envelope of varying amplitude formed by the superposition of two or more waves propagates .

$$V_g = \underset{dk\to 0}{Lt}\frac{dw}{dk}$$

Relation between phase velocity and group velocity:

We have phase velocity $V_p = \frac{w}{k}$

$$V_g = \underset{dk\to 0}{Lt}\frac{dw}{dk} = \frac{d}{dk}(kV_p) = V_p + K.\frac{dV_P}{dk} \quad \ldots\ldots\ldots (1)$$

But $\frac{dV_P}{dk} = \frac{dV_p}{d\lambda} \times \frac{d\lambda}{dk}$ and $k = \frac{2\pi}{\lambda}$; $\frac{dk}{d\lambda} = \frac{-2\pi}{\lambda^2}$

Substituting these in equation (1) we get

$$V_g = V_p - \lambda \frac{dV_P}{d\lambda}$$

2.b. {1+1+2}

Waves associated with the moving particles are called matter waves

Debroglie wavelength

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.62X10^{-34}}{\sqrt{2X9.1X10^{-31}X1.6X10^{-19}X100000}}$$

$$\lambda = 0.388x10^{-11}m$$

3.a. {7}

To Show that electron does not exist inside the nucleus:

We know that the diameter of the nucleus is of the order of 10^{-15}m.If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

$$\Delta x \leq 10^{-14} m$$

Now the uncertainty in momentum is

$$\Delta p \geq \frac{h}{4\pi.\Delta x}$$

$$\Delta p \geq \frac{6.62 \times 10^{-34}}{4\pi \times 10^{-15}}$$

$$\Delta p \geq 0.5 \times 10^{-20} Ns$$

Then the momentum of the electron can at least be equal to the uncertainty in momentum.

$$p \geq 0.5 \times 10^{-20} Ns$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by - for small velocities (non-relativistic) case- For high velocities,

$$E = \left(\frac{1}{\sqrt{1 - \left(\frac{v^2}{c^2} \right)}} \right) m_o c^2 \qquad \text{..........(1)}$$

$$E^2 = \left(\frac{1}{1 - \left(\frac{v^2}{c^2} \right)} \right) m^2{}_o c^4$$

Momentum $p = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right) m_o v$

$$p^2 = \left(\frac{1}{1 - \left(\frac{v^2}{c^2} \right)} \right) m^2{}_o v^2$$

$$p^2 c^2 = \left(\frac{1}{1 - \left(\frac{v^2}{c^2} \right)} \right) m^2{}_o v^2 c^2 \qquad \text{..............................(2)}$$

From (1) and (2) $E^2 = p^2c^2 + m^2{}_o c^4$

On substitution

$$E = (9.1x10^{-31})^2(3x10^8)^2 + (0.5x10^{-19})^2(3x10^8)^4 \quad = 9.4 MeV$$

This relativistic expression for energy yields a value of 9.4 Mev. The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of decay of nucleus

($n \rightarrow p + e + \bar{\nu}$ / $p \rightarrow n + e^{+} + \nu$).

3.b. {1+2}

The required expression is

$$(\Delta E).(\Delta t) \geq \frac{h}{4\pi}$$

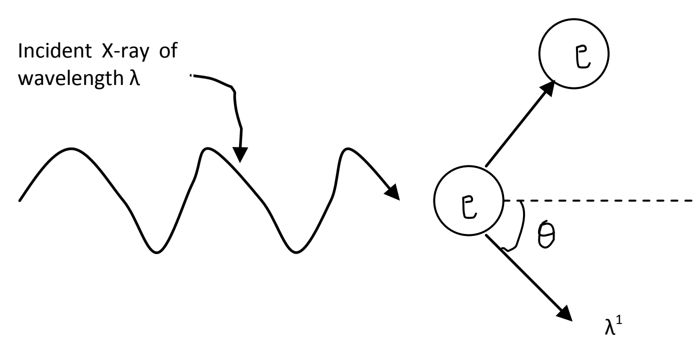
$$h.\Delta f.\Delta t = \frac{h}{4\pi}$$

Given $\Delta t = 10X10^{-6} s$

$$\Delta f = 0.79X10^4 \text{ Hz}$$

4.a. **Compton Effect:** {1+3}

This effect deals with the interaction between X rays and electrons in an atom. When x rays are incident on an electron, the scattered radiation will have the wavelength equal to or greater than the incident wavelength.



Change in wavelength of x ray after scattering =

$$d\lambda = \lambda^1 - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

Here h is plancks constant

m_0 is rest mass of electron

c is velocity of light

θ is angle of scattering

Change in wavelength of x ray after Compton scattering is given by

$$d\lambda = \lambda^1 - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Here , when θ=180^0,
Wavelength of scattered photon

$$\lambda^1 = \lambda + \frac{h}{m_0 c}(1-(-1)) = 1.548\,\mathring{A}$$

4.b. {6}

Time independent Schrödinger equation

A matter wave can be represented in complex form as

$$\Psi = Ae^{i(wt-kx)}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = -ikA\psi$$

$$\frac{d^2\Psi}{dx^2} = -k^2\Psi \quad \text{........................ (1)}$$

From debroglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\Pi p}{h}$$

$$k^2 = 4\Pi^2 \frac{p^2}{h^2} \quad \text{.......................... (2)}$$

Total energy of a particle E = Kinetic energy + Potential Energy

$$E = \frac{1}{2} m v^2 + V$$

$$E = \frac{p^2}{2m} + V$$

$$p^2 = (E-V)2m$$

Substituting in (2)

$$k^2 = \frac{4\Pi^2(E-V)2m}{h^2}$$

∴ From (1)

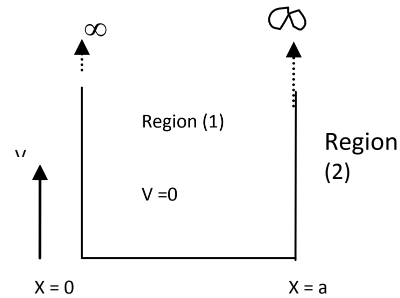
$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m(E-V)\Psi}{h^2} = 0}$$

(For one dimension)

5.a. {7}

Particle in an infinite potential well problem:

Consider a particle of mass m moving along X-axis in the region from X=0 to X=a in a one dimensional potential well as shown in the diagram. The potential energy is zero inside the region and infinite outside the region.



Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \qquad \because V = 0$$

But $k^2 = \frac{8\Pi^2 mE}{h^2}$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

The general solution to this expression is given by

$$\Psi = C\cos kx + D\sin kx$$

At x=0, $\Psi = 0 \quad \therefore C = 0$

At x=a, $\Psi = 0$ D sin ka = 0 $\Rightarrow$ ka = nΠ where n = 1, 2 3∴

$$\Psi = D\sin\left(n\frac{\Pi}{a}\right)x$$

$$E = \frac{n^2 h^2}{8ma^2}$$

To evaluate the constant D:

Normalisation : For one dimension

$$\int_0^a \Psi^2 dx = 1$$

$$\int_0^a D^2 \sin^2\left(\frac{n\Pi}{a}\right)x dx = 1$$

But $\cos 2\theta = 1 - 2\sin^2\theta$

$$\int_0^a D^2 \frac{1}{2}\left(1-\cos 2\left(\frac{n\Pi}{a}\right)x\right)dx = 1$$

$$\int_0^a \frac{D^2}{2}dx - \int_0^a \frac{1}{2}\cos 2\left(\frac{n\Pi}{a}\right)x)dx = 1$$

$$\frac{D^2 a}{2} - \quad [\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}]_0^a = 1$$

$$D^2 \frac{a}{2} - 0 = 1$$

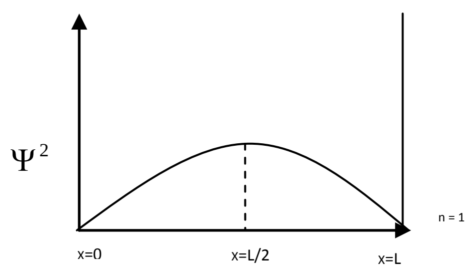
D = $\sqrt{\frac{2}{a}}$

$$\therefore \Psi_n = \sqrt{\frac{2}{a}} \sin\left(n\frac{\Pi}{a}\right)x$$

For n = 1, First state

$$\therefore \Psi_1 = \sqrt{\frac{2}{a}} \sin\left(1.\frac{\Pi}{a}\right)x$$

The graph of Ψ^2 $versus$ x is shown below.

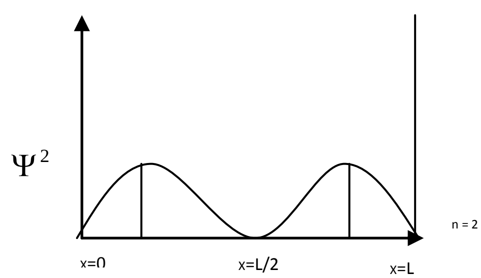


It is seen from the graph that probability density is maximum at the centre for the particle in the first state.

For n = 2, Second state

$$\therefore \Psi_2 = \sqrt{\frac{2}{a}} \sin\left(2.\frac{\Pi}{a}\right)x$$

The graph of Ψ^2 $versus$ x is shown below.



It is seen from the graph that probability density is maximum at x = L / 4 and x = 3L/4 for the particle in the second state.

5.b. {1+1+1}

Energy of a particle in an infinite potential well E = $\frac{n^2 h^2}{8ma^2}$

Ground state corresponds to n= 1

$$E = \frac{1^2 h^2}{8.(9.1x10^{-31}).(1x10^{-9})^2} = 0.6x10^{-19} J = 0.376eV$$

For Second excited state , n = 3

$$E = \frac{3^2 h^2}{8.(9.1x10^{-31}).(1x10^{-9})^2} = 5.4X10^{-19} J = 3.375eV$$

6.a. {2+2+2}

Success of quantum theory:

1. Specific heat:

Classical theory predicted high values of specific heat for metals on the basis of the assumption that all the conduction electrons are capable of absorbing the heat energy as per Maxwell -

Boltzmann distribution i.e., $C_V = \frac{3}{2}kT$

But according to the quantum theory, only those electrons occupying energy levels close to Fermi energy (E_F) are capable of absorbing heat energy to get excited to higher energy levels. Thus only a small percentage of electrons are capable of receiving the thermal energy and specific heat value becomes small.

It can be shown that $C_V = (2k/E_f)\, RT = 10^{-4} R\,T$.

This is in conformity with the experimental values.

2. **Temperature dependence of electrical conductivity.**

According to classical free electron theory,

Electrical conductivity $\propto \frac{1}{\sqrt{Temperature}}$

Where as from quantum theory

Electrical conductivity

$$\propto \frac{1}{collisional\ area\ of\ cross\ section\ of\ lattice\ atoms}$$

$$\alpha \frac{1}{vibrational\ energy}$$

$$\alpha \frac{1}{Temperature}$$

This is in agreement with experimental values.

3. **Dependence of electrical conductivity on electron concentration:**

According to classical theory,

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto n$$

But it has been experimentally found that Zinc which is having higher electron concentration

than copper has lower Electrical conductivity.

According to quantum free electron theory,

Electrical conductivity $\sigma = \frac{ne^2}{m}\left(\frac{\lambda}{V_F}\right)$ where V_F is the Fermi velocity.

Zinc possesses lesser conductivity because it has higher Fermi velocity.

6.b. {1+1+2}

It represents the probability of occupation of an energy state.

Fermi probability factor

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)} + 1}$$

For E- E_F=0.02eV=0.02x1.6x10^{-19}J ;

T=300K

f(E) =0.32

7.a. {7}

Expression for Electrical conductivity:

Imagine a conductor across which an electric field E is applied. The equation of motion for an electron moving under the influence of external field is given by

F= dp/dt = eE

Let the wave number change from k_1 to k_2 in time interval τ_F in the presence of electric field.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{2\pi p}{h}$$

$$p = \frac{hk}{2\pi}$$

$$\frac{dp}{dt} = \frac{h}{2\pi}\left(\frac{dk}{dt}\right)$$

$$dk = \frac{2\pi}{h} eEdt$$

On integration $k_2 - k_1 = \Delta k = \frac{2\pi . eE . \tau_F}{h}$ ....(1)

From quantum theory, conductivity J = $\Delta k . ne . \frac{h}{2\pi . m}$ ....(2)

Substituting (1) in (2)

We get J = $\frac{ne^2 \tau_F}{m^*} E$ ...(3)

Since from Ohm's, J = σE, conductivity σ can be written as

$$\sigma = \frac{ne^2}{m^*}\frac{\lambda}{v_F} \quad \textbf{where} \quad \tau = \frac{\lambda}{v_F}$$

7.b. {1+1+1}

Classical free electron theory:(Drude – Lorentz theory)

Postulates:

1. A metal is assumed to possess a three dimensional array of ions in between which there are freely moving valence electrons confined to the metallic boundary.
2. These free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic energy of gases. In the absence of any electric field the energy associated with electrons is equal to

Kinetic energy = $\frac{3}{2}kT$

3. The electric current in a metal is due to the drift of electrons in a direction opposite to Electric field.

4. The electric field due to all the ions is assumed to be constant.