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CMRIT — CMR INSTITUTE OF TECHNOLOGY, BENGALURU. ACCREDITED WITH A+ GRADE BY NAAC

Internal Assessment Test I

Sub	ENGINEERING MATHEMATICS IV			Code	15MAT41	
Date 12 / 03 / 2018	Duration 90 mins	Max Marks 50	Sem IV	Branch	EC B, EE B	
	Question 1 is compulsory. Answer any SIX questions from the rest.			Marks	OBE	
	REGULAR				CO	RBT
1.	Use Adams-Bashforth method to find y(0.3) from $y' = x^2 + y^2, y(0) = 1$, after computing $y(-0.1), y(0), y(0.1)$ *and* $y(0.2)$ by Taylor's series method. Apply the corrector formula once.			08	1	L3
2.	Using Milne method solve the differential equation $y'' + y' = 2e^x$ for y and y' at x = 0.4 given that y(0) = 2, y(0.1) = 2.01, y(0.2) = 2.04, y(0.3) = 2.09 y'(0) = 0, y'(0.1) = 0.2, y'(0.2) = 0.4, y'(0.3) = 0.6			07	1	L3
3.	Given that x satisfies the differential equation $x''(t) - t\,x'(t) + 4x = 0$ and the initial conditions x = 3 and $x'(t) = 0$ at t = 0. Obtain the values of x and $x'(t)$ at t = 0.1, using Runge-Kutta method. (Take h = 0.1)			07	1	L3
4.	Derive the Cauchy-Riemann equations in polar form.			07	3	L4

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5.	Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The DE is $y' = -ky$ where k = 0.01, $t_0 = 0$, $y_0 = 100$. Determine how much substance will remain at the moment t = 100 sec. Find the solution by modified Euler method taking h = 50.(Do 2 stages and use Modified Euler formula once)	07	1	L4
6.	Find the analytic function $f(z)$ given $v = (r - \frac{1}{r}\sin\theta)$ $r \neq 0$	07	3	L3
7.	If $u = x^2/y$, $y \neq 0$ and $v = x^2 + 2y^2$, show that the curves u = constant and v = constant are orthogonal but $f(z)$ *is not an analytic function.*	07	3	L3
8.	If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$ find $f(z)$ in terms of z by Milne Thomson method.	07	3	L3
9.	An electrostatic field in the xy plane is given by the potential function $\phi = 3x^2y - y^3$. Show that ϕ is harmonic and find the complex function $w = \phi + i\psi$.	07	3	L3

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15MAT41 IAT 1 Solution

1. $y' = x^2 + y^2$, $y(0) = 1$

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!}y'(0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \cdots$$

$$y(x) = y(0) + \frac{(x-0)}{1!}y'(0) + \frac{(x-0)^2}{2!}y''(0) + \frac{(x-0)^3}{3!}y'''(0) + \cdots$$

$$y(x) = y(0) + \frac{x}{1!}y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \cdots$$

$y' = x^2 + y^2$ $\qquad y'(0) = 0^2 + 1^2 = 1$

$y'' = 2x + 2yy'$

$y''(0) = 2(0) + 2(0)(1) = 2$

$y''' = 2 + 2(y'y' + yy'') = 2 + 2(y'^2 + yy'')$

$y'''(0) = 2 + 2(1.1 + 1.2) = 8$

$y'''' = 2\{2y'y'' + y'y'' + yy'''\}$

$y''''(0) = 2\{2.1.2 + 1.2 + 1.8\}$

$= 2(14) = 28$

Taylor's Series

$$y(x) = 1 + \frac{x}{1!}(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(8) + \frac{x^4}{4!}(28) + \cdots$$

$$y(-0.1) = 1 - 0.1 + (-0.1)^2 + \frac{4}{3}(-0.1)^3 + \frac{7}{6}(0.1)^4$$

$$= 1 - 0.1 + 0.01 - \frac{4}{3}(.001) + \frac{7}{6}(.0001)$$

$$= 0.91 - \frac{0.004}{3} + \frac{0.0007}{6}$$

$$= 0.91 - 0.0013 + 0.00012$$

$$= 0.9088$$

$$y(0) = 1$$

$$y(0.1) = 1 + 0.1 + (0.1)^2 + \frac{4}{3}(0.1)^3 + \frac{7}{6}(0.1)^4$$

$$= 1.1114$$

$$y(0.2) = 1 + 0.2 + (0.2)^2 + \frac{4}{3}(0.2)^3 + \frac{7}{6}(0.2)^4$$

$$= 1.2 + 0.04 + \frac{4}{3}(0.008) + \frac{7}{6}(.0016)$$

$$= 1.24 + 0.0107 + 0.0019$$

$$= 1.2526$$

3M

x	y	$y' = x^2 + y^2$
x_0 -0.1	y_0 0.9088	0.8359
x_1 0	y_1 1	1
x_2 0.1	y_2 1.1114	1.2453
x_3 0.2	y_3 1.2526	1.6088
0.3	?	

$$y_4^p = y_3 + \frac{h}{24}\left(55f_3 - 59f_2 + 37f_1 - 9f_0\right)$$

$$= 1.2526 + \frac{0.1}{24}\left\{55(1.6088) - 59(1.2453) + 37(1) - 9(.8359)\right\}$$

$$= 1.2526 + 0.1854 = \underline{1.438}$$

$$f(x_4, y_4^p) = f(0.3, 1.438)$$

$$= (0.3)^2 + (1.438)^2 = 0.09 + 2.0678$$

$$= 2.1578$$

$$y_4^C = y_3 + \frac{h}{24}\left(f_1 - 5f_2 + 19f_3 + 9f_4^p\right)$$

$$= 1.2526 + \frac{0.1}{24}\left\{1 - 5(1.2453) + 19(1.6088) + 9(2.1578)\right\}$$

$= 1.2526 + \frac{0.1}{24}$

4 M

$1.2526 + 0.1865 = 1.4391$

Ans $\underline{1.4391}$

1 M

2. Let $z = \frac{dy}{dx}$

$y'' + y' = 2e^{x}$

$z' + z = 2e^{x}$

2 M

$z' = 2e^{x} - z$

$x_0 = 0 \quad x_1 = 0.1 \quad x_2 = 0.2 \quad x_3 = 0.3$

$y_0 = 2 \quad y_1 = 2.01 \quad y_2 = 2.04 \quad y_3 = 2.09$

$z_0 = 0 \quad z_1 = 0.2 \quad z_2 = 0.4 \quad z_3 = 0.6$

$z_0' = 2e^{x} - z_0 = 2e^{0} - 0 = 2$

$z_1' = 2e^{x_1} - z_1 = 2e^{0.1} - 0.2 = 2.0104$

$z_2' = 2e^{x_2} - z_2 = 2e^{0.2} - 0.4 = 2.0428$

$z_3' = 2e^{x_3} - z_3 = 2e^{0.3} - 0.6 = 2.0998$

Milne's Predictor

$$y_4^P = y_0 + \frac{4h}{3}\left(2z_1 - z_2 + 2z_3\right)$$

$$= 2 + \frac{4}{3}(0.1)\left\{2(0.2) - 0.4 + 2(0.6)\right\}$$

$$= 2.16$$

$$z_4^P = z_0 + \frac{4h}{3}\left(2z_1' - z_2' + 2z_3'\right)$$

$$= 0 + \frac{4}{3}(0.1)\left\{2(2.0104) - 2.0428 + 2(2.0998)\right\}$$

$$= 0.8237$$

$$z_4' = 2e^{z_4} - z_4^P = 2e^{0.4} - 0.8237$$

$$= 2.1599$$

Corrector formula

$$y_4^C = y_2 + \frac{h}{3}\left(z_2 + 4z_3 + z_4^P\right)$$

$$= 2.04 + \frac{0.1}{3}\left\{0.4 + 4(0.6) + 0.8237\right\}$$

$$= 2.1608$$

$$z_4^C = z_2 + \frac{h}{3}\left(z_2' + 4z_3' + z_4'\right)$$

$$= 0.4 + \frac{0.1}{3}\left\{2.0428 + 4(2.0998) + 2.1599\right\}$$

$$= 0.8201$$

4M

$y(0.4) = y_4^C = 2.1608$

$y'(0.4) = z_4^C = 0.8201$ (1M)

3. $\frac{d^2x}{dt^2} - t\frac{dx}{dt} + 4x = 0$

Let $z = \frac{dx}{dt}$

$\frac{dz}{dt} - tz + 4x = 0$

$\frac{dz}{dt} = tz - 4x$ (2M)

$x_0 = x(t_0) = 3$, $z_0 = z(t_0) = 0$; $t_0 = 0$

$k_1 = h z_0 = 0$

$p_1 = h\,\phi(t_0\ x_0\ z_0) = h(t_0 z_0 - 4x_0)$

$= 0.1(0 - 4(3)) = -1.2$

$k_2 = h(z_0 + p_1/2) = 0.1(0 - 0.6) = -0.06$

$p_2 = h\,\phi\left(t_0 + \frac{h}{2}\ \ x_0 + \frac{k_1}{2}\ \ z_0 + \frac{p_1}{2}\right)$

$= 0.1\{(0.05)(-0.6) - 4(3+0)\} = -1.203$

$k_3 = h(z_0 + p_2/2) = 0.1(0 - .6015)$

$= -0.06015$

$p_3 = h\,\phi(t_0 + h/2,\ x_0 + k_2/2,\ z_0 + p_2/2)$

$= 0.1\ \{(0.05)(-0.6015) - 4(3 - 0.03)\}$

$= -1.191$

$k_4 = h(z_0 + p_3) = 0.1(-1.191) = -0.1191$ (4)

$p_4 = h\phi(t_0 + h \quad x_0 + k_3 \quad z_0 + p_3)$

$= 0.1\{(0.1)(-1.191) - 4(3 - .06015)\}$

$= -1.18785$

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = -0.0599$ (4M)

$p = \frac{1}{6}(p_1 + 2p_2 + 2p_3 + p_4) = -1.195975$

$x(0.1) = x_0 + k = 3 - .0599 = 2.9401$ (1M)

$\frac{dx}{dt}(0.1) = z_0 + p = 1.195975$

4. $z = x + iy, \quad x, y \in \mathbb{R} \quad i = \sqrt{-1}$

$z = re^{i\theta}$ (3M)

$f(z) = u + iv$

$u(r, \theta) + iv(r, \theta) = f(re^{i\theta})$

$\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r} = f'(re^{i\theta})e^{i\theta}$ — (1)

$\frac{\partial u}{\partial \theta} + i\frac{\partial v}{\partial \theta} = f'(re^{i\theta})\, ie^{i\theta}r$ — (2)

$= ir\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right)$

$$u_\theta + i v_\theta = ir(u_r + i v_r)$$

$$u_\theta + i v_\theta = i r u_r + i^2 r v_r$$

$$u_\theta + i v_\theta = i r u_r - r v_r$$

(3M)

$$u_\theta = -r v_r \qquad v_\theta = r u_r$$

$u_r = \frac{1}{r} v_\theta$ & $v_r = -\frac{1}{r} v_\theta$ are the (1M)

CR eqns in polar form.

5. IVP $y' = -ky$ $\{h = 50\}$

$y(0) = 100$
$t_0 \quad y_0$

$$\frac{dy}{dt} = -ky$$

Euler's $y_1^F = y_0 + h f(x_0, y_0)$ at $t_1 = t_0 + h$
$= 0 + 50$
$= 50$

$$= 100 + 50(-0.01 y_0)$$

$$= 100 + 50\{-0.01 \times 100\} = 50$$

$y_1^F = 50$ at $t_1 = 50$

<u>ME</u> $y_1^{(1)} = y_0 + \frac{h}{2}\{f(x_0 y_0) + f(x_1 y_1^F)\}$

$100 + \frac{50}{2}\{-0.01 \times 100 + (-0.01 \times 50)\}$

$100 + 25\{-1 - 0.5\} = 62.5$

(3M)

II stage $x_0 = 50 \quad y_0 = 62.5$

$$y' = -ky$$
$$= -0.01(62.5) = -0.625$$

$$y_1^E = y_0 + h f(x_0\, y_0)$$
$$62.5 + 50(-0.625) = 31.25$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left\{f(x_0\, y_0) + f(x_1, y_1^E)\right\}$$
$$= 62.5 + \frac{50}{2}\left\{-0.625 + (-0.01)31.25\right\}$$
$$= 62.5 + 25\left\{-0.625 - .3125\right\}$$ 3M
$$62.5 - 15.9375 = 46.5625$$
at $x_1 = 100$ 1M

$y_1 = 46.5625$ at $x_1 = 100$

6. $v = r - \frac{1}{r}\sin\theta, \quad r \neq 0$

$$v_r = 1 + \frac{1}{r^2}\sin\theta \qquad v_\theta = -\frac{1}{r}\cos\theta$$ 1M

$$f'(z) = e^{-i\theta}(u_r + i v_r)$$
$$e^{-i\theta}\left(\frac{1}{r}v_\theta + i v_r\right)$$
$$= e^{-i\theta}\left(\frac{1}{r}\left(-\frac{1}{r}\cos\theta\right) + i\left(1 + \frac{1}{r^2}\sin\theta\right)\right)$$ 2M

Milne Thomson $r \longleftrightarrow z$, $\theta \longleftrightarrow 0$

~~$f'(z) = \frac{1}{z} - \frac{1}{z} + i\left(1 + \frac{1}{z^2}(0)\right)$~~

$$f'(z) = e^{-i\theta}\left(-\frac{1}{r^2}\cos\theta + i\left(1 + \frac{1}{r^2}\sin\theta\right)\right)$$

$$f'(z) = -\frac{1}{z^2} + i \quad = -z^{-2} + i$$

(3 M)

(1M)

Intg $f(z) = z^{-1} + iz$

$= (re^{i\theta})^{-1} + ire^{i\theta}$

$= \frac{1}{re^{i\theta}} + ire^{i\theta}$

$\frac{1}{r}(\cos\theta - i\sin\theta) + ir(\cos\theta + i\sin\theta)$

$= \frac{1}{r}\cos\theta - \frac{i}{r}\sin\theta + ir\cos\theta - r\sin\theta$

$= \left(\frac{1}{r}\cos\theta - r\sin\theta\right) + i\left(r\cos\theta - \frac{1}{r}\sin\theta\right)$

7. $u = x^2/y \quad y \neq 0 \quad v = x^2 + 2y^2$

$u = \frac{x^2}{y} \quad y \neq 0$

diff w.r.t x

$\frac{2x}{y} - \frac{x^2}{y^2}\frac{dy}{dx} = 0 \quad \frac{dy}{dx} = \frac{2y}{x} = m_1$

$v = x^2 + 2y^2$

$2x + 4y\frac{dy}{dx} = 0$

$x + 2y\frac{dy}{dx} = 0 \quad \frac{dy}{dx} = -\frac{x}{2y} \quad m_2$

$m_1 m_2 = -1$

∴ The curves u = constant and v = constant are orthogonal (3M)

$u_x = \frac{2x}{y} \quad u_y = -\frac{x^2}{y^2}$

$v_x = 2x \quad v_y = 4y$

$u_x \neq v_y \quad u_y \neq -v_x$

CR eqns are not satisfied.

$f(z) = u + iv$ is not analytic (4M)

8) $u - v = (x-y)(x^2+4xy+y^2)$

$u - v = x^3 + 4x^2y + xy^2 - yx^2 - 4xy^2 - y^3$

$u_x - v_x = 3x^2 + 8xy + y^2 - 2xy - 4y^2$

$u_y - v_y = 4x^2 + 2xy - x^2 - 8xy - 3y^2$

$u_x - v_x = 3x^2 + 6xy - 3y^2$ (1)

$-u_x + v_x = 3x^2 - 6xy - 3y^2$ (2)

(1) + (2) $\quad -2v_x = 6x^2 - 6y^2$

$v_x = -3x^2 + 3y^2$

(1) − (2) $\quad 2u_x = 12xy$ (4M)

$u_x = 6xy$

$f'(z) = u_x + i v_x$

$= 6xy + i(3y^2 - 3x^2)$

Milne Thomson $\quad x \longleftrightarrow z,\ y \longleftrightarrow 0$

$f'(z) = 6(z)(0) + i(3(0) - 3z^2)$ (2M)

$f'(z) = -3iz^2$ (1M)

Intg $\quad f(z) = -iz^3 + c$

$= -i(x+iy)^3 + c$

(13)

$$f(z) = -i\{x^3 + 3x^2 iy + 3x(i^2y^2) + (iy)^3\} + c$$

$$f(z) = -i\{x^3 + 3ix^2y - 3xy^2 - iy^3\}$$

$$= -ix^3 - 3i^2x^2y + 3ixy^2 + i^2y^3$$

$$= -ix^3 + 3x^2y + 3ixy^2 - y^3$$

$$= \underbrace{(3x^2y - y^3)}_{u} + i\underbrace{(3xy^2 - x^3)}_{v}$$

$$u - v = x^3 - y^3 + 3x^2y - 3xy^2$$

9. $\phi(x,y) = 3x^2y - y^3$

$\phi_x = 6xy \qquad \phi_y = 3x^2 - 3y^2$

$\phi_{xx} = 6y \qquad \phi_{yy} = -6y$

$\phi_{xx} + \phi_{yy} = 0 \Rightarrow \phi$ is harmonic (3M)

$$f'(z) = \phi_x + i\psi_x$$

$$= \phi_x - i\phi_y$$

$$= 6xy - i(3x^2 - 3y^2)$$

Milne Thomson $x \leftrightarrow z \quad y \leftrightarrow 0$ (3M)

$$f'(z) = -i(3z^2)$$

(1M)

Intg $f(z) = -iz^3$

$$= -i(x+iy)^3$$

$$= \underbrace{3x^2y - y^3}_{\phi} + \underbrace{i(3xy^2 - x^3)}_{+i\psi}$$