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Internal Assesment Test - I

Sub:	Engineering Mathematics-II					Code:	17MAT21
Date:	12 / 03 /2018	Duration: 90 mins	Max Marks: 50		Sem: 2	Sections	All sections.

NOTE: First question is compulsory. Answer any six questions from the rest.

		Marks	OBE	
			CO	RBT
1	Solve the equation $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ by the method of variation of parameters	[8]	CO1	L3
2	Solve the equation $\frac{d^2y}{dx^2} - 4y = Cosh\ (2x-1) + 3^x$.	[7]	CO1	L3
3	Solve the equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 1 + 3x + x^2$.	[7]	CO1	L3

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4 Solve the equation $(D^2 - 4D + 4)y = 8(e^{2x} + \sin x)$. [7] CO1 L3

5\. Solve the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2 + e^x$ by the method of undetermined coefficients. [7] CO1 L3

6\. Solve the equation $(2x+3)^2 y'' - (2x+3)y' - 12y = 6x$. [7] CO1 L3

7 Solve the equation $x^3 y''' + 3x^2 y'' + xy' + 8y = 65\,Cos(\log x)$. [7] CO1 L3

8 Solve the equation $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$. [7] CO1 L3

9 Solve the equation $(px - y)(py + x) = 2p$ by reducing it into Clairaut's form taking $X = x^2$ and $Y = y^2$. [7] CO1 L3

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1. $y'' - 6y' + 9y = \dfrac{e^{3x}}{x^2}$

Auxiliary equation is $m^2 - 6m + 9 = 0$

$\Rightarrow (m-3)^2 = 0$

$\Rightarrow m = 3, 3$

C.F, $y_c = (c_1 + c_2 x)e^{3x}$

$= c_1 e^{3x} + c_2 x e^{3x} = c_1 y_1 + c_2 y_2$ — 2

The general solution is $y = A(x) y_1 + B(x) y_2$.

Here $y_1 = e^{3x} \Rightarrow y_1' = 3e^{3x}$

$y_2 = x e^{3x} \Rightarrow y_2' = e^{3x} + 3x e^{3x}$.

$W = y_1 y_2' - y_2 y_1' = e^{3x}(e^{3x} + 3x e^{3x}) - x e^{3x}(3e^{3x})$

$= e^{6x} + 3x e^{6x} - 3x e^{6x} = e^{6x}$. — 2

$A(x) = -\int \dfrac{y_2 \phi(x)}{W} \cdot dx + k_1$

$= -\int \dfrac{x e^{3x} \cdot e^{3x}}{e^{6x} \cdot x^2} \cdot dx + k_1 = -\int \dfrac{1}{x} \cdot dx + k_1$

$= -\log x + k_1$

$B(x) = \int \dfrac{y_1 \phi(x)}{W} dx + k_2$

$= \int \dfrac{e^{3x} e^{3x}}{e^{6x} \cdot x^2} dx + k_2 = \int \dfrac{1}{x^2} dx + k_2$

$= \dfrac{-1}{x} + k_2$ — 3

The general soln is $y = (-\log x + k_1) e^{3x} + \left(\dfrac{-1}{x} + k_2\right) x e^{3x}$.

$= k_1 e^{3x} + k_2 x e^{3x} - e^{3x} \log x - e^{3x}$

— 1

2. $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$

$(D^2-4)y = \cosh(2x-1) + 3^x$

A.E is $m^2 - 4 = 0 \Rightarrow m = 2, -2$

$y_c = c_1 e^{2x} + c_2 e^{-2x}$ — 2

$y_p = \frac{1}{D^2-4}\left[\cosh(2x-1) + 3^x\right]$

$= \frac{1}{D^2-4}\left[\frac{e^{2x-1} + e^{-2x+1}}{2} + e^{x\log 3}\right]$ — 1

$= \frac{1}{2}\left[\frac{1}{D^2-4}e^{2x-1} + \frac{1}{D^2-4}e^{-2x+1}\right] + \frac{1}{D^2-4}e^{x\log 3}$

$= \frac{1}{2}\left[\frac{x}{2D}e^{2x-1} + \frac{x}{2D}e^{-2x+1}\right] + \frac{1}{(\log 3)^2-4}3^x$

$= \frac{x}{8}e^{2x-1} - \frac{x}{8}e^{-(2x-1)} + \frac{3^x}{(\log 3)^2-4}$ — 3

$\therefore$ Complete solution is,

$y = y_c + y_p$

$= C_1 e^{2x} + C_2 e^{-2x} + \frac{x}{8}e^{2x-1} - \frac{x}{8}e^{-(2x-1)} + \frac{3^x}{(\log 3)^2-4}$ — 1

$= C_1 e^{2x} + C_2 e^{-2x} + \frac{x}{4}\sinh(2x-1) + \frac{3^x}{(\log 3)^2-4}$

(3)

3. $y'' + 3y' + 2y = 1 + 3x + x^2$

$(D^2 + 3D + 2)\, y = 1 + 3x + x^2$

A.E is $m^2 + 3m + 2 = 0$

$m = -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$ — (2)

$y_p = \dfrac{1}{D^2 + 3D + 2}(x^2 + 3x + 1)$ — (1)

$2 + 3D + D^2)\ x^2 + 3x + 1\ \left(\dfrac{x^2}{2}\right.$ (3)

$x^2 + 3x + 1$

$(-)\ (-)\ (-)$

0

$\therefore\ y = c_1 e^{-x} + c_2 e^{-2x} + \dfrac{x^2}{2}$ — (1)

4. $(D^2 - 4D + 4)\, y = 8(e^{2x} + \sin x)$

A.E is $m^2 - 4m + 4 = 0$

$m = 2, 2$

$y_c = (c_1 + c_2 x) e^{2x}$ — (2)

$y_p = 8 \dfrac{1}{D^2 - 4D + 4} e^{2x} + 8 \dfrac{1}{D^2 - 4D + 4} \sin x$

$= 8 \cdot \dfrac{x}{2D - 4} e^{2x} + 8 \times \dfrac{1}{-4D + 3} \sin x$

$= 8 \cdot \dfrac{x^2}{2} e^{2x}$ — (2) $+ 8 \times \dfrac{(-4D - 3)}{16D^2 - 9} \sin x$ — (2)

$$= 4x^2e^{2x} + 8\frac{(-4D-3)}{-25}\sin x$$

$$= 4x^2e^{2x} - \frac{8}{25}\{-4\cos x - 3\sin x\}$$

$$= 4x^2e^{2x} + \frac{32}{25}\cos x + \frac{24}{25}\sin x$$

$\therefore$ The complete soln. is

$$y = y_c + y_p$$

$$= (C_1 + C_2x)e^{2x} + 4x^2e^{2x} + \frac{32}{25}\cos x + \frac{24}{25}\sin x \quad — 1$$

5. $y'' - 3y' + 2y = x^2 + e^x \quad —(1)$

$(D^2 - 3D + 2)y = x^2 + e^x$

A.E is $m^2 - 3m + 2 = 0$

$m = 1, 2$

$y_c = C_1e^x + C_2e^{2x} \quad — 2$

Let $y_p = Ax^2 + Bx + C + Dxe^x \quad —(2) \quad — 1$

$y_p' = 2Ax + B + D(e^x + xe^x) \quad —(3)$

$y_p'' = 2A + D(e^x + e^x + xe^x) \quad —(4)$

Using (2), (3), (4) in (1),

$$2A + 2De^x + \cancel{Dxe^x} - 6Ax - 3B - 3De^x - \cancel{3Dxe^x}$$
$$+ 2Ax^2 + 2Bx + 2C + \cancel{2Dxe^x} = x^2 + e^x$$

(4)

$x^2: \; 2A = 1 \Rightarrow A = 1/2$

$x: \; -6A + 2B = 0$

$\Rightarrow -6\left(\frac{1}{2}\right) + 2B = 0$

$\Rightarrow 2B = 3$

$\Rightarrow B = \frac{3}{2}$.

Const: $2A - 3B + 2C = 0$

$\Rightarrow 2\left(\frac{1}{2}\right) - 3\left(\frac{3}{2}\right) + 2C = 0$

$\Rightarrow 1 - \frac{9}{2} + 2C = 0$ — 3

$\Rightarrow 2C = \frac{9}{2} - 1$

$\Rightarrow C = \frac{9}{4} - \frac{1}{2}$

$\Rightarrow C = \frac{9-2}{4} = \frac{7}{4}$

Also, $2D - 3D = 1$

$-D = 1$

$D = -1$.

$\therefore y_p = \frac{1}{2}x^2 + \frac{3}{2}x + \frac{7}{4} - xe^x$.

$\therefore$ G.S: $y = y_c + y_p$

$= c_1 e^x + c_2 e^{2x} + \frac{1}{2}x^2 + \frac{3}{2}x + \frac{7}{4} - xe^x$.

— 1

6. $(2x+3)^2 y'' - (2x+3)y' - 12y = 6x$ — (1)

– Legendre's linear equation.

Let $2x+3 = e^t \Rightarrow t = \log(2x+3)$ — 1

$(1) \Rightarrow$

$$4D(D-1)y - 2Dy - 12y = 6\left(\frac{e^t - 3}{2}\right)$$

$$\Rightarrow (4D^2 - 4D - 2D - 12)y = 3(e^t - 3)$$

$$\Rightarrow (4D^2 - 6D - 12)y = 3(e^t - 3)$$

$$\Rightarrow \left(D^2 - \frac{3}{2}D - 3\right)y = \frac{3}{4}(e^t - 3)$$ — $1\frac{1}{2}$

A.E is $m^2 - \frac{3}{2}m - 3 = 0 \Rightarrow m = \frac{3 \pm \sqrt{57}}{4}$.

$$y_c = c_1 e^{\left(\frac{3+\sqrt{57}}{4}\right)t} + c_2 e^{\left(\frac{3-\sqrt{57}}{4}\right)t}$$ — 1

$$P.I = \frac{3}{4}\left[\frac{e^t}{D^2 - \frac{3}{2}D - 3} - 3.\frac{e^{0t}}{D^2 - \frac{3}{2}D - 3}\right]$$

$$= \frac{3}{4}\left[\frac{e^t}{1 - \frac{3}{2} - 3} - 3.\frac{1}{-3}\right]$$

$$= \frac{3}{4}\left[-\frac{2}{7}e^t + 1\right]$$

$$= -\frac{3}{14}e^t + \frac{3}{4}$$ $2\frac{1}{2}$

$$\therefore y = y_c + y_p = c_1 e^{\left(\frac{3+\sqrt{57}}{4}\right)t} + c_2 e^{\left(\frac{3-\sqrt{57}}{4}\right)t} - \frac{3}{14}e^t + \frac{3}{4}$$ $\frac{1}{2}$

$$= c_1 (2x+3)^{\left(\frac{3+\sqrt{57}}{4}\right)} + c_2 (2x+3)^{\left(\frac{3-\sqrt{57}}{4}\right)} - \frac{3}{14}(2x+3) + \frac{3}{4}$$ $1\frac{1}{2}$

7. $x^3y''' + 3x^2y'' + xy' + 8y = 65\cos(\log x)$. —(1) (5)

This is a Cauchy's linear equation.

Let $x = e^t \Rightarrow \log x = t$. — 1 $D = \frac{d}{dt}$.

Then (1) becomes,

$$D(D-1)(D-2)y + 3D(D-1)y + Dy + 8y = 65\cos t$$

$$\Rightarrow D(D^2-3D+2)y + (3D^2-3D)y + (D+8)y = 65\cos t$$

$$\Rightarrow (D^3-3D^2+2D+3D^2-3D+D+8)y = 65\cos t$$

$$\Rightarrow (D^3+8)y = 65\cos t \quad —(2)$$ — 1

A.E is $m^3+8=0$

$\Rightarrow m = -2,\ 1 \pm i\sqrt{3}$.

$$y_c = c_1e^{-2t} + e^t(c_2\cos\sqrt{3}t + c_3\sin\sqrt{3}t) \quad — 1\tfrac{1}{2}$$

$$= c_1\left(\frac{1}{x^2}\right) + x\left[c_2\cos\{\sqrt{3}(\log x)\} + c_3\sin\{\sqrt{3}(\log x)\}\right].$$

$$y_p = \frac{1}{D^3+8}65\cos t = 65\frac{1}{D^2 \cdot D+8}\cos t$$

$$= 65\frac{1}{-D+8}\cos t.$$

$$= 65\frac{1}{8-D}\frac{(8+D)}{(8+D)}\cos t$$

$$= \frac{65(8+D)}{64-D^2}\cos t = (8+D)\cos t$$ 2

$$= 8\cos t - \sin t.$$

$$= 8\cos(\log x) - \sin(\log x)$$

$\therefore$ The general solution is,

$$y = y_c + y_p$$ $1\tfrac{1}{2}$

$$= \frac{c_1}{x^2} + x\left[c_2\cos(\sqrt{3}\log x) + c_3\sin(\sqrt{3}\log x)\right] + 8\cos(\log x) - \sin(\log x).$$

8. $xyp^2+(3x^2-2y^2)p-6xy=0$

- Solvable for p

$$p=\frac{-(3x^2-2y^2)\pm\sqrt{(3x^2-2y^2)^2-4(xy)(-6xy)}}{2xy}$$

$$=\frac{-(3x^2-2y^2)\pm\sqrt{9x^4+2y^4-12x^2y^2+24x^2y^2}}{2xy}$$

$$=\frac{-(3x^2-2y^2)\pm\sqrt{(3x^2+2y^2)^2}}{2xy}$$

$$=\frac{-(3x^2-2y^2)\pm(3x^2+2y^2)}{2xy} \quad - 3$$

$$p=\frac{-(3x^2-2y^2)+(3x^2+2y^2)}{2xy} \quad (\text{or}) \quad p=\frac{-3x^2+2y^2-3x^2-2y^2}{2xy}$$

$$=\frac{4y^2}{2xy} \quad (\text{or}) \quad p=\frac{-6x^2}{2xy}$$

The component eqns are,

$$p=\frac{2y}{x} \quad -(1) \qquad \text{and} \qquad p=-\frac{3x}{y} \quad -(2)$$

1

Consider $p=\frac{2y}{x}$

$$\frac{dy}{dx}=\frac{2y}{x} \Rightarrow \frac{dy}{y}=2\frac{dx}{x}.$$

Integrating both sides, $\log y = 2\log x + \log c$

$$\Rightarrow y=cx^2 \Rightarrow y-cx^2=0. \quad - 1$$

(6)

Consider $p = -\frac{3x}{y}$

$$\Rightarrow \frac{dy}{dx} = -\frac{3x}{y} \Rightarrow y\,dy = -3x\,dx \quad - 1$$

Integrating both sides,

$$\frac{y^2}{2} = -\frac{3}{2}x^2 + c$$

$$\Rightarrow y^2 + 3x^2 - 2c = 0$$

$\therefore$ The general solution is,

$$(y - cx^2)(y^2 + 3x^2 - 2c) = 0. \quad - 1$$

9. $(px - y)(py + x) = 2p.$

Consider $X = x^2$ $\qquad$ $Y = y^2$

$$\Rightarrow \frac{dX}{dx} = 2x \;\; \tfrac{1}{2} \qquad \Rightarrow \frac{dY}{dy} = 2y. \;\; \tfrac{1}{2}$$

We have $p = \frac{dy}{dx}$

$$= \frac{dy}{dY}\,\frac{dY}{dX}\,\frac{dX}{dx}$$

$$= \frac{1}{2y}\,P\,2x, \quad \text{where } P = \frac{dY}{dX}.$$

$$= \frac{x}{y}P = \sqrt{\frac{X}{Y}}\,P \qquad 2$$

$- 3$

Consider the given equation,

$$(px - y)(py + x) = 2p.$$

$$\Rightarrow \left(\sqrt{\frac{X}{Y}}P\sqrt{X} - \sqrt{Y}\right)\left(\sqrt{\frac{X}{Y}}P\sqrt{Y} + \sqrt{X}\right) = 2\sqrt{\frac{X}{Y}}\,P$$

$- 4$

$$\Rightarrow \left(\frac{X}{\sqrt{Y}} P - \sqrt{Y}\right)\left(\sqrt{X} P + \sqrt{X}\right) = 2\sqrt{\frac{X}{Y}}\, P$$ $1\frac{1}{2}$

$$\Rightarrow (PX - Y)\sqrt{X}(P+1) = 2\sqrt{X}\, P.$$

$$\Rightarrow (PX - Y)(P+1) = 2P$$

$$\Rightarrow PX - Y = \frac{2P}{P+1} \Rightarrow Y = PX - \frac{2P}{P+1} \quad \text{--- (1)}$$

Eq^n (1) is of clairaut's form. 1

Hence, its general solution is $Y = CX - \frac{2C}{C+1}$,

where C is an arbitrary constant.

$\therefore$ The general solⁿ of given D.E is,

$$y^2 = Cx^2 - \frac{2C}{C+1}$$ $1\frac{1}{2}$