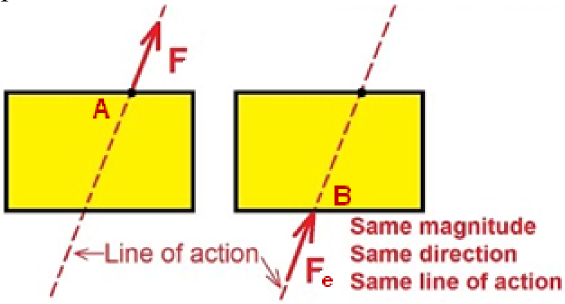
Question bank with solutions for IAT – Elements of civil engineering and engineering mechanics

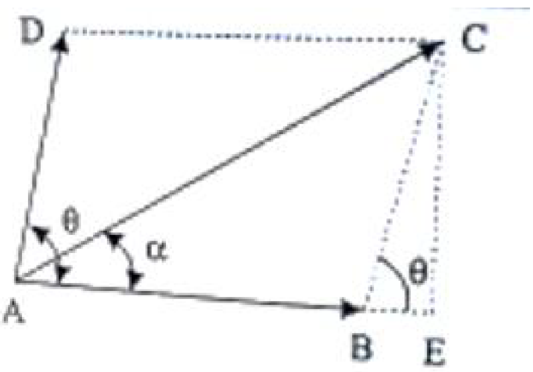
1. State and explain the principle of transmissibility of forces

The principle of transmissibility of force applicable only to rigid bodies states that any force acting at a point can be replaced by the same magnitude of force at some other point on the same line of action in the same direction. This action of shifting the force from one point to the other does not change the condition of motion of the body.
Consider a force F acting at a point A on the line of action displayed in the dashed line as shown in figure. This force can be replaced by the same magnitude of force F_e at some other point on the same line of action as shown. This change in the position of the force does not alter the effect of the force acting at the point A



2. State and prove parallelogram law of forces.

This law is the basic law in mechanics for finding the resultant of concurrent forces. This law was formulated based on experimental results. Though Stevinces employed it in 1586, the credit of presenting it as a law goes to Varignon and Newton (1687). This law states that *if two forces acting simultaneously on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces.*



Now the resultant R of F_1 and F_2 is given by

$$R = AC$$
$$= \sqrt{AE^2 + CE^2}$$
$$= \sqrt{(AB + BE)^2 + CE^2}$$

But

$$AB = F_1$$
$$BE = BC \cos\theta = F_2 \cos\theta$$

$$CE = BC \sin\theta = F_2 \sin\theta$$

$$R = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

$$= \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta}$$

$$= \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$$

The inclination of the resultant to force F_1 is given by α, where

$$\tan\alpha = \frac{CE}{AE} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

Thus $$\alpha = \tan^{-1}\frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

3. State and prove Varignon's theorem.

The algebraic sum of moments of a system of coplanar forces about a moment centre is equal to the moment of their resultant force about the same moment centre.

Proof: Referring to Fig. 6.1, let R be the resultant of forces F_1 and F_2 and 'O' be the moment centre. Let d, d_1 and d_2 be the moment arms of the forces R, F_1 and F_2 respectively. Then in this case we have to prove

$$Rd = F_1d_1 + F_2d_2 \qquad \text{Eqn. (6.1)}$$

Join OA and consider it as y-axis. Draw x-axis to it with A as origin [Ref. Fig. 6.1(b)]. Let resultant make an angle θ with x-axis. Noting that angle AOB is also θ, we can write

$$Rd = R \times AO.\cos\theta$$

$$= AO \times (R\cos\theta)$$

$$= AO \times R_x \qquad ...(i)$$

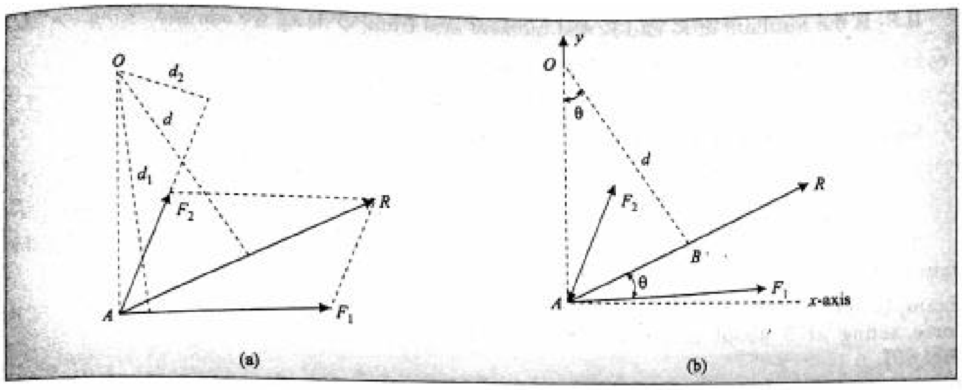


Fig. 6.1

where R_x denotes the component of R in x-direction. Similarly, if F_{1x} and F_{2x} are the components of F_1 and F_2 in x-direction,
then,

$$F_1 d_1 = AO \times F_{1x} \quad \text{...(ii)}$$

and

$$F_2 d_2 = AO \times F_{2x} \quad \text{...(iii)}$$

From eqns. (ii) and (iii), we get

$$F_1 d_1 + F_2 d_2 = AO \times (F_{1x} + F_{2x})$$

$$= AO \times R_x \quad \text{...(iv)}$$

From eqns. (i) and (iv), we observe

$$F_1 d_1 + F_2 d_2 = Rd$$

Thus we find sum of the moment of forces about a moment centre is same as moment of their resultant about the same centre.

If a system of forces consists of more than two forces, the result can be extended as given below:

Let F_1, F_2, F_3 and F_4 be four concurrent forces and R be their resultant. Referring to Fig. 6.2, d_1, d_2, d_3, d_4 and 'a' be moment arms of F_1, F_2, F_3, F_4 and R about moment centre 'O'.

If R_1 is the resultant of forces F_1 and F_2 and its moment arm is a_1, then from the above proof for two force system, we get

$$R_1 a_1 = F_1 d_1 + F_2 d_2$$

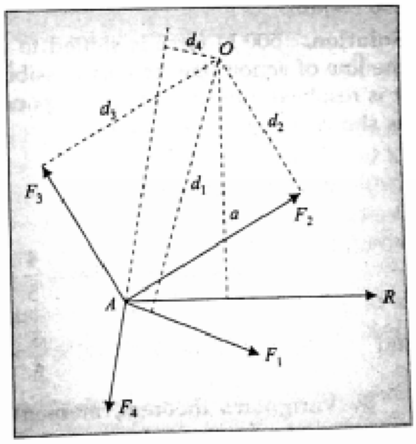
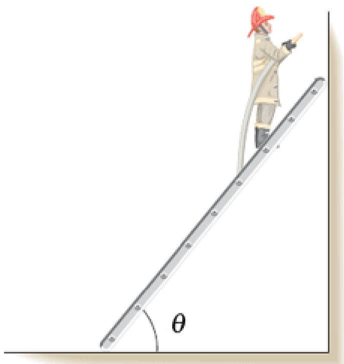


Fig. 6.2

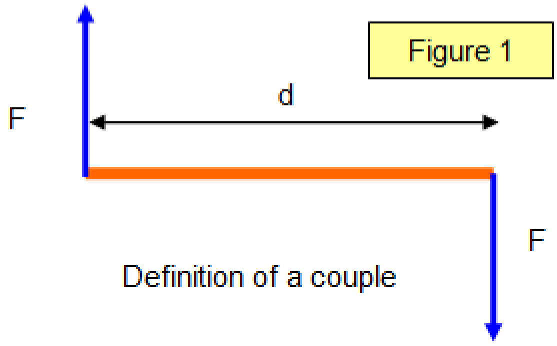
4. Define force. Also mention characteristics of force.
   An external agency which tries to change state of rest or state of uniform motion of body, according to Newton's law definition of unit force required to produce unit acceleration in a body of unit mass.
   Characteristics of force
   - magnitude
   - point of application
   - Line of action
   - Direction
   - AB is a ladder kept against a wall. At C person weighing 600N is standing

Then 600N is magnitude, point of application is at C which is 3m along ladder, Line of action is vertical and direction is downward



5. Define couple. Explain the characteristics of a couple.

Two parallel forces equal in magnitude and opposite in direction and seoerated by a definite distance are said form a couple.fig.1 shows the representation

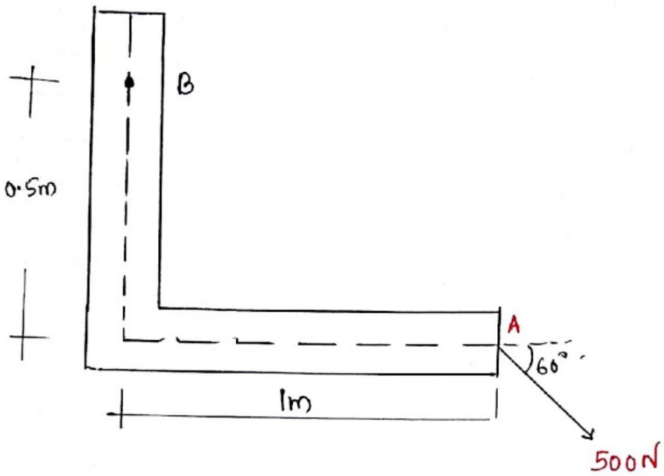


Definition of a couple

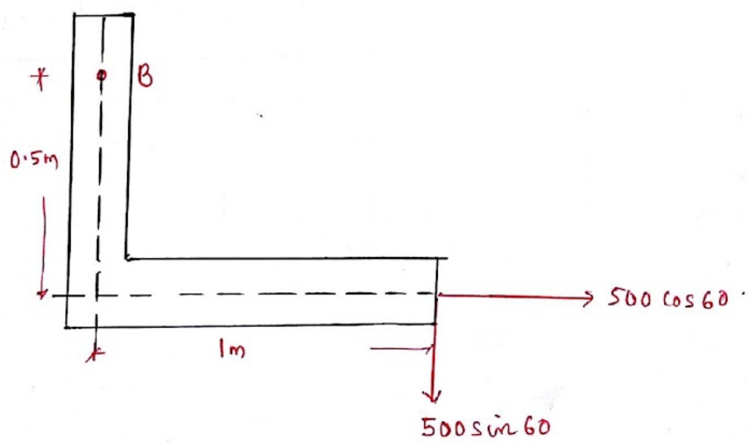
Characteristics of a couple

- The sum of forces forming a couple in any direction is zero, which means the translator effect of the couple is zero.
- The rotational effect of couple on the body is zero.
- The rotational effect of a couple about any point is a constant and it is equal to the product of the magnitude of the forces and perpendicular distance between the two forces.
- The effect of couple is unchanged if couple is rotated, shifted and replaced by another pair of forces whose rotational effects are the same.

3a) A 500 N force is applied to a point A of a L Shaped plate. Find the equivalent force-couple system at B.

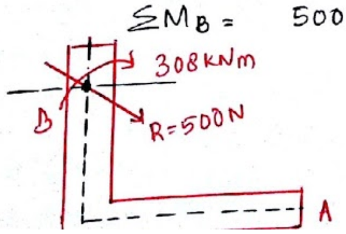


Solution :~



Taking moment about B

$$\Sigma M_B = 500 \sin 60 \times 1 - 500 \cos 60 \times 0.5 = 308 \text{ kNm } (\circlearrowright)$$



Equivalent force couple system at B.

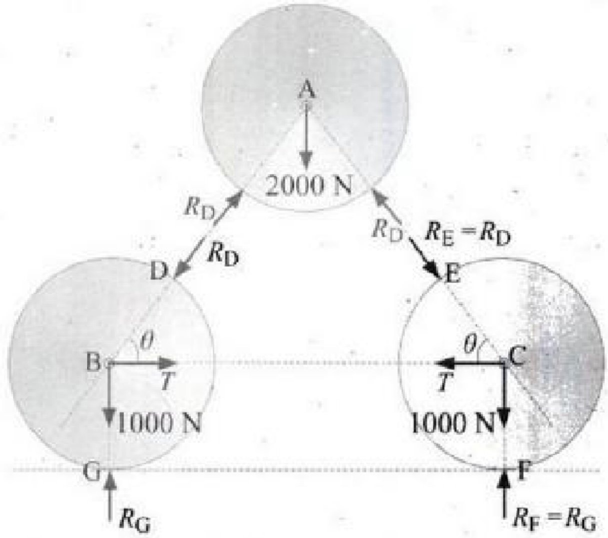


FIGURE 1.16(b) |

An equilibrium of vertical forces requires

$$\sum F_y = 0 \Rightarrow R_D \sin\theta + R_E \sin\theta - 2000 = 0$$

or

$$2R_D \sin\theta = 2000$$

$$[R_E = R_D]$$

or

$$R_D = \frac{2000}{\sin\theta}$$

$$\left[\sin\theta = \frac{AH}{AB} = \frac{\sqrt{500^2 - 400^2}}{500} = \frac{3}{5} \Rightarrow \cos\theta = \frac{4}{5}\right]$$

or

$$R_D = \frac{1000}{\frac{3}{5}} = \frac{5000}{3}\text{ N}$$

$$R_D = R_E = \frac{5000}{3}\text{ N}$$

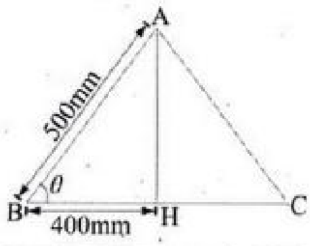


Figure 1.16(c)

Equilibrium of Cylinder B

Equilibrium of horizontal forces requires [see Fig. 1.16(c)]

$$\sum F_x = 0 \Rightarrow T - R_D\cos\theta = 0$$

or

$$T = R_D\cos\theta = \frac{5000}{3} \times \frac{4}{5}$$

$$T = \frac{4000}{3}\text{ N}$$

Equilibrium of vertical forces requires

$$\sum F_y = 0 \Rightarrow R_G - 1000 - R_D\sin\theta = 0$$

or

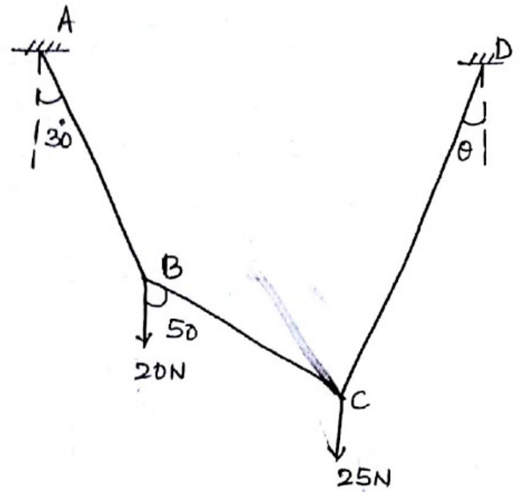
$$R_G - 1000 - \frac{5000}{3} \times \frac{3}{5} = 0$$

or

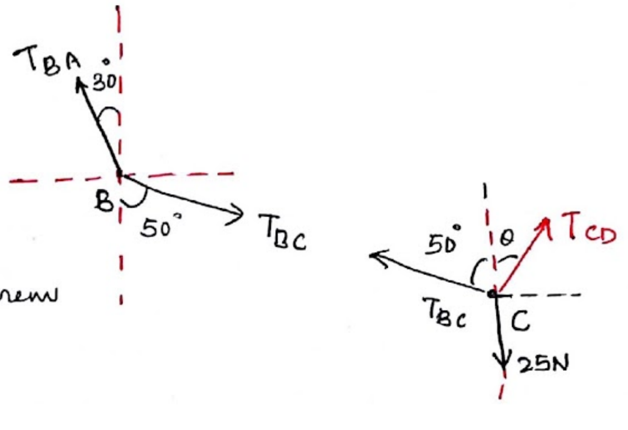
$$R_G = 2000\text{N}$$

$$R_G = R_E = 2000\text{N}$$

6(a) Determine the angle θ for the system of strings ABCD shown in fig 6.a to be in equilibrium.



The FBD of B and C are shown as below.



Using Lamis theorem for B

$$\frac{T_{BA}}{\sin 50} = \frac{T_{BC}}{\sin 150} = \frac{20}{\sin 160}$$

$T_{BA} = 44.8N$

$T_{BC} = 29.24N$

For C,

$\Sigma F_x = 0 \Rightarrow T_{CD} \sin\theta - T_{CB} \sin 5$ —①

$\Sigma F_y = 0 \Rightarrow T_{CD} \cos\theta + T_{CB} \cos 50 -$ —②

Rearranging & dividing ①/②

$$\tan\theta = \frac{29.24 \sin 50}{25 - 29.24 \cos 50}, \quad \theta = 74.52^\circ$$