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Internal Assessment Test II

Sub	ENGINEERING MATHEMATICS IV			Code	15MAT41
Date 16 / 04 / 2018	Duration 90 mins	Max Marks 50	Sem IV	Branch	EC B, EE B

Question 1 is compulsory. Answer any SIX questions from the rest.

	REGULAR	Marks	OBE CO	RBT
1.	Define Conformal transformation and discuss the transformation $w = z^2$.	08	3	L3
2.	Derive the mean and variance of Poisson distribution.	07	4	L3
3.	State and prove Cauchy theorem.	07	3	L3
4.	Using Cauchy integral formula, evaluate $\oint_C \frac{\cos(\pi z^2)}{(z-1)(z-2)} dz$ where C is the circle $\lvert z \rvert = 3$.	07	3	L3
5.	Using Cauchy n^th integral formula evaluate $\int_C \frac{z^4}{(3z+1)^4} dz$ where C is the circle $\lvert z \rvert = 1$.	07	3	L3

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6. If C is the circle $|z| = 2$, using Cauchy residue theorem, prove that $\int_C \tan z\, dz = -4\pi i$ — 07 | 3 | L3

7. If A random variable has the following density function: $f(x) = \begin{cases} kx^2 & -3 \le x \le 3 \\ 0 & elsewhere \end{cases}$

Evaluate k and find the mean and variance of the distribution . Also find $P(X \le 2)$ — 07 | 4 | L3

8. A box contains 12 items of which 4 are defective. A sample of 3 items is selected from the box. Let X denote the number of defective items in the sample. Find the probability distribution of X. Find the mean and standard deviation of the distribution. — 07 | 4 | L3

9. Alpha particles are emitted by a radioactive source at an average rate of 5 in a 20 minute interval. Using Poisson distribution, find the probability that there will be a) exactly 2 emissions b) at least 2 emissions, in a randomly chosen 20 minutes interval. — 07 | 4 | L3

10. 4 coins are tossed 100 times and the following results are obtained. Fit a binomial distribution for the data and calculate the theoretical frequencies. — 07 | 4 | L3

Number of heads	0	1	2	3	4
Frequency	5	29	36	25	5

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Engineering Maths IV
II Internal Test

Consider the transformation

$$w = z^2 \quad (1)$$

$$f(z) = z^2 \quad f'(z) = 2z$$

(1) is conformal for $z \neq 0$

Take $z = x + iy \quad w = u + iv$

$$u + iv = (x + iy)^2$$

$$u + iv = x^2 + i2xy - y^2$$

$$u + iv = (x^2 - y^2) + i2xy \quad (2)$$

$$u = x^2 - y^2 \quad v = \cancel{2}xy$$

If u = constant 'A', $x^2 - y^2 = A$ rep a rectangular hyperbola

v = constant say $2B$ $xy = B$ is also a rectangular hyperbola

Under $w = z^2$, rectangular hyperbolas $x^2 - y^2 = A$ and $xy = B$ in z plane transform to the st lines $u = A$ and $v = 2B$ in the w plane.

Consider a line parallel to y axis of the form $x = a$

$$u = a^2 - y^2$$

$$v = 2ay$$

$$v^2 = 4a^2y^2 = -4a^2(u-a^2) \quad (3)$$

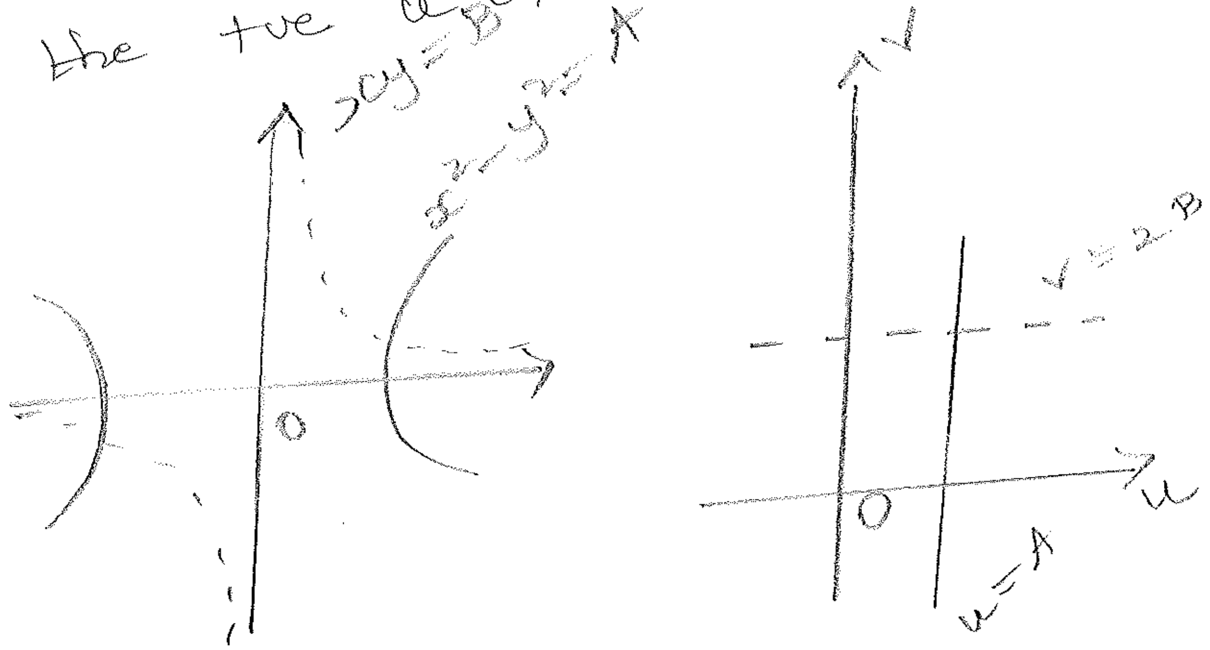
(3) represent a ~~hyperbola~~ parabola in the w plane having vertex at $(a^2, 0)$ -ve u axis as its axis

Consider a line parallel to the x axis. Its eqn is of the form $y = b$, b is a constant.

$$u = x^2 - b^2, \quad v = 2xb$$

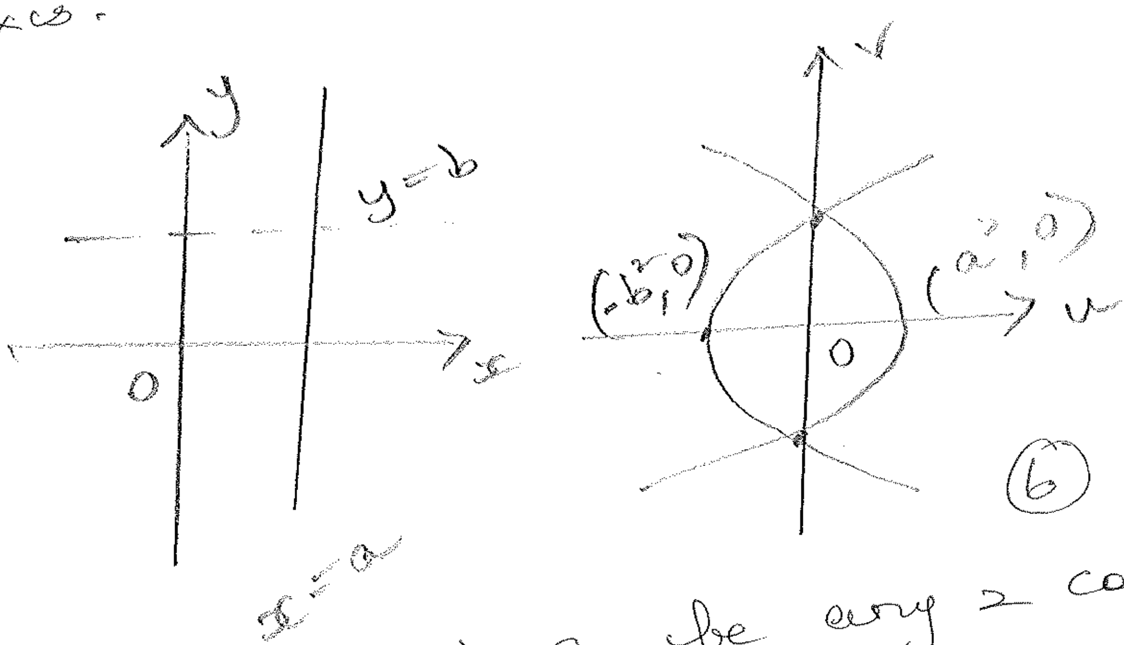
$$v^2 = 4x^2b^2 = 4b^2(u+b^2)$$

This represent a parabola in the w plane having vertex at $(-b^2, 0)$ as the axis and the +ve u axis



(3)

$w = z^2$ transforms st lines $\parallel^{l}$ to the y axis to parabolas having $-$ve u axis as their common axis and st lines $\parallel^{l}$ to x axis to parabolas having $+$ve u axis as their common axis.



(6)

Let C_1 and C_2 be any 2 continuous curves in the z plane intersecting at a pt z_0. Suppose (1) $w = f(z)$ transforms C_1 and C_2 to C_1' and C_2' which intersect at $Q = w_0 = f(z_0)$ in the w plane.

$w = f(z)$ is called Conformal Transformation at the pt z_0.

(2)

$6 + 2 = 8M$

2. Mean and Variance of Poisson

$$\mu = \sum x\, p(x)$$

$$= \sum x \frac{e^{-m} m^{x}}{x!}$$

$$\sum \frac{e^{-m} m^{x}}{(x-1)!} = e^{-m} \sum_{x=1}^{\infty} \frac{m^{x}}{(x-1)!}$$

$$= e^{-m} \left[\frac{m}{0!} + \frac{m^2}{1!} + \frac{m^3}{2!} + \frac{m^4}{3!} + \cdots \right]$$

$$= m e^{-m} \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots \right]$$

$$= m e^{-m} \cdot e^{m} = m e^{0} = m$$ (3 M)

Variance $\sigma^2 = \sum (x-\mu)^2 p(x)$

$$= \sum (x^2 - 2\mu x + \mu^2)\, p(x)$$

$$= \sum x^2 p(x) - 2\mu \sum x\, p(x) + \mu^2 \sum p(x)$$

$$= \sum x^2 p(x) - \mu^2$$

$$= \sum x^2 \frac{e^{-m} m^{x}}{x!} - \mu^2$$

$$\sigma^2 = e^{-m} \sum \frac{x^2 m^x}{x!} - \mu^2$$

$$= e^{-m} \sum \frac{m^x}{(x-2)!} - \mu^2$$

$$= e^{-m} \left\{ \frac{m^2}{0!} + \frac{m^3}{1!} + \frac{m^4}{2!} + \frac{m^5}{3!} + \cdots \right\} - \mu^2$$

$$= e^{-m} m^2 \left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots \right) - \mu^2$$

$$= e^{-m} \cdot m^2 \cdot e^m - \mu^2$$

$$\sigma^2 = e^{-m} \sum \frac{(x^2 + x - x)}{x!} m^x - \mu^2$$

$$= e^{-m} \cdot \sum \frac{(x^2 - x)}{x!} m^x + e^{-m} \sum \frac{x m^x}{x!} - \mu^2$$

$$= e^{-m} \sum \frac{m^x}{(x-2)!} + e^{-m} \sum \frac{m^x}{(x-1)!} - \mu^2$$

$$e^{-m} \left\{ \frac{m^2}{0!} + \frac{m^3}{1!} + \frac{m^4}{2!} + \cdots \right\} + e^{-m} \left\{ \frac{m}{0!} + \frac{m^2}{1!} + \cdots \right\} - \mu^2$$

$$= e^{-m} \cdot m^2 \cdot e^m + e^{-m} \cdot m \cdot e^m - m^2$$

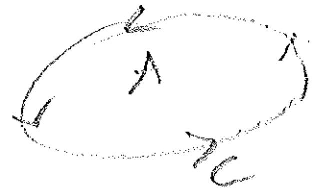
$$= m \qquad \boxed{\sigma^2 = m} \qquad 4M$$

3. Cauchy Theorem If a complex function $f(z)$ is analytic on and within a simple closed curve C then $\int_C f(z)dz = 0$ (2M)

<u>Proof</u> Let $f(z) = u+iv$ $\quad z = x+iy$, $dz = dx+idy$

$$\int_C f(z)\,dz = \int_C (u+iv)(dx+idy)$$

$$= \int_C (udx - vdy) + i(vdx + udy)$$



If A is the region bounded by C, by Green's theorem

$$= \iint\left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy$$

$$+ i\iint\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dx\,dy$$

$$= 0 + i0 = 0$$

$$\therefore \oint_C f(z)\,dz = 0 \qquad \text{(5M)}$$

4. $\oint_C \frac{\cos(\pi z^2)}{(z-1)(z-2)}\, dz$ C is $|z|=3$

Consider $\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$

$1 = A(z-2) + B(z-1)$

$A + B = 0$ $-A = 1 \Rightarrow \boxed{A=-1}$

$-2A - B = 1$ $\boxed{B=1}$ (2M)

We K.T $\oint_C \frac{f(z)}{z-a}\, dz = 2\pi i f(a)$ (1M)

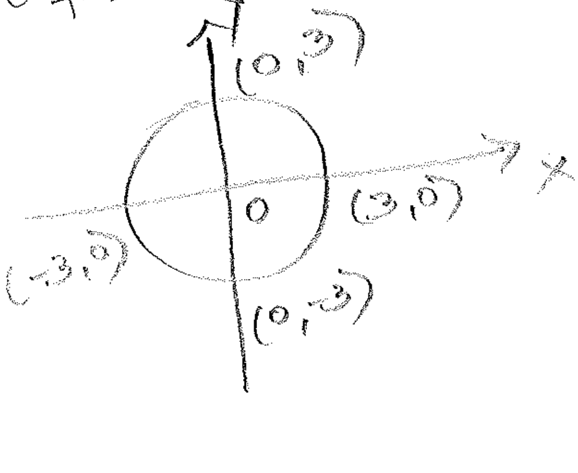
$-\oint_C \frac{\cos(\pi z^2)}{z-1}\, dz + \oint_C \frac{\cos(\pi z^2)}{z-2}\, dz$

$a = 1, 2$ lie inside C

$= -2\pi i f(1) + 2\pi i f(2)$ (2M)

$= -2\pi i (\cos \pi) + 2\pi i (\cos 4\pi)$

$= 2\pi i + 2\pi i = 4\pi i$ (1M)



5. $\int_C \frac{z^4}{(3z+1)^4}\,dz \qquad C$ is $|z|=1$

$$\oint_C \frac{f(z)}{(z-a)^{n+1}}\,dz = \frac{2\pi i}{n!} f^n(a) \quad (1M)$$

$a = -1/3$ lie inside $|z|=1$ (1M)

$$\oint_C \frac{z^4}{3^4(z-(-1/3))^{3+1}}\,dz = \frac{2\pi i}{81} f'''(-1/3)\frac{1}{3!} \quad (1) \quad (2M)$$

$f(z) = z^4 \quad f'(z) = 4z^3 \quad f''(z) = 12z^2$

$f'''(z) = 24z$ (2M)

RHS of (1) $\frac{2\pi i}{81} f^3(-1/3)\cdot\frac{1}{6}$

$= \frac{2\pi i\, 24(-1/3)}{(81)(6)} = \frac{-16\pi i}{(81)(6)}$ (1M)

6. C is $|z|=2$

Consider $\int_C \frac{\sin z}{\cos z}\,dz$

Let $f(z) = \frac{\sin z}{\cos z}$ (1M)

poles of $f(z)$ are the roots of the eqn $\cos z = 0 \Rightarrow z = \pm\frac{\pi}{2}, \pm 3\pi/2, \ldots$

Only $z = \pm\frac{\pi}{2}$ lie within C (1M)

Residue at $\frac{\pi}{2} = \lim_{z \to \pi/2} (z - \pi/2) f(z)$

$$= \lim_{z \to \pi/2} (z - \pi/2) \frac{\sin z}{\cos z} \quad \left(\frac{0}{0}\right)$$

$$\lim_{z \to \pi/2} \frac{(z - \pi/2)\cos z + \sin z}{-\sin z} \quad \text{L'Hopital's rule}$$

$$= -1$$

Res at $-\pi/2 = -1$ (5M)

By Residue theorem

$$\int_C \tan z \, dz = 2\pi i(-1-1) = -4\pi i \quad \text{(1M)}$$

7. We K.T $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-3}^{3} kx^2\,dx = 1 \Rightarrow k\left(\frac{x^3}{3}\right)_{-3}^{3} = 1$$

$$\boxed{k = \frac{1}{18}} \quad \text{(2M)}$$

$$\text{Mean } \mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\mu = \int_{-3}^{3} \frac{1}{18} x^2 \cdot x \, dx$$

$$\mu = \frac{1}{18}\left(\frac{x^4}{4}\right)_{-3}^{3} = 0.$$ (1M)

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$= \int_{-3}^{3} x^2 \frac{1}{18} x^2 \, dx = \frac{1}{18}\left(\frac{x^5}{5}\right)_{-3}^{3}$$

$$= \frac{1}{18}\left(3^5 - (-3)^5\right)$$

$$= \frac{486}{18} = \frac{243}{9}$$ (2M)

$$P(x \le 2) = \int_{-3}^{2} \frac{1}{18} x^2 \, dx$$

$$= \frac{1}{18}\left(\frac{x^3}{3}\right)_{-3}^{2} = \frac{1}{54}\left(2^3 - (-3)^3\right)$$

$$= \frac{35}{54}$$ (2M)

8. Let X be the no. of defective items

X takes the values 0, 1, 2, 3 (1M)

$$P(X=0) = \frac{8C_3}{12C_3} = \frac{14}{55}$$

$$P(X=1) = \frac{4C_1 \times 8C_2}{12C_3} = \frac{28}{55}$$

$$P(X=2) = \frac{4C_2 \times 8C_1}{12C_3} = \frac{12}{55}$$

$$P(X=3) = \frac{4C_3}{12C_3} = \frac{1}{55}$$

X	0	1	2	3
$P(X)$	$\frac{14}{55}$	$\frac{28}{55}$	$\frac{12}{55}$	$\frac{1}{55}$

(4M)

We find 1) $P(X) \geq 0$

2) $\sum P(X) = 1$

Mean $\mu = \sum X P(X)$

$$= \left(0 \times \frac{14}{55}\right) + \left(1 \times \frac{28}{55}\right) + \left(2 \times \frac{12}{55}\right) + \left(3 \times \frac{1}{55}\right) = 1$$

(1M)

$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

$$\sigma^2 = \sum (x-1)^2 p(x)$$

$$= (0-1)^2 \frac{14}{55} + (1-1)^2 \frac{28}{55} + (2-1)^2 \frac{12}{55}$$

$$+ (3-1)^2 \frac{1}{55} = \frac{30}{55}$$

$$\boxed{\sigma = 0.7385}$$ (1M)

9. $\mu = 5$ (1M) $\quad p(5, x) = \frac{e^{-5} 5^x}{x!}$ (1M)

$$p(2 \text{ emissions}) = \frac{e^{-5} 5^2}{2!} = 0.0842$$ (2M)

p(at least 2 emissions)

$$p(x \geq 2) = 1 - p(x < 2)$$

$$= 1 - \{p(0) + p(1)\}$$

$$= 1 - (e^{-5} + 5e^{-5}) = 0.9596$$ (3M)

10. Let x denote the no. of heads.
f the corresponding frequency (1M)

$$\text{Mean } \mu = \frac{\sum xf}{\sum f}$$

$$= \frac{0 + 29 + 72 + 75 + 20}{100} = 1.96$$

$\mu = np$

$4p = 1.96 \quad p = 0.49$

$q = 0.51$

$P(x) = nC_x p^x q^{n-x}$ 4M

$4C_x (0.49)^x (0.51)^{4-x}$

Theoretical frequencies are

$F(x) = 100 \; 4C_x (0.49)^x (0.51)^{4-x}$

$F(0) = 7$

$F(1) = 26$ $\quad F(4) = 6$ 2M

$F(2) = 37 \quad F(3) = 24$

Theoretical frequencies are

7, 26, 37, 24, 6