

**ENGG PHYSICS IAT-2
SCHEME OF EVALUATION**

1. a. [6]

Conductivity of Intrinsic semiconductors: [3]

Current density $J = n_e e V_d$

For a semiconductor, $J = n_e e V_d(e) + n_h e V_d(h)$ (1)

But drift velocity $V_d = \mu E = \mu J / \sigma$

Using (1), $\sigma = n_e e \mu_e + n_h e \mu_h$

In an intrinsic semiconductor, number of holes is equal to number of electrons.

$$\sigma_{int} = n_e e [\mu_e + \mu_{hole}]$$

n_e is the electron concentration

n_p is the hole concentration

μ_e is the mobility of electrons

μ_h is the mobility of holes

Law of mass action: [3]

The product of electron and hole concentration is constant at any given temperature independent of Fermi energy & type of doping. This principle is used to calculate hole and electron densities.

$$n_e n_h = 4 \left(\frac{kT}{2\pi h^2} \right)^3 (m_e^* m_h^*)^2 e^{-\frac{E_g}{kT}} = a \text{ constant}$$

$$\text{Electron concentration } n_e = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{\frac{3}{2}} e^{-\left(\frac{E_c - E_F}{kT}\right)}$$

$$\text{Hole concentration } n_h = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{\frac{3}{2}} e^{-\left(\frac{E_F - E_{VF}}{kT}\right)}$$

For Intrinsic semiconductor $n_e = n_h = n_i$
hence

$$n_i^2 = n_e n_h = 4 \left(\frac{kT}{2\pi h^2} \right)^3 (m_e^* m_h^*)^2 e^{-\frac{E_g}{kT}} = a \text{ constant}$$

1. b. [4]

Conductivity of an Intrinsic semiconductor is given by

$$\sigma_{int} = n_e e [\mu_e + \mu_{hole}]$$

On substitution, we get

$$\rho_{int} = \frac{1}{\sigma} = 1.6 \times 10^{-3} \Omega m$$

2. a. [6]

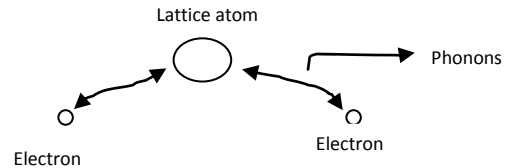
It is a phenomenon in which some materials lose their resistance completely below certain temperature. [1]

BCS Theory : [Bardeen, Cooper, Schrieffer] [5]

According to this theory superconductivity occurs when an attractive interaction known as electron-lattice-electron interaction is established resulting in the formation of Cooper pairs.

In a lattice, an electron passing close to a lattice atom is attracted towards it and displaces it. This lattice atom will interact with another electron and in turn

forms an **electron – lattice – interaction**. This system of two electrons of equal and opposite momentum attached to a lattice atom is known as a **Cooper pair**. The electrons are bound to the lattice atom through the exchange of phonons (Lattice vibrations). Collective flow of Cooper pairs under the influence of applied electric field reduces resistance. The energy gap between the normal state and superconducting state is of the order of milli electron volts. The thermal energy at low temperatures is not sufficient to break Cooper pair interaction. Cooper pairs are represented collectively by a coherent matter wave function.



2b. [4]

High temperature superconductivity:

These are generally alloy materials with critical temperature greater than 90K. They possess perovskite structure. Here Cooper pairs do not possess equal and opposite spin. So net angular momentum is non zero.

Ex: $YBa_2Cu_3O_7$

BCS theory cannot explain the behaviour of high-temperature superconductors, discovered in 1986, which have transition temperatures as high as 138 K. These 'cuprates' consist of parallel planes of copper oxide in which the copper atoms lie on a square lattice and where the charge is carried by "holes" sitting on oxygen sites. Each copper atom has an unpaired electron, and hence a magnetic moment or "spin", and some researchers believe that it is the coupling between these spins that gives rise to superconductivity in these materials. This is also connected with high frequency electron-phonon-electron vibration. Effective mass of electrons is found to be less.

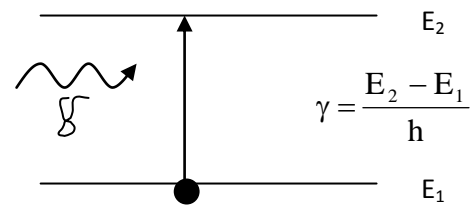
3. a. [6]

Expression for energy density:

Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E_1 and E_2 be the energy levels in an atom and N_1 and N_2 be the number density in these levels respectively. Let U_ν be the energy density of the radiation incident.



Rate of absorption is proportional to the number of atoms in lower state and also on the energy density U_ν .

$$\text{Rate of absorption} = B_{12} N_1 U_\nu$$

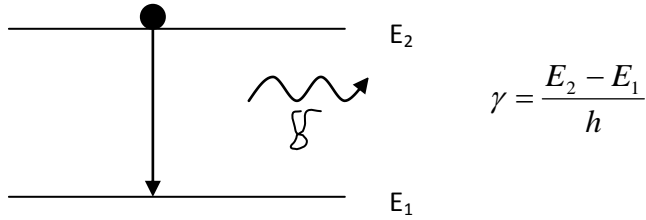
Here B_{12} is a constant known as Einstein's coefficient of spontaneous absorption.

Spontaneous emission:

It is a process in which atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

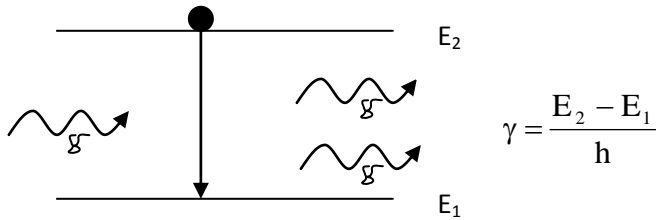
$$\text{Rate of spontaneous absorption} = A_{21} N_2$$

Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.



Stimulated emission:

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U_γ .

Rate of stimulated emission = $B_{21} N_2 U_\gamma$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12} N_1 U_\gamma = A_{21} N_2 + B_{21} N_2 U_\gamma$$

$$U_\gamma = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Rearranging this, we get

$$U_\gamma = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right]$$

By Boltzmann's law, $\frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$

Hence

$$U_\gamma = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\gamma}{kT}} - 1} \right]$$

From Planck's radiation law,

$$U_\gamma = \frac{8\pi h \gamma^3}{c^3} \left[\frac{1}{e^{\frac{h\gamma}{kT}} - 1} \right]$$

Comparing these expressions, we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h \gamma^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1$$

$$\therefore U_\gamma = \frac{A}{B} \left[\frac{1}{e^{\frac{h\gamma}{kT}} - 1} \right]$$

3.b. [4]

$$\text{Number of Photons} = n = \frac{P X \lambda}{hc} = 1.71 \times 10^{16} / s$$

4.a. [7]



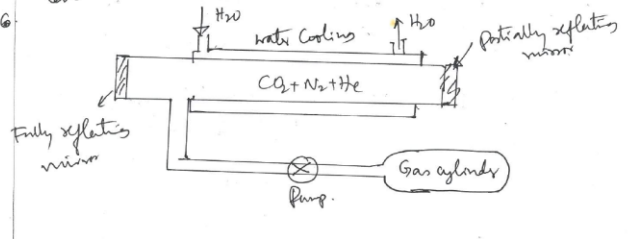
CO₂ LASER

It belongs to the category of gas lasers with 4 level laser system.

Active medium: Mixture of CO₂, N₂ & He in the ratio 1:2:8. Nitrogen is used as Molecular nitrogen absorbs energy more efficiently from electric discharge than CO₂ does.

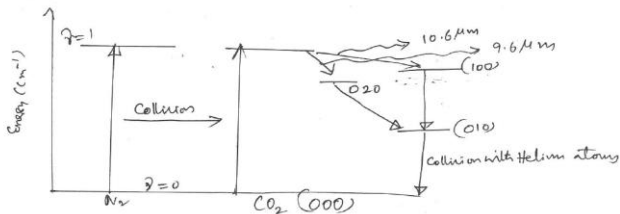
Construction:

1. A glass plasma tube of 10-15mm diameter with an integral water cooling jacket.
2. Partially reflecting and fully reflecting mirrors are mounted on the ends of the tube.
3. A coaxial water jacket around the plasma tube serves to remove heat through the tube walls.
4. Optical pumping is achieved by electric discharge caused by applying a p.d of over 1000V.
5. Helium gas conducts away heat generated and also catalyzes collisional deexcitation of CO₂ molecules from lower laser level.



Working:

- ① CO_2 is linear molecule and has three fundamental modes of vibration: Symmetric stretching, bending and asymmetric stretching represented as (001) , (100) & (010) with corresponding energy states E_1, E_2, E_3, \dots .
- ② The energy level 001 corresponding to asymmetric stretching is the upper laser level. The energy levels (100) , (010) are lower and are the lower laser states.
- ③ During electric discharge, electrons released due to ionization excite N_2 molecules to the first vibrational level which is close to upper laser level of CO_2 .
- ④ N_2 molecules undergo collision with CO_2 molecules and cause their excitation leading to population inversion.
- ⑤ Lasing action usually takes place due to transitions from 001 to 100 , (010) to (010) corresponding to a wavelength $\lambda = 10.6 \mu m$ and $9.6 \mu m$ respectively.
- ⑥ The CO_2 molecules deexcite to ground state by collision with Helium atoms.



4.b. [3]

Laser Welding:

Lasers are used for welding metals, by virtue of its ability to focus large power in a small region.

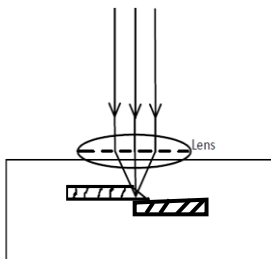
In laser welding, the laser beam is focussed to the spot to be welded by means of a lens and a very high temperature is generated at this spot by virtue of its high intensity.

Due to the heat, the metal used for welding is melted and a strong, homogeneous joint is formed.

Advantages:

- a) It is a contactless process. Therefore, unwanted materials like oxides can be eliminated.
- b) Only the focussed region is heated and so it can be used in micro-electronics, where heat-sensitive components are involved.
- c) There is no mechanical stress on the components involved, thus there is no deformation.
- d) It can be used to weld joints where man cannot physically be present, for example, in nuclear power plants.

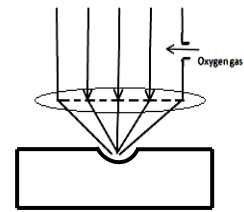
Carbon di-oxide lasers are the most popular one in this particular applica



Laser Cutting:

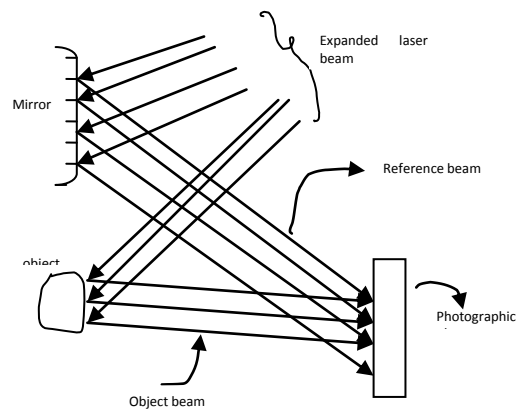
A laser beam, assisted by a jet of gas is used for cutting materials. The laser beam is surrounded by a nozzle into which oxygen gas is fed. The gas helps in combustion and also assists by blowing out the molten metal. The flowing action increases the depth and also the speed of cutting. The cutting accuracy is well controlled.

Laser cutting is used in the tailoring industry where large number of layers of cloth is stockpiled. In this case the laser beam is focussed on the pile and moved along the path, along which the cut is to be made.



5.a. [7]

Recording of the image of an object: [4]

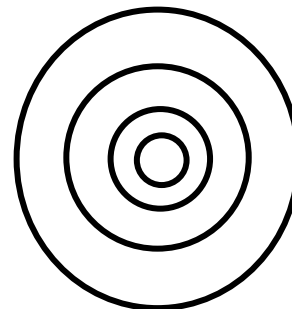


The given object, mirror and photographic plates are arranged as shown in the figure. An expanded laser beam is directed on this arrangement in which a part of the beam is incident on the mirror and the rest falls on the object.

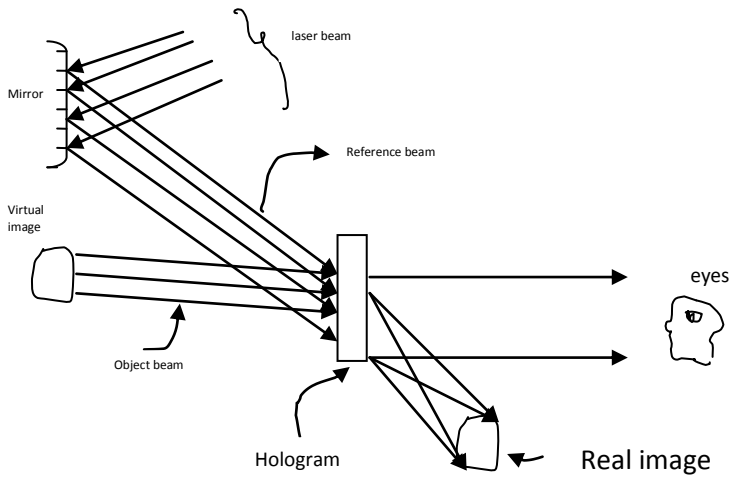
The photographic plate is placed such that it receives the reflected beam (reference beam) and light scattered from the object (object beam).

Due to the interference between plane wavefronts of reference beam and spherical wavefronts of object beam, an interference pattern is formed on the photographic plate. This will be called as a hologram.

Hologram :



Reconstruction of the image: [3]



The image of the object is reconstructed by passing the reference beam from the same laser through the hologram, which is oriented with respect to the reference beam. The reference beam is diffracted and two images of the object, real image and virtual image are seen.

5b. [3]

$$\frac{N_2}{N_1} = \frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)}} = 1.6 \times 10^{-30}$$

Substitute for T = 330k, we get $\lambda = 709\text{nm}$

6.a. [6]

Expression for condition for Numerical Aperture:

Consider a light ray falling in to the optical fibre at an angle of incidence θ_0 equal to acceptance angle. Let n_0 be the refractive index of the surrounding medium.

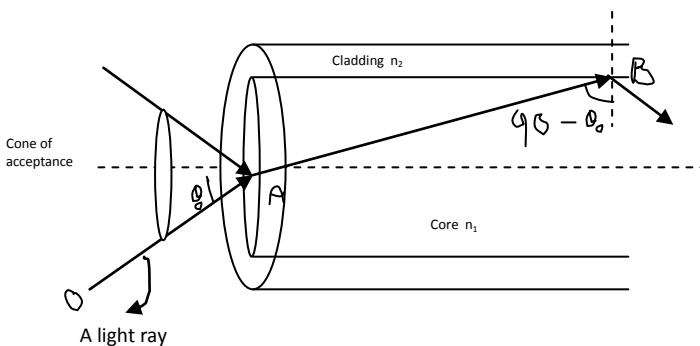
Let n_1 be the refractive index of the core.

Let n_2 be the refractive index of the cladding.

From Snell's Law:

For the ray OA $n_0 \sin \theta_0 = n_1 \sin \theta_1$

$$= n_1 (1 - \cos^2 \theta_1)^{\frac{1}{2}} \dots \dots \dots (1)$$



For the ray AB

$$n_1 \sin(90 - \theta_1) = n_2 \sin 90$$

[here the angle of incidence is $(90 - \theta_1)$ for which angle of refraction is 90°].

$$n_1 \cos \theta_1 = n_2$$

Substituting for $\cos \theta_1$ in equation (1)

$$n_0 \sin \theta_0 = n_1 \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If the medium surrounding the fibre is air then $n_0 = 1$,

$$\text{Numerical aperture} = \sin \theta_0 = \sqrt{n_1^2 - n_2^2}$$

The total internal reflection will take place only if the angle of incidence $\theta_i < \theta_0$

$$\therefore \sin \theta_i < \sin \theta_0$$

$$\sin \theta_i < \sqrt{n_1^2 - n_2^2}$$

This is the condition for propagation.

6.b [4]

$$\sin \theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$\sin 30 = 0.5 = \frac{\sqrt{n_1^2 - n_2^2}}{1} \dots \dots \dots (1)$$

For Water, $\sin \theta = \frac{\sqrt{n_1^2 - n_2^2}}{1.33} \dots \dots \dots (2)$

From (1) and (2) $\theta = 23^\circ 7'$

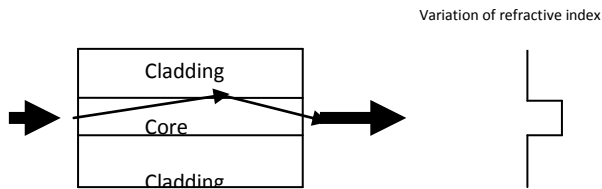
7.a [7]

Mode: [1] It represents a specific electric field and magnetic field distribution pattern that is repeated along the fiber at regular intervals. Only a certain discrete number of modes are capable of propagating along the fiber. These are the electromagnetic waves that satisfy the homogeneous wave equation and the boundary conditions.

TYPES:

1. Single mode fiber: [2]

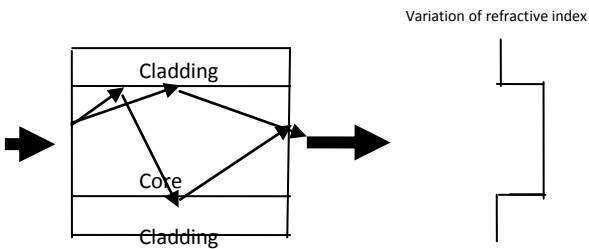
It consists of a core which is made of glass having refractive index n_1 . The core is surrounded by a cladding made of glass which is of refractive index n_2 where $n_1 > n_2$. Core diameter is around 5-10 μm . The core is narrow and hence it can guide just a single mode.



- No modal dispersion
- Difference between n_1 & n_2 is less. Critical angle is high. Low numerical aperture.
- Low Attenuation -0.35db/km
- Bandwidth -100GHz

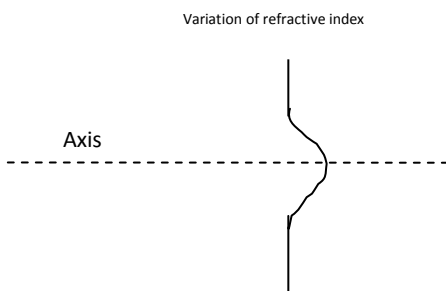
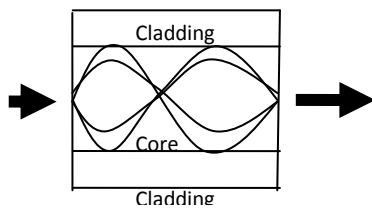
Step index multimode fibre : [2]

- Here the diameter of core is larger so that large number of rays can propagate. Core diameter is around 50. μm .
- High modal dispersion
- Difference between n_1 & n_2 is high. Low Critical angle. Large numerical aperture.
- Low Attenuation -0.35db/km
- Bandwidth -500MHz
- Allows several modes to propagate



Graded index multimode fiber: [2]

In this type, the refractive index decreases in the radially outward direction from the axis and becomes equal to that of the cladding at the interface. Modes travelling close to the axis move slower whereas the modes close to the cladding move faster. As a result the delay between the modes is reduced. This reduces modal dispersion.



- Low modal dispersion
- High data carrying capacity.
- High cost
- Many modes propagate
- Bandwidth -10GHz

7.b. [3]

$$\text{Normalized frequency number } V = \frac{2\pi r}{\lambda} (\text{NA})$$

$$\text{Mode number } N = V^2/2$$

On substitution, we get $V = 230$

$$N = 26568$$