USN	CMRIT

Internal Assessment Test – II

Sub:	**Engineering Mathematics - IV**					Sub Code:	15MAT41	Branch:	**ECE, IS**	
Date:	16.04.2018	Duration:	90 min	Max Marks:	50	Sem / Sec:	**IV/ EC A & C, IS A & B**		OBE	
Answer Question 1 or Question 2 and any SIX questions from Question 3 to 10.								MARKS	CO	RBT

1. Discuss the transformation $w = z + \frac{1}{z}, z \neq 0$. [08] CO3 L3

2. Apply Milne's method to compute y(0.8) given $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and the following table of initial values: [08] CO1 L3

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

3. Using Taylor series method, compute y(0.2) and y(0.4) given $\frac{dy}{dx} = x^2y+1$, with $y(0) = 1$. Consider up to fourth degree terms. [07] CO1 L3

4. Solve the differential equation $\frac{dy}{dx} = \log(x + y)$ under the initial condition $y(0) = 2$, by using modified Euler's method. Find y(0.4) by taking $h = 0.2$. [07] CO1 L3

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5. Approximate the solution of the initial value problem $\frac{dy}{dx} = \frac{y-x}{y+x}$, at $x = 0.2$ by Runge-Kutta 4th order method, given $y(0) = 1$. Take $h = 0.2$ [07] CO1 L3

6. Derive mean and variance of the Poisson distribution. [07] CO4 L3

7. A random variable has the following density function: $f(x) = \begin{cases} kx^2 & -3 \le x \le 3 \\ 0 & elsewhere \end{cases}$ [07] CO4 L3

Evaluate k and find the mean and variance of the distribution . Also find $P(X \le 2)$

8. The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men now aged 60, (i) exactly 9 will live to be 70 (ii) at most 9 will live to be 70 (iii)at least 7 will live to be 70? [07] CO4 L3

9. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D, given that φ (0.5)= 0.19 and φ (1.4)=0.42 where φ (z) is the area under the standard normal curve from 0 to z. [07] CO4 L3

10. The sale per day in a shop is exponentially distributed with average sale amounting to Rs.1000 and net profit 8%. Find the probability that the net profit exceeds Rs. 300 on a day. [07] CO4 L3

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Solution — IAT-2 (April 2018)

Engineering mathematics – 4

1. The transformation $w = z + \frac{1}{z}$, $z \neq 0$

<u>Conformality</u> : $f(z) = z + \frac{1}{z}$

$$f'(z) = 1 - \frac{1}{z^2} = \frac{z^2 - 1}{z^2}$$

$$= \frac{(z+1)(z-1)}{z^2}$$

$f'(z) = 0 \Rightarrow z = \pm 1$

∴ the mapping is conformal at all points in the domain of defn. except $z = \pm 1$

Writing $w = u + iv$ and $z = r(\cos\theta + i\sin\theta)$, — (1)

$$u + iv = r(\cos\theta + i\sin\theta) + \frac{1}{r}(\cos\theta - i\sin\theta)$$

$$= \left(r + \frac{1}{r}\right)\cos\theta + i\left(r - \frac{1}{r}\right)\sin\theta$$

$\Rightarrow u = \left(r + \frac{1}{r}\right)\cos\theta$ and $v = \left(r - \frac{1}{r}\right)\sin\theta$ — I

— (1)

Eliminating θ from I,

$$\frac{u^2}{\left(r + \frac{1}{r}\right)^2} + \frac{v^2}{\left(r - \frac{1}{r}\right)^2} = 1 \quad \text{— (1)}$$

When r = a constant ($\neq 1$), the above eqn. represents an ellipse with foci — (1)

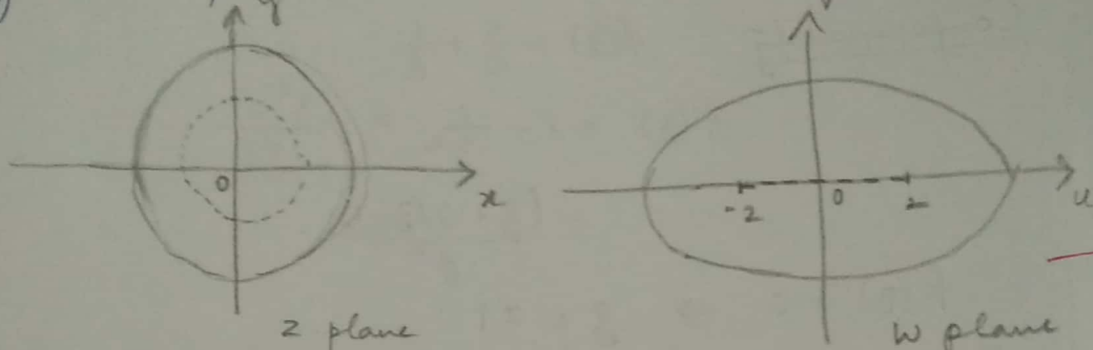
$\left(\pm\sqrt{\left(r + \frac{1}{r}\right)^2 - \left(r - \frac{1}{r}\right)^2},\ 0\right) = (\pm 2, 0)$ which is ind. of r.

∴ the image of a family of concentric circles $r = c$ is a family of confocal ellipses

When $r = 1$, $v = 0$ and $u = 2\cos\theta$

Since $-1 \le \cos\theta \le 1$, we have $-2 \le u \le 2$

$\therefore$ The image of the unit circle is the segment of the real axis from -2 to 2



—— (1)

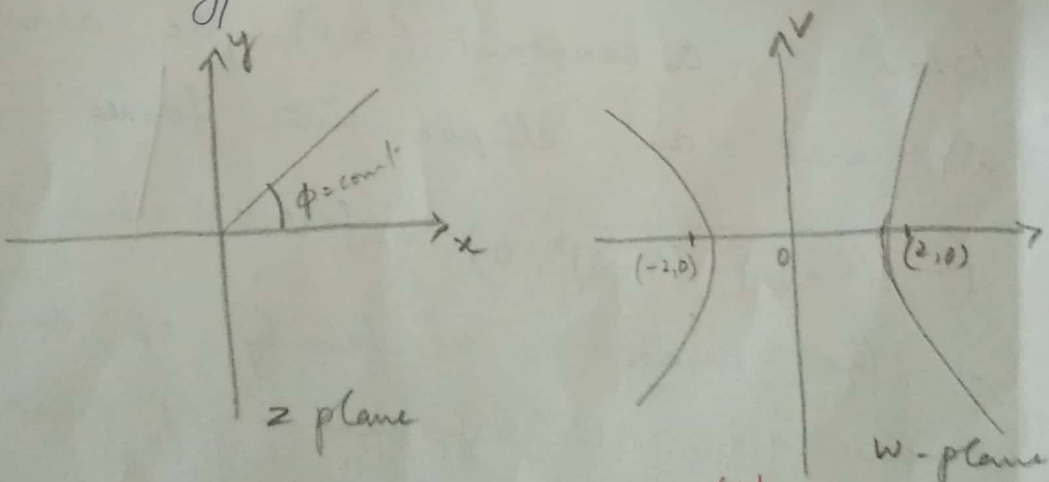
Eliminating 'r' from I, we get

$$\frac{u^2}{\cos^2\theta} - \frac{v^2}{\sin^2\theta} = 4 \quad \text{i.e} \quad \frac{u^2}{4\cos^2\theta} - \frac{v^2}{4\sin^2\theta} = 1$$ —— (1)

When θ = a constant, the above eqn represents a hyperbola with focus —— (1)

$$\left(\pm\sqrt{4\cos^2\theta + 4\sin^2\theta},\ 0\right) = (\pm 2, 0)$$

$\therefore$ the image of ~~a~~ radial lines are ~~a~~ confocal hyperbolae.



—— (1)

2.

x	y	$y' = z$	$y'' = 1 - 2yy' = z'$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$	$z_0' = 1$
$x_1 = 0.2$	$y_1 = 0.02$	$z_1 = 0.1996$	$z_1' = 0.992$
$x_2 = 0.4$	$y_2 = 0.0795$	$z_2 = 0.3937$	$z_2' = 0.9374$
$x_3 = 0.6$	$y_3 = 0.1762$	$z_3 = 0.5689$	$z_3' = 0.7995$

— (1)

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3)$$

$$= 0 + \frac{4 \times 0.2}{3}(2 \times 0.1996 - 0.3937 + 2 \times 0.5689)$$

$$= 0.3049$$ —— $(1\tfrac{1}{2})$

$$z_4^{(P)} = z_0 + \frac{4h}{3}(2z_1' - z_2' + 2z_3')$$

$$= 0 + \frac{4 \times 0.2}{3}(2 \times 0.992 - 0.9374 + 2 \times 0.7995)$$

$$= 0.7055$$ —— (1)

$$y_4^{(c)} = y_2 + \frac{h}{3}(z_2 + 4z_3 + z_4)$$

$$= 0.0795 + \frac{0.2}{3}(0.3937 + 4 \times 0.5689 + 0.7055)$$

$$= 0.3045$$ —— $(1\tfrac{1}{2})$

$$z_4' = 1 - 2y_4^{(P)} z_4^{(P)} = 1 - 2 \times 0.3049 \times 0.7055$$ —— (1)

$$= 0.5698$$

$$z_4^{(c)} = z_2 + \frac{h}{3}(z_2' + 4z_3' + z_4')$$

$$= 0.3937 + \frac{0.2}{3}(0.9374 + 4 \times 0.7995 + 0.5698)$$

$$= 0.7074$$ —— (1)

Applying corrector formula again,

$$y_4^{(c)} = 0.0795 + \frac{0.2}{3}(0.3937 + 4 \times 0.5689 + 0.7074) = 0.3046$$

$$\boxed{y(0.8) = 0.3046}$$ —— (1)

3. $\frac{dy}{dx} = x^2y - 1, \quad y(0) = 1 \quad$ ie $\quad x_0 = 0, \; y_0 = 1$

$$y' = x^2y - 1 \qquad y'(0) = -1$$

$$y'' = x^2y' + 2xy \qquad y''(0) = 0 \text{ —— (1)}$$

$$y''' = x^2y'' + 2xy' + 2xy' + 2y$$

$$= x^2y'' + 4xy' + 2y \qquad y'''(0) = 2 \text{ —— (1)}$$

$$y^{iv} = x^2y''' + 2xy'' + 4xy'' + 4y' + 2y'$$

$$= x^2y''' + 6xy'' + 6y' \qquad y^{iv}(0) = -6 \text{ —— (1)}$$

$$y(x) = y(x_0) + (x - x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \cdots$$

$$= 1 + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) + \frac{x^4}{4!}y^{iv}(0) \text{ —(1)}$$

(neglecting higher pwrs of x)

$$= 1 - x + \frac{x^3}{3} - \frac{x^4}{4} \text{ —— (1)}$$

$$y(0.2) = 0.80227 \text{ —— (1)}$$

$$y(0.4) = 0.61493 \text{ —— (1)}$$

4. $\frac{dy}{dx} = \log(x+y), \quad y(0) = 2$

<u>I Stage</u>

$$x_0 = 0 \qquad y_0 = 2 \qquad h = 0.2$$

$$f(x,y) = \log(x+y) \text{ —— (1/2)}$$

$$f(x_0, y_0) = 0.6931, \quad x_1 = x_0 + h = 0.2$$

$$y_1^{(0)} = y_0 + h\, f(x_0, y_0)$$

$$= 2 + 0.2 \times 0.6931$$

$$= 2.1386 \text{ —— (1)}$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 2 + 0.1\left[0.6931 + f(0.2, 2.1386)\right]$$

$$= 2.1543 \quad \text{—— } (1)$$

$$y_1^{(2)} = 2 + 0.1\left[0.6931 + f(0.2, 2.1543)\right]$$

$$= 2.1549 \quad \text{—— } (1/2)$$

$$y_1^{(3)} = 2 + 0.1\left[0.6931 + f(0.2, 2.1549)\right]$$

$$= 2.1550$$

$$y(0.2) = 2.1550 \quad \text{—— } (1)$$

<u>Stage 2</u>

$x_0 = 0.2 \quad y_0 = 2.1550, \quad h = 0.2$

$f(x_0, y_0) = \log(0.2 + 2.1550) = 0.8565$ —— (1/2)

$x_1 = x_0 + h = 0.4$

$y_1^{(0)} = y_0 + h\, f(x_0, y_0) = 2.3263$ —— (1/2)

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 2.1550 + 0.1\left[0.8565 + f(0.4, 2.3263)\right]$$

$$= 2.3409 \quad \text{—— } (1/2)$$

$$y_1^{(2)} = 2.1550 + 0.1\left[0.8565 + f(0.4, 2.3409)\right]$$

$$= 2.3414 \quad \text{—— } (1/2)$$

$$y_1^{(3)} = 2.1550 + 0.1\left[0.8565 + f(0.4, 2.3414)\right]$$

$$= 2.3414$$

$$y(0.4) = 2.3414 \quad \text{—— } (1)$$

Q.5 Approximate the solution of the initial value Problem $\frac{dy}{dx} = \frac{y-x}{y+x}$ at $x=0.2$ by R-K 4th order method, given $y(0)=1$. Take $h=0.2$

here $f(x,y) = \frac{y-x}{y+x}$, $x_0=0$, $y_0=1$, $h=0.2$

$k_1 = h\,f(x_0, y_0) = (0.2)\,f(0,1) = (0.2)\left(\frac{1-0}{1+0}\right) \boxed{= 0.2}$

$k_2 = h\,f\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$ —— (1)

$= (0.2)\,f(0.1, 1.1)$

$= (0.2)\left(\frac{1.1-0.1}{1.1+0.1}\right)$

$\boxed{k_2 = 0.1667}$ —— (1)

$k_3 = h\,f\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$

$= (0.2)\,f(0.1, 1.08335)$

$= (0.2)\left[\frac{1.08335-0.1}{1.08335+0.1}\right]$

$\boxed{k_3 = 0.1662}$ —— (1)

$k_4 = h\,f(x_0+h, y_0+k_3)$

$= (0.2)\,f(0.2, 1.1662)$

$= (0.2)\left[\frac{1.1662-0.2}{1.1662+0.2}\right]$

$\boxed{k_4 = 0.1414}$ —— $(1\frac{1}{2})$

$\therefore\ y(0.2) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$ —— $(1\frac{1}{2})$

$= 1 + \frac{1}{6}[0.2 + 2(0.1667) + 2(0.1662) + 0.1414]$

$\boxed{y(0.2) = 1.1679}$ or $\boxed{1.16786}$ —— (1)

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(6.) Mean of Poisson Distribution –

Mean (μ) = $\sum_{x=0}^{\infty} x\, P(x)$

$$= \sum_{x=0}^{\infty} x \frac{e^{-m} \cdot m^x}{x!} = \sum_{x=1}^{\infty} \frac{e^{-m} m^x}{(x-1)!}$$

$$= e^{-m}\left[m + \frac{m^2}{1!} + \frac{m^3}{2!} + \frac{m^4}{4!} + \cdots\right] = m e^{-m}\left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots\right)$$

$$\mu = m e^{-m} \cdot e^{m} = m$$

$$\boxed{\therefore \quad \mu = m = np} \quad \text{—— (3)}$$

Variance:

$$V = \sum_{x=0}^{\infty} x^2 P(x) - \mu^2$$

$$= \sum_{0}^{\infty} (x^2 - x + x) P(x) - \mu^2$$

$$= \sum_{0}^{\infty} x(x-1) P(x) + \sum_{0}^{\infty} x\, P(x) - \mu^2$$

$$= \sum_{0}^{\infty} x(x-1) \frac{e^{-m} m^x}{x!} + \mu - \mu^2$$

$$= \sum_{x=2}^{\infty} \frac{e^{-m} m^x}{(x-2)!} + \mu - \mu^2$$

$$= e^{-m}\left[\frac{m^2}{0!} + \frac{m^3}{1!} + \frac{m^4}{2!} + \cdots\right] + \mu - \mu^2$$

$$= m^2 e^{-m}\left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots\right) + \mu - \mu^2$$

$$= m^2 e^{-m} \cdot e^{m} + \mu - \mu^2 = m^2 + m - m^2$$

$$\therefore \quad \boxed{V = m = np} \quad \text{—— (4)}$$

(7)

$$f(x) = \begin{cases} kx^2 , & -3 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

we have $f(x) \geq 0$, if $k \geq 0$.

& we must have $\int_{-\infty}^{\infty} f(x)\,dx = 1$ —— (1)

$$\Rightarrow \int_{-\infty}^{-3} 0\,dx + \int_{-3}^{3} kx^2\,dx + \int_{3}^{\infty} 0\,dx = 1$$

$$k \left.\frac{x^3}{3}\right|_{-3}^{3} = 1$$

$$\Rightarrow \frac{k}{3}\left[(3)^3 - (-3)^3\right] = 1 \Rightarrow \frac{k}{3}(54) = 1$$

$$k = \frac{1}{18}$$ —— (1)

Mean:

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-3}^{3} x\left(\frac{1}{18}x^2\right)dx = \frac{1}{18}\int_{-3}^{3} x^3\,dx$$

$$= \frac{1}{18}\left.\left(\frac{x^4}{4}\right)\right|_{-3}^{3} = \frac{1}{72}\left[(3)^4 - (-3)^4\right] = 0$$ —— (2)

Variance:

$$V = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \int_{-3}^{3} \frac{1}{18} x^4\,dx - 0$$ —— (2)

$$= \frac{1}{18}\cdot\left.\frac{x^5}{5}\right|_{-3}^{3} = \frac{1}{90}\left((3)^5 - (-3)^5\right) = \frac{486}{90} = 5.4$$

$$P(X<2) = \int_{-3}^{2} \frac{1}{18}x^2\,dx = \frac{5}{12}$$ —— (1)

8. The Probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men now aged 60, (i) exactly 9 will live to be 70. (ii) atmost 9 will live to be 70 iii) at least 7 will live to be 70?

Let X be the no. of persons aged 60 years living up to 70 years. —— (1)

$p = 0.65$, hence, $q = 0.35$, $n = 10$ —— (1)

by Binomial distribution,

we have $P(X) = {}^{n}C_{x} \, p^{x} q^{n-x}$

$P(x) = {}^{10}C_{x} (0.65)^{x} (0.35)^{10-x}$ —— (1)

③

probability of exactly 9 men will live to be 70 is

$$P(x=9) = {}^{10}c_9 (0.65)^9 (0.35)^1$$

$$= \frac{10!}{9!\,1!} (0.65)^9 (0.35)$$

$$= 10 \times (0.65)^9 \times (0.35)$$

$$\boxed{P(x=9) = 0.0724}$$ —— (1)

ii) prob. of atmost 9 men will live to be 70 is

$$P(x \leq 9) = 1 - p(x=10)$$

$$= 1 - \left[{}^{10}c_{10} (0.65)^{10} (0.35)^0\right]$$

$$= 1 - 0.0134$$

$$\boxed{P(x \leq 9) = 0.9865}$$ —— (1)

iii) Pro. of atleast 7 men will live to be 70 is

$$P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$$

$$= {}^{10}c_7 (0.65)^7 (0.35)^3 + {}^{10}c_8 (0.65)^8 (0.35)^2$$

$$+ {}^{10}c_9 (0.65)^9 (0.65)^1 + {}^{10}c_{10} (0.65)^{10} (0.35)^0$$

$$= (120)(0.65)^7 (0.35)^3 + 45 (0.65)^8 (0.35)^2$$

$$+ 10 (0.65)^9 (0.35) + (0.65)^{10}$$

$$\boxed{P(x \geq 7) = 0.5138}$$ —— (1+1)

(9) Given

$P(x<45) = .31$

& $P(x>64) = 0.08$ } — (1)

we have

$z = \frac{x-\mu}{\sigma}$

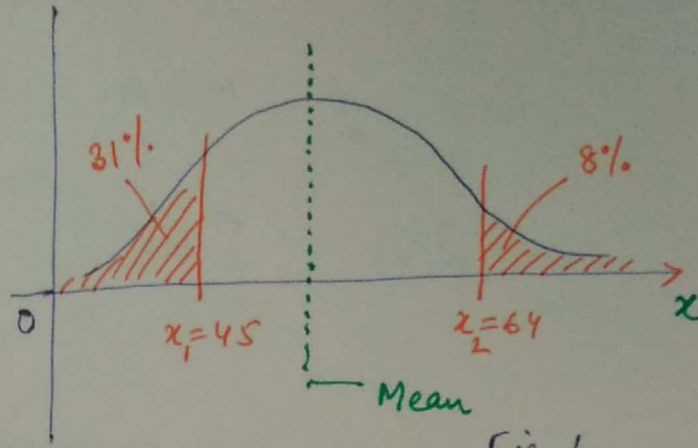

Fig. 1

Let $z_1 = \frac{x_1-\mu}{\sigma} = \frac{45-\mu}{\sigma}$ — (1)

— (1/2)

& $z_2 = \frac{x_2-\mu}{\sigma} = \frac{64-\mu}{\sigma}$ — (2)

— (1/2)

— (1)

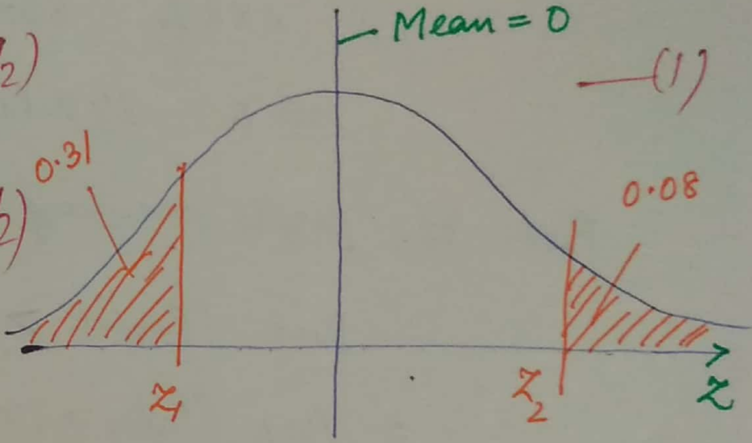

Fig. 2

From fig. (2)

$0.5 - A(z_1 < z < 0) = 0.31$

$A(z_1 < z < 0) = 0.5 - 0.31 = 0.19$

$\Rightarrow z_1 = -0.5$ (neg. sign as z_1 is left to the mean)

— (1)

&

$0.5 - A(0 < z < z_2) = 0.08$

$A(0 < z < z_2) = 0.42 \Rightarrow \phi(z_2) = 0.42$

$\Rightarrow z_2 = 1.4$ — (1)

Sub. z_1 & z_2 in (1) & (2) respectively

$\frac{45-\mu}{\sigma} = -0.5$ & $\frac{64-\mu}{\sigma} = 1.4$

$\Rightarrow \mu = 50$ & $\sigma = 10$

— (1) — (1)

(10) X = Sales in the shop ——(1)

Given $\frac{1}{\alpha} = 1000$ Rs. $\Rightarrow \alpha = \frac{1}{1000 \text{ Rs.}}$ ——(1)

$$\therefore f(x) = \begin{cases} \frac{1}{1000} e^{-x/1000}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$ ——(1)

Let A be the amount for which profit is 8%

$\therefore$ $A \times 8\% = 300$

$\Rightarrow A = 3750$ Rs. ——(1)

Prob. of profit exceeding Rc. 300

= Prob. (Profit > 300) ——(1)

= Prob. (Sales > 3750) or $P(x > 3750)$

$$= \int_{3750}^{\infty} (0.001) e^{-(.001)x} \, dx$$ ——(1)

$= e^{-3.75}$ (on a single day) ——(1)

~~The prob. that it repeats on the following day is~~

$$\cancel{= e^{-3.75} \times e^{-3.75}}$$