USN

CELEBRATING 25 YEARS
CMRIT
CMR INSTITUTE OF TECHNOLOGY, BENGALURU.
ACCREDITED WITH A+ GRADE BY NAAC

Internal Assessment Test 3 – May 2018

Sub:	Engineering Mathematics II				Sub Code:	17MAT21		
Date:	21/05/2018	Duration:	90 mins	Max Marks: 50	Sem / Sec:	II/A,E&G	OBE	

Question 1 is compulsory and answer any SIX questions from the rest.

		MARKS	CO	RBT
1.	(a) Evaluate $L\left\{\frac{\cos 2t-\cos 3t}{t}\right\}$ (b) Evaluate $L\{t^2e^{-3t}\sin 2t\}$.	[08]	CO6	L3
2.	A periodic function of period 2a is defined by, $f(t)=\begin{cases} E & for\ 0\le t\le a \\ -E & for\ a<t\le 2a\end{cases}$ where E is a constant. Show that $L\{f(t)\}=\frac{E}{s}\tanh\left(\frac{as}{2}\right)$.	[07]	CO6	L3
3.	Express $f(t)=\begin{cases}\cos t & 0<t<\pi \\ \cos 2t & \pi<t<2\pi \\ \cos 3t & t>2\pi\end{cases}$ in terms of unit step function and hence find its Laplace transform.	[07]	CO6	L3
4.	Solve $y''(t)-2y'(t)+y(t)=e^t$ subject to the conditions, y(0)=2, $y'(0)=-1$ by using Laplace transform.	[07]	CO6	L3
5.	Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+9)}\right\}$ by using Convolution theorem.	[07]	CO6	L3

USN

CELEBRATING 25 YEARS
CMRIT
CMR INSTITUTE OF TECHNOLOGY, BENGALURU.
ACCREDITED WITH A+ GRADE BY NAAC

Internal Assessment Test 3 – May 2018

Sub:	Engineering Mathematics II				Sub Code:	17MAT21		
Date:	21/05/2018	Duration:	90 mins	Max Marks: 50	Sem / Sec:	II/A,E&G	OBE	

Question 1 is compulsory and answer any SIX questions from the rest.

		MARKS	CO	RBT
1.	(a) Evaluate $L\left\{\frac{\cos 2t-\cos 3t}{t}\right\}$ (b) Evaluate $L\{t^2e^{-3t}\sin 2t\}$.	[08]	CO6	L3
2.	A periodic function of period 2a is defined by, $f(t)=\begin{cases} E & for\ 0\le t\le a \\ -E & for\ a<t\le 2a\end{cases}$ where E is a constant. Show that $L\{f(t)\}=\frac{E}{s}\tanh\left(\frac{as}{2}\right)$.	[07]	CO6	L3
3.	Express $f(t)=\begin{cases}\cos t & 0<t<\pi \\ \cos 2t & \pi<t<2\pi \\ \cos 3t & t>2\pi\end{cases}$ in terms of unit step function and hence find its Laplace transform.	[07]	CO6	L3
4.	Solve $y''(t)-2y'(t)+y(t)=e^t$ subject to the conditions, y(0)=2, $y'(0)=-1$ by using Laplace transform.	[07]	CO6	L3
5.	Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+9)}\right\}$ by using Convolution theorem.	[07]	CO6	L3

6. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dxdydz$. [7] CO3 L3

7. Evaluate the double integral $\int_{0}^{a}\int_{y}^{a}\frac{x}{x^2+y^2}$ dxdy by changing the order of integration. [7] CO3 L3

8. Find the area enclosed by the curve r = a (1+cosθ) above the initial line. [7] CO3 L3

9. Evaluate $\int_{0}^{\infty}\int_{0}^{\infty}e^{-(x^2+y^2)}dxdy$ by changing into polar coordinates. [7] CO3 L3

10. (a) Find $L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\}$ (b) Find $L^{-1}\left\{\log\frac{s^2+1}{s(s+1)}\right\}$. [7] CO6 L3

6. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dxdydz$. [7] CO3 L3

7. Evaluate the double integral $\int_{0}^{a}\int_{y}^{a}\frac{x}{x^2+y^2}$ dxdy by changing the order of integration. [7] CO3 L3

8. Find the area enclosed by the curve r = a (1+cosθ) above the initial line. [7] CO3 L3

9. Evaluate $\int_{0}^{\infty}\int_{0}^{\infty}e^{-(x^2+y^2)}dxdy$ by changing into polar coordinates. [7] CO3 L3

10. (a) Find $L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\}$ (b) Find $L^{-1}\left\{\log\frac{s^2+1}{s(s+1)}\right\}$. [7] CO6 L3

SOLUTIONS TO IAT-03 (1)

10. (a) To find $L^{-1}\left\{\dfrac{4S+5}{(S+1)^2(S+2)}\right\}$

$$\frac{4S+5}{(S+1)^2(S+2)} = \frac{A}{S+1} + \frac{B}{(S+1)^2} + \frac{C}{S+2} \quad \text{—— (1)}$$

$\Rightarrow 4S+5 = A(S+1)(S+2) + B(S+2) + C(S+1)^2$

$\Rightarrow 4S+5 = A(S^2+3S+2) + BS + 2B + CS^2 + 2CS + C$

Equating the coefficients of S^2, S and constants.

$S^2 : 0 = A + C$

$S : 4 = 3A + B + 2C$

Const : $5 = 2A + 2B + C$

$A = 3$, $B = 1$ and $C = -3$.

$$\therefore \frac{4S+5}{(S+1)^2(S+2)} = \frac{3}{S+1} + \frac{1}{(S+1)^2} - \frac{3}{S+2}. \quad \text{—— (2)}$$

$$\therefore L^{-1}\left\{\frac{4S+5}{(S+1)^2(S+2)}\right\} = 3L^{-1}\left(\frac{1}{S+1}\right) + L^{-1}\left(\frac{1}{(S+1)^2}\right) - 3L^{-1}\left(\frac{1}{S+2}\right)$$

$$= 3e^{-t} + te^{-t} - 3e^{-2t} \quad \text{—— (1)}$$

1. (a) To evaluate: $L\left\{\dfrac{\cos 2t - \cos 3t}{t}\right\}$

$$L(\cos 2t - \cos 3t) = \frac{S}{S^2+4} - \frac{S}{S^2+9}. \quad \text{—— (1)}$$

$$\therefore L\left\{\frac{\cos 2t - \cos 3t}{t}\right\} = \int_S^\infty \left(\frac{S}{S^2+4} - \frac{S}{S^2+9}\right) dS \quad \text{—— (1)}$$

$$= \frac{1}{2}\log\left(\frac{S^2+9}{S^2+4}\right) \quad \text{—— (3)}$$

(b) To evaluate: $L\{t^2 e^{-3t} \sin 2t\}$

$$L(\sin 2t) = \frac{2}{s^2+4} \quad \text{—— (1)}$$

$$L(t^2 \sin 2t) = \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right) = 2\frac{d}{ds}\left[\frac{-1}{(s^2+4)^2} \times 2s\right] \quad \text{—— (1)}$$

$$= -4\,\frac{d}{ds}\left\{\frac{s}{(s^2+4)^2}\right\}$$

$$= -4\left[\frac{(s^2+4)^2 - s \times 2(s^2+4) \times 2s}{(s^2+4)^4}\right]$$

$$= \frac{-4(s^2+4)}{(s^2+4)^4}\left\{s^2+4-4s^2\right\}$$

$$= \frac{-4}{(s^2+4)^3}\left(-3s^2+4\right) \quad \text{—— (1)}$$

$$L(e^{-3t} t^2 \sin 2t) = \frac{-4\left[-3(s+3)^2+4\right]}{\left[(s+3)^2+4\right]^3}. \quad \text{—— (1)}$$

2. $$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt \quad \text{—— (1)}$$

$$= \frac{1}{1-e^{-2as}} \int_0^{2a} e^{-st} f(t)\,dt \quad \text{—— (1)}$$

$$= \frac{1}{1-e^{-2as}}\left[\int_0^a e^{-st} E\,dt + \int_a^{2a} e^{-st}(-E)\,dt\right] \quad \text{—— (1)}$$

$$= \frac{1}{1-e^{-2as}}\left[E\left(\frac{e^{-st}}{-s}\right)_0^a - E\left(\frac{e^{-st}}{-s}\right)_a^{2a}\right]$$

—— (1)

$$= \frac{E}{S} \times \frac{1}{1-e^{-2as}} \left[\left\{ e^{-st} \right\}_a^{2a} - \left\{ e^{-st} \right\}_0^{a} \right] \quad (2)$$

$$= \frac{E}{S} \times \frac{1}{1-e^{-2as}} \left[\left(e^{-2as} - e^{-as} \right) - \left(e^{-as} - 1 \right) \right]$$

$$= \frac{E}{S} \times \frac{1}{1-e^{-2as}} \left(e^{-2as} - 2e^{-as} + 1 \right)$$

$$= \frac{E}{S} \times \frac{1}{1-e^{-2as}} \left(e^{-as} - 1 \right)^2 \quad \text{--- } (1)$$

$$= \frac{E}{S} \times \frac{1}{(1-e^{-as})(1+e^{-as})} \left(e^{-as} - 1 \right)^2$$

$$= \frac{E}{S} \frac{(1-e^{-as})}{(1+e^{-as})} \times \frac{e^{as/2}}{e^{as/2}} = \frac{E}{S} \left\{ \frac{e^{as/2} - e^{-as/2}}{e^{as/2} + e^{-as/2}} \right\}$$

$$= \frac{E}{S} \tanh\left(\frac{as}{2}\right) \quad \text{--- } (2)$$

3. $f(t) = \cos t + (\cos 2t - \cos t)\, u(t-\pi) + (\cos 3t - \cos 2t)\, u(t-2\pi)$

$$L[f(t)] = L(\cos t) + L(\cos 2t - \cos t)\, u(t-\pi) + L(\cos 3t - \cos 2t)\, u(t-2\pi) \quad \text{--- } (2)$$

$$= L_1 + L_2 + L_3$$

L_1 : $L(\cos t) = \dfrac{S}{S^2+1}$ --- (1)

L_2 : $L(\cos 2t - \cos t)\, u(t-\pi)$

$F(t) = \cos 2t - \cos t$

$t \to t + \pi$

$F(t+\pi) = \cos 2(t+\pi) - \cos(t+\pi) = \cos 2t + \cos t$ --- (2)

$\therefore f(t) = \cos 2t + \cos t$

$\Rightarrow \bar{f}(s) = \dfrac{s}{s^2+4} + \dfrac{s}{s^2+1}.$

$\therefore L_2 = \left(\dfrac{s}{s^2+4} + \dfrac{s}{s^2+1}\right) e^{-\pi s}$

$L_3 : L(\cos 3t - \cos 2t)\, u(t - 2\pi)$

$F(t) = \cos 3t - \cos 2t$

$t \to t + 2\pi$

$\therefore f(t) = \cos 3(t+2\pi) - \cos 2(t+2\pi)$

$= \cos 3t - \cos 2t$

$\bar{f}(s) = \dfrac{s}{s^2+9} - \dfrac{s}{s^2+4}$

$L_3 = \left(\dfrac{s}{s^2+9} - \dfrac{s}{s^2+4}\right) e^{-2\pi s}$

$\therefore L[f(t)] = \dfrac{s}{s^2+1} + \left(\dfrac{s}{s^2+4} + \dfrac{s}{s^2+1}\right) e^{-\pi s}$

$+ \left(\dfrac{s}{s^2+9} - \dfrac{s}{s^2+4}\right) e^{-2\pi s}$ ——— 2

4. $y''(t) - 2y'(t) + y(t) = e^t$ ——— 1

$L(y'') - 2L(y') + L(y) = L(e^t)$

$\Rightarrow s^2\bar{y}(s) - sy(0) - y'(0) - 2\{s\bar{y}(s) - y(0)\} + \bar{y}(s) = \dfrac{1}{s-1}$ ——— 1

$\Rightarrow (s^2 - 2s + 1)\bar{y}(s) - 2s + 1 + 4 = \dfrac{1}{s-1}$

$\Rightarrow (s^2 - 2s + 1)\bar{y}(s) - 2s + 5 = \dfrac{1}{s-1}$

$\Rightarrow (s^2 - 2s + 1)\bar{y}(s) = \dfrac{1}{s-1} + (2s - 5)$ ——— 1

$\Rightarrow$

$$\bar{y}(s) = \frac{1}{(s-1)^3} + 2\frac{s}{(s-1)^2} - \frac{5}{(s-1)^2}$$ — 1 3

$$y(t) = L^{-1}\left[\frac{1}{(s-1)^3}\right] + 2L^{-1}\left[\frac{s-1+1}{(s-1)^2}\right] - L^{-1}\left[\frac{5}{(s-1)^2}\right]$$

$$= e^{t} \times \frac{t^2}{2} + 2L^{-1}\left[\frac{1}{s-1}\right] + 2L^{-1}\left[\frac{1}{(s-1)^2}\right] - e^{t}t \times 5$$

$$= \frac{t^2e^t}{2} + 2e^t - 3te^t$$ — 3

5. To find $L^{-1}\left\{\frac{1}{(s+1)(s^2+9)}\right\}$

$$\bar{f}(s) = \frac{1}{s+1} \qquad \bar{g}(s) = \frac{1}{s^2+9}$$ — 1

$$\Rightarrow f(t) = e^{-t} \qquad g(t) = \frac{1}{3}\sin 3t$$ — 1

By Convolution theorem,

$$L^{-1}\{\bar{f}(s)\,\bar{g}(s)\} = \int_{u=0}^{t} f(u)\,g(t-u)\,.du$$ — 1

$$= \frac{1}{3}\int_{u=0}^{t} e^{-u}\sin 3(t-u)\,.du$$ — 1

$$= \frac{1}{3}\int_{u=0}^{t} e^{-u}\sin(3t-3u)\,.du$$

$$= -\frac{1}{3}\int_{u=0}^{t} e^{-u}\sin(3u-3t)\,.du \qquad \begin{matrix} a=-1 \\ b=3 \end{matrix}$$

$$= -\frac{1}{3}\left[\frac{1}{1+9}e^{-u}\left(-\sin(3u-3t) - 3\cos(3u-3t)\right)\right]_0^t$$ — 2

$$= +\frac{1}{30}\left[e^{-u}\left\{\sin(3u-3t) + 3\cos(3u-3t)\right\}\right]_0^t$$

$$= \frac{1}{30}\left(e^{-t} + \sin 3t - 3\cos 3t\right). \quad \text{—— (1)}$$

6. $$\int_{z=-c}^{c}\int_{y=-b}^{b}\int_{x=-a}^{a} (x^2+y^2+z^2)\,dx\,dy\,dz$$

$$= \int_{-c}^{c}\int_{-b}^{b}\left(\frac{x^3}{3} + xy^2 + xz^2\right)_{-a}^{a} dy\,dz \quad \text{—— (1)}$$

$$= \int_{-c}^{c}\int_{-b}^{b}\left[\left\{\frac{a^3}{3} + ay^2 + az^2\right\} - \left\{-\frac{a^3}{3} - ay^2 - az^2\right\}\right] dy\,dz$$

$$= 2\int_{-c}^{c}\int_{-b}^{b}\left(\frac{a^3}{3} + ay^2 + az^2\right) dy\,dz \quad \text{—— (1)}$$

$$= 2\int_{-c}^{c}\left(\frac{a^3 y}{3} + \frac{ay^3}{3} + ayz^2\right)_{-b}^{b} dz \quad \text{—— (1)}$$

$$= 2\int_{-c}^{c}\left[\left\{\frac{a^3 b}{3} + \frac{ab^3}{3} + abz^2\right\} - \left\{-\frac{a^3 b}{3} - \frac{ab^3}{3} - abz^2\right\}\right] . dz$$

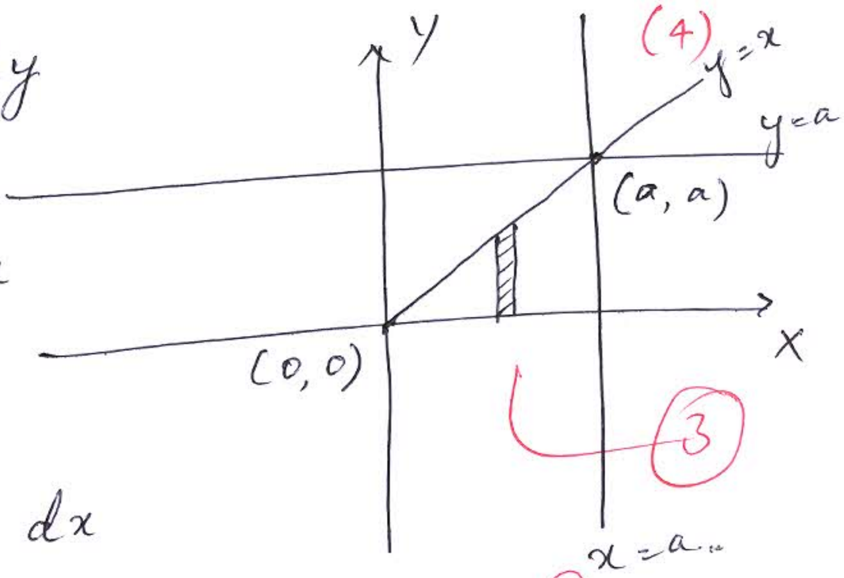
$$= 4\int_{-c}^{c}\left(\frac{a^3 b}{3} + \frac{ab^3}{3} + abz^2\right) dz \quad \text{—— (1)}$$

$$= 4\left[\frac{a^3 b}{3} z + \frac{ab^3}{3} z + ab\frac{z^3}{3}\right]_{-c}^{c} \quad \text{—— (1)}$$

$$= 4\left[\left\{\frac{a^3 bc}{3} + \frac{ab^3}{3}c + \frac{abc^3}{3}\right\} - \left\{-\frac{a^3 bc}{3} - \frac{ab^3 c}{3} - \frac{abc^3}{3}\right\}\right] \quad \text{—— (1)}$$

$$= \frac{8abc}{3}\left(a^2+b^2+c^2\right) \quad \text{—— (1)}$$

7. $I = \int_{y=0}^{a} \int_{x=y}^{a} \frac{x}{x^2+y^2}\, dx\, dy$

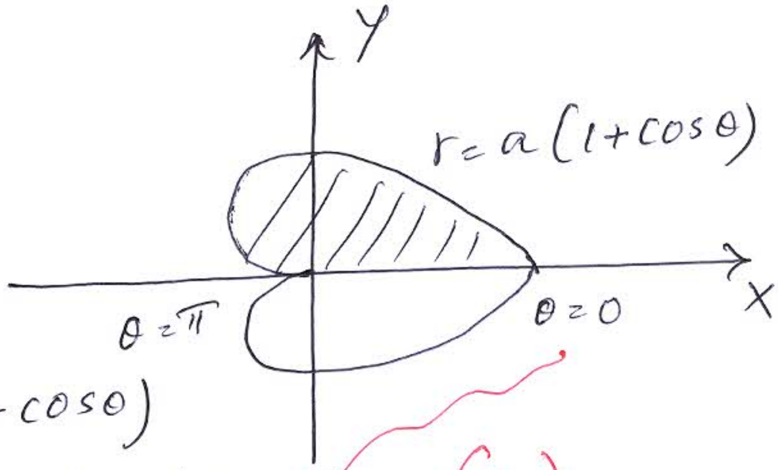


$= \int_{x=0}^{a} \left(\int_{y=0}^{x} \frac{x}{x^2+y^2}\, dy \right) dx$

$= \int_{x=0}^{a} \cancel{x} \times \frac{1}{\cancel{x}} \tan^{-1} \frac{y}{x} \Big|_{y=0}^{x} . dx$

$= \int_{x=0}^{a} \left(\frac{\pi}{4} \right) . dx = \frac{\pi}{4} (x)_0^a = \frac{\pi a}{4}.$ (4)

8. $r = a(1+\cos\theta)$



$\theta : 0 \to \pi$

$r : 0 \to a(1+\cos\theta)$

$\therefore A = \int_{\theta=0}^{\pi} \int_{r=0}^{a(1+\cos\theta)} r\, dr\, d\theta$

$= \frac{1}{2} \int_{\theta=0}^{\pi} a^2 (1+\cos\theta)^2 . d\theta$

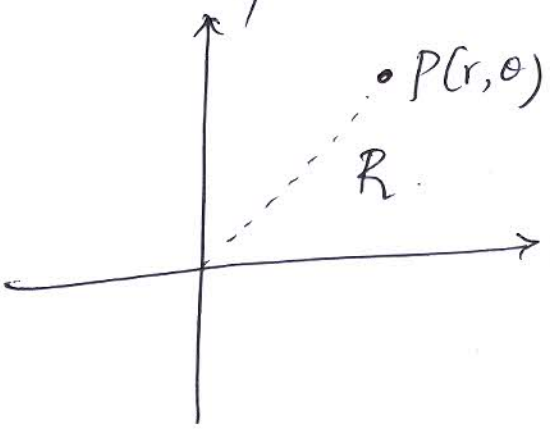
$= \frac{a^2}{2} \int_{\theta=0}^{\pi} (1 + \cos^2\theta + 2\cos\theta) . d\theta$

$= \frac{a^2}{2} \int_{\theta=0}^{\pi} \left\{ 1 + \frac{1}{2}(1+\cos 2\theta) + 2\cos\theta \right\} . d\theta$ (4)

$= \frac{a^2}{2} \left[\left\{ \pi + \frac{\pi}{2} \right\} + 0 \right] = \frac{3a^2}{4} \pi$ Sq. units.

9. $I = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \cdot dx\, dy$

$= \int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} e^{-r^2} \cdot r \cdot dr\, d\theta$ (1)



$x = r\cos\theta$

$y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$ (3)

$r^2 = t$

$\Rightarrow r \cdot dr = \frac{dt}{2}$ — (1)

$t : 0 \to \infty$

$\therefore I = \int_{\theta=0}^{\pi/2} \int_{t=0}^{\infty} e^{-t} \cdot \frac{dt}{2} \cdot d\theta$ (1)

$= -\frac{1}{2} \int_{\theta=0}^{\pi/2} \left(e^{-t}\right)_0^\infty \cdot d\theta = -\frac{1}{2}(-1)\frac{\pi}{2} = \frac{\pi}{4}$ (1)

10.(b) To find $L^{-1}\left\{\log \frac{s^2+1}{s(s+1)}\right\}$.

$\bar{f}(s) = \log(s^2+1) - \log s - \log(s+1)$

$\frac{d}{ds}\bar{f}(s) = \frac{2s}{s^2+1} - \frac{1}{s} - \frac{1}{s+1}$ — (1)

$-\frac{d}{ds}\bar{f}(s) = \frac{-2s}{s^2+1} + \frac{1}{s} + \frac{1}{s+1}$

$L^{-1}\left[-\frac{d}{ds}\bar{f}(s)\right] = -2L^{-1}\left(\frac{s}{s^2+1}\right) + L^{-1}\left(\frac{1}{s}\right) + L^{-1}\left(\frac{1}{s+1}\right)$ — (1)

$\Rightarrow t f(t) = -2\cos t + 1 + e^{-t}$

$\Rightarrow f(t) = \frac{1}{t}\left(1 - 2\cos t + e^{-t}\right)$. — (1)