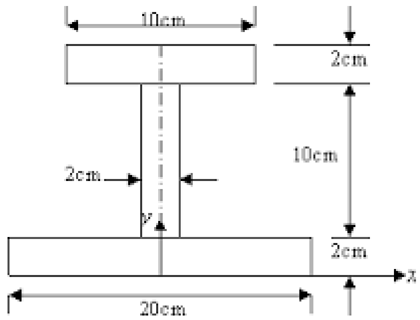
ELEMENTS OF CIVIL ENGINEERING AND ENGINEERING MECHANICS

IMPORTANT QUESTIONS WITH SOLUTIONS FOR IAT 3

1. Determine the moment of inertia of the area shown in Fig. 1a about given X and Y axis.



Sol. First of all find the location of centre of gravity of the given figure. The given section is symmetrical about the axis Y-Y and hence the C.G. of the section will lie on Y-Y axis. The given section is split up into three rectangles ABCD, EFGH and JKLM. The centre of gravity of the section is obtained by using

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} \quad \text{... (i)}$$

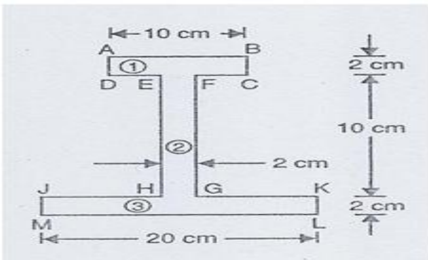


Fig. 4.32

where y = Distance of the C.G. of the section from the bottom line ML.

a_1 = Area of rectangle ABCD = $10 \times 2 = 20$ cm^2

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y_1 = Distance of the C.G. of the rectangle ABCD from the bottom line ML

$= 2 + 10 + \frac{2}{2} = 12 + 1 = 13$ cm

a_2 = Area of rectangle EFGH = $10 \times 2 =$ cm^2

y_2 = Distance of the C.G. of rectangle EFGH from the bottom line ML

$= 2 + \frac{10}{2} = 2 + 5 = 7$ cm

a_3 = Area of rectangle JKLM = $20 \times 2 = 40$ cm^2

y_3 = Distance of the C.G. of rectangle JKLM from the bottom line ML

$= \frac{2}{2} = 1.0$ cm

Substituting the above values in equation (i), we get

$$\bar{y} = \frac{20 \times 13 + 20 \times 7 + 40 \times 1}{20 + 20 + 40}$$

We want to find the moment of inertia of the given section about a horizontal axis passing through the C.G. of the given section.

Let I_{G_1}

= Moment of inertia of rectangle ① about the horizontal axis and passing through it

I_{G_2}

= Moment of inertia of rectangle ② about the horizontal axis and passing through

the C. G. of the rectange

h_1

= The distance between the C. G. of the rectangle ①and the C. G. of the given section

$= y_1 \quad \bar{y} = 13.0 \quad 5.50 = 7.50$ cm

h_2

= The distance between the C. G. of the rectangle ② and the C. G. of the given section

$= y_2 - y = 7.0 - 5.50 = 1.50$ cm.

h_3

= The distance between the C. G. of the rectangle ③ and the C. G. of the given section

$= \bar{y} \quad y_3 = 5.50 \quad 1.10 = 4.5$ cm.

$= \bar{y} \quad y_3 = 5.50 \quad 1.10 = 4.5$ cm.

Now $I_{G_1} = \dfrac{10 \times 2^3}{12} = 6.667\ cm^4$.

$I_{G_2} = \dfrac{2 \times 10^3}{12} = 166.667\ cm^4$.

$I_{G_3} = \dfrac{20 \times 2^3}{12} = 13.333\ cm^4$.

From the theorem of parallel axes, the moment of inertia of the rectangle ? about the horizontal axis passing through the C.G. of the given section

$= I_{G_1} + a_1 h_1^2 = 6.667 + 20 \times (7.5)^2$

$= 6.667 + 1125 = 1131.667\ cm^4$.

Similarly, the moment of inertia of the rectangle ? about the horizontal axis passing through the C.G. of the given section

$= I_{G_2} + a_2 h_2^2 = 166.667 + 20 \times (1.5)^2$

$= 166.667 + 45 = 211.667\ cm^4$.

And moment of inertia of the rectangle ? about the horizontal axis passing through the C.G. of the given section

$= I_{G_3} + a_3 h_3^2 = 13.333 + 40 \times (4.5)^2$

$= 13.333 + 810 = 823.333\ cm^4$.

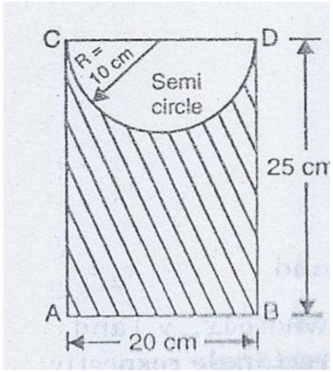
Now moment of inertia of the given section about the horizontal axis, passing through the C.G. of the given section

= Sum of the moment of inertia of the rectangles ? , ? and ? about the horizontal axis, passing through the C.G. of the given section

$= 1131.667 + 211.667 + 823.333$

$= 2166.667\ cm^4$. Ans.

2. Determine polar moment of inertia of the section about AB shown in Fig. 2a (All dimension are in **mm**)



Sol. Given:

Radius of semi circle, $R = 10$ cm

Width of rectangle, $b = 20$ cm

Depth of rectangle, $d = 25$ cm

Moment of inertia of the shaded portion about AB

= M.O.I. of rectangle ABCD about AB − M.O.I. of semi − circle on DC about AB

M. O.I. of rectangle ABCD about AB

$$= \frac{bd^3}{3}$$

$$= \frac{20 \times 25^3}{12} = 104.167 \text{ cm}^4$$

M. O.I. of semi circle about DC

$$= \frac{1}{2} \times [\text{M. O. I. of a circle of radius 10 cm about a diameter}]$$

$$- \frac{1}{2} \times \left[\frac{\pi}{64} d^4\right]$$

$$- \frac{1}{2} \times \frac{\pi}{64} \times 20^4 - 3.925 \text{ cm}^4$$

Distance of C. G. of semi − circle from DC

$$= \frac{4r}{3\pi} = \frac{4 \times 10}{3\pi} = 4.24 \text{ cm}$$

Area of semi circle,

$$A = \frac{\pi r^2}{2} = \frac{\pi \times 10^2}{2} = 157.1 \text{ cm}^2$$

M. O.I. of semi circle about a line through its C. G. parallel to CD

= M.O.I. of semi circle about CD Area × [Distance of C. G. of semi − circle from DC]^2

$= 3925 \quad 157.1 \times 4.24^2$

$= 3925 \quad 2824.28$

$= 1100.72 \text{ cm}^4$

Distance of C. G. of semi circle about AB

$= 25 \quad 4.24 = 20.76$ cm

M. O.I. of semi − circle about AB

$= 1100.72 + 157.1 \times 20.76^2$

$= 1100.72 + 67706.58 = 68807.30 \text{ cm}^4$

3. Derive an expression for moment of inertia of a rectangular plate by method of integration.

4. State perpendicular axis theorem.

The moment of inertia (MI) of a plane area about an axis normal to the plane is equal to the sum of the moments of inertia about any two mutually perpendicular axes lying in the plane and passing through the given axis. That means the Moment of Inertia $I_z = I_x + I_y$.

5. Define (i) Velocity (ii) Distance and (iii) Acceleration

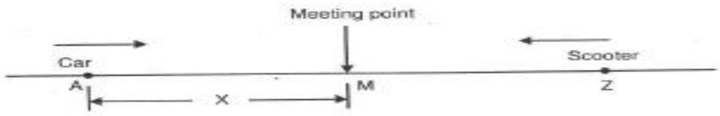
Velocity is speed in a given direction. Speed describes only how fast an object is moving, whereas velocity gives both the speed and direction of the object's motion being a vector quantity. The standard unit to measure velocity is meter per second (m/s or ms^{-1}). For example, "10 meters per second" is scalar and "10 meters per second West" is vector.

Distance is the actual length traveled from initial to final position where as displacement is the shortest distance between the two positions. Both, displacement and distance has standard unit meters.

Acceleration is defined as the rate of change of velocity with time. Acceleration is the rate at which an object speeds up or slows down. If the object speeds up it is said to have positive acceleration. If the object slows down it is said to have negative acceleration also called as retardation. Acceleration is also a vector quantity. The standard unit to measure acceleration is meter per second squares (m/s^2 or ms^{-2}).

6. A car starts from rest and travels on a straight road with a constant acceleration of $1.2 m/s^2$. After some time a scooter passes by it travelling in the opposite direction with a uniform velocity of 24 kmph. The scooter reaches the starting position of the car 30 sec after car had left from there. Determine when and where two vehicles passed each other.

Solution:



Refer figure. At t = 0, let the car be at position A and the scooter be at position Z. Let the two vehicles pass each other at M which is x metres from A, t sec later.

Motion of car
Uniform acceleration

(A to M)
$u = 0$
$v = -$ *
$s = x$
$a = 1.2 m/s^2$
$t = t$ sec
using $s = ut + \frac{1}{2} at^2$
$x = 0 + \frac{1}{2} \times 1.2 \times t^2$
$x = 0.6 t^2$ .... (1)

Motion of scooter
Uniform velocity
(Z to A)
$v = 36$ kmph $= 10 m/s$
$s = ?$
$t = 30$ sec
using $v = \frac{s}{t}$
$10 = \frac{s}{30}$
$s = 300$ m
∴ the scooter was 300 m away from the car when the car started its motion

(Z to M)
$v = 10 m/s$
$s = (300 - x)$ metres
$t = t$ sec
using $v = \frac{s}{t}$
$10 = \frac{300 - x}{t}$
$x = 300 - 10t$ .... (2)

Solving equations (1) and (2)
$t = 15.53$ sec ..... **Ans.**
$x = 144.71$ m ..... **Ans.**

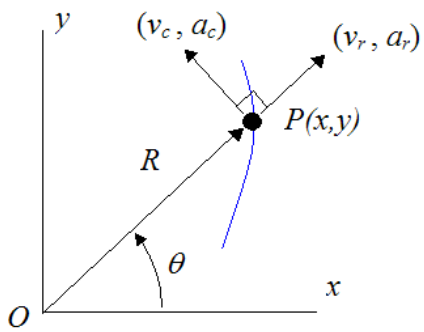
7. Define (i) Rectilinear motion (ii) Curvilinear motion (iii) Motion under gravity

Rectilinear motion is another name for straight-line motion. This type of motion describes the movement of a particle or a body.

A body is said to experience rectilinear motion if any two particles of the body travel the same distance along two parallel straight lines. The figures below illustrate rectilinear motion for a particle and body.

Rectilinear motion for a particle:

Curvilinear motion is defined as motion that occurs when a particle travels along a curved path. The curved path can be in two dimensions (in a plane), or in three dimensions. This type of motion is more complex than **rectilinear (straight-line) motion**.



A particle which is moving under gravity alone is moving in a straight line under constant acceleration, so all the concepts applied to linear motion under constant acceleration may equally be applied here.
On the ordinary level paper the acceleration due to gravity is taken as 10 m/s^2, whereas on the higher level the value for g is 9.8 m/s^2.
We adopt the same sign convention here as in Cartesian geometry, namely "up" is positive, "down" is negative.

The equations of motion are now:

$$v = u - gt$$

$$s = \left(\frac{u+v}{2}\right).t$$

$$s = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gs$$

8. With the help of neat sketch explain the following terms angle of projection, time of flight, horizontal range and maximum height attained by the particle.

Time of Flight

The time of flight of a projectile motion is the time from when the object is projected to the time it reaches the surface.

Maximum Height

The maximum height is reached when Vy=0. Using this we can rearrange the velocity equation to find the time it will take for the object to reach maximum height. From the displacement equation we can find the maximum height

$y= u_y t - (1/2)gt^2$

At $t=T/2$, $y=H$

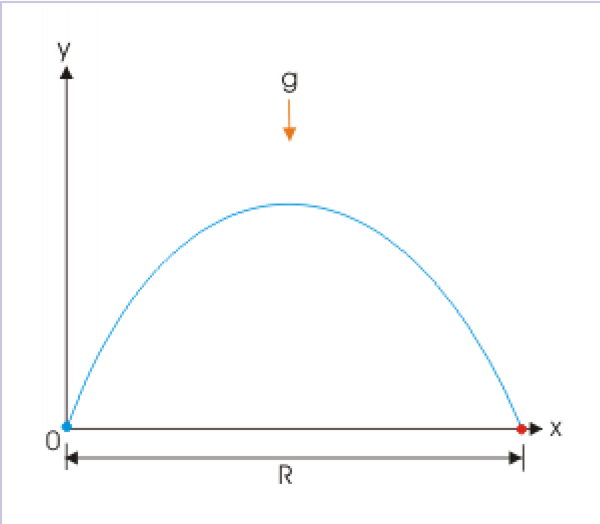
$H= u\sin\Phi . T/2 - (1/2)g(T/2)^2$

substituting T

$H= u\sin\Phi . u\sin\Phi/g - (1/2)g(u\sin\Phi/g)^2$

$= (u^2 \sin^2)/g - (u^2 \sin^2)/2g$

$H= (u^2 \sin^2)/2g$



Range is the total horizontal distance covered during the time of flight.

From equation for horizontal motion, $x=u_x t$

When $t=T$, $x=R$

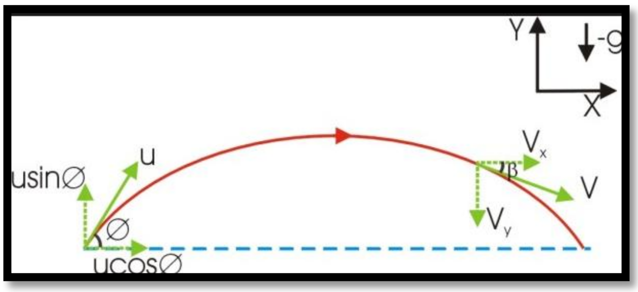
$R= u_x T = u\cos . 2u\sin\Phi/g$

$= u^2 2\sin\Phi\cos\Phi/g = u^2\sin 2\Phi/g$ using $2\sin\Phi\cos\Phi= \sin 2\Phi$

$R= (u^2 \sin 2\Phi)/g$

Angle of projection

The "**angle** of reach" is the **angle** (θ) at which a **projectile** must be launched in order to go a distance d , given the initial velocity v .



9. A stone is thrown vertically upwards and returns to the ground in 6s. How high will it go and also initial velocity of projection?

Solution: Let the stone be projected with an initial velocity v_0 from A. Let it travel h_{max} metres, to the peak at B and return to ground at C. The stone's displacement is zero between A and C. Also the stone's velocity at the peak is zero.

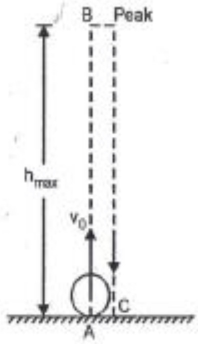
Motion of stone

Entire motion (A – C)
M.U.G ↑ + ve
$u = v_0$
$v = -$
$s = 0$
$a = -9.81 \text{ m/s}^2$
$t = 6$ sec.

Using $s = ut + \frac{1}{2}at^2$
$0 = v_0 \times 6 + \frac{1}{2}(-9.81) \times 6^2$
$\therefore v_0 = 29.43 \text{ m/s}$ ... **Ans.**

Ground to peak (A – B)
M.U.G ↑ + ve
$u = 29.43 \text{ m/s}$
$v = 0$
$s = h_{max}$
$a = -9.81 \text{ m/s}^2$
$t = -$

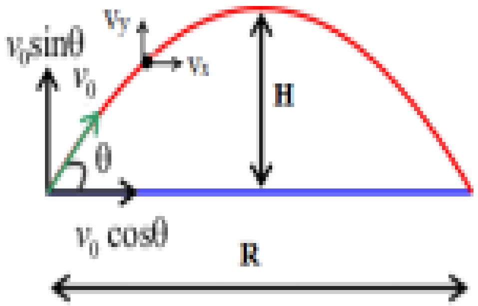
Using $v^2 = u^2 + 2as$
$0 = 29.43^2 + 2 \times (-9.81) \times h_{max}$
$\therefore h_{max} = 44.145 \text{ m}$ ...... **Ans.**



10. Derive the equation of path of the projectile.

Equation of Path of Projectile:

- Let v_0 = Velocity of projection and θ = Angle of projection. Resolving v_0 into two component. viz. $v_0 \cos\theta$ the horizontal component. And $v_0 \sin\theta$ the vertical component. Consider vertical Component $v_0 \sin\theta$. Due to this component, there is the vertical motion of the body.



- Let us consider a rectangular Cartesian system of axes such that the origin lies at the point of projection and x-axis is along horizontal and in the plane of projection. Let P(x, y) be the position of the particle after time t from the time of projection.

x coordinate of the position of a particle after time 't' is the horizontal distance travelled by the projectile

$$x = v_0 \cos\theta . \ t$$

$$t = \frac{x}{V_o \cos\theta} \ .... (1)$$

y coordinate of the position of a particle after time 't' is the vertical distance travelled by the projectile

$$y = v_0 \sin\theta . t - g t^2 \ ....(2)$$

Substituting values of equation (1) in (2) we get

$$y = V_0 . \sin\theta \left(\frac{x}{V_o \cos\theta} \right) - \frac{1}{2} g \left(\frac{x}{V_o \cos\theta} \right)^2$$

$$y = x . \tan\theta - \frac{1}{2} \left(\frac{g x^2}{V_o^2 \cos^2\theta} \right)$$

This relation is called as the equation of the trajectory of a particle performing projectile motion.

- In this equation v_0, g and q are constant. This equation is in the form $y = a + bx^2$. Where a and b are constant. Thus the trajectory of the projectile is a parabola.

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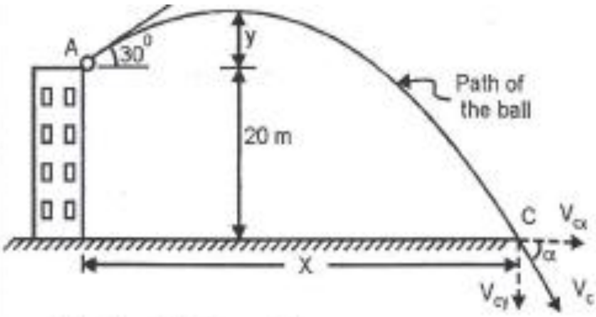
11. A projectile is fired from the top of a cliff 20m height with an initial velocity of 15m/sec at an upward angle of 30^0 to horizontal. Neglecting air resistance determine, the horizontal distance from the gun point to the point where the projectile strikes the ground.

the total time of flight.

Solution: Refer figure. Let x be the distance from foot of the building. Let the ball reach a height y metres above the building.



Projectile motion (A – B)

Horizontal Motion

Vertical Motion ↑ +ve

u = 15 sin 30 = 7.5 m/s

v = 0

s = y

a = - 9.81 m/s²

t = –

(1) using $v^2 = u^2 + 2as$

$0 = (7.5)^2 + 2(-9.81)\,y$

∴ y = 2.867 m.

Maximum height attained above ground = 2.867 + 20 = 22.867 m ...... **Ans.**

Projectile Motion (A – C)

Horizontal Motion

v = 15 cos30 = 13 m/s

s = x

t = t sec

Using $v = \frac{s}{t}$

$13 = \frac{x}{t}$ .... (1)

substituting t = 2.924 sec obtained from vertical motion

x = 38.01 m .... **Ans.**

Vertical Motion ↑ +ve

u = 7.5 m/s

$v = -v_{Cy}$

s = – 20 m

a = – 9.81 m/s²

t = t sec

(3) Using $s = ut + \frac{1}{2}at^2$

$-20 = 7.5\,t - \frac{1}{2} \times 9.81 \times t^2$

∴ t = 2.924 sec ...... **Ans.**

(4) using v = u + at

$-v_{Cy} = 7.5 - 9.81 \times 2.924$

$v_{Cy} = 21.18$ m/s

Velocity of landing at C

Since velocity does not change in the x direction, $v_{Cx} = 13$ m/s →

Also $v_{Cy} = 21.18$ m/s ↓

∴ (5) $v_c = \sqrt{(v_{cx})^2 + (v_{cy})^2} = \sqrt{(13)^2 + (21.18)^2} = 24.85$ m/s

also $\tan\alpha = \frac{v_{cy}}{v_{cx}} = \frac{21.18}{13}$ ∴ $\alpha = 58.46°$

∴ $v_C = 24.85$ m/s, $\alpha = 58.46°$ ↘ ...... **Ans.**

12. The acceleration, *a*, of a particle, at time *t* seconds is given by: $a = 4 - \frac{t}{20}\ \text{ms}^{-2}$. This model is valid for $0 \le t \le 80$. Given that the particle starts at rest, find the distance travelled by the particle when *t*= 80.

Solution

First integrate the acceleration to obtain the velocity.

$$v = \int \left(4 - \frac{t}{20}\right) dt = 4t - \frac{t^2}{40} + c_1$$

To find the value of the constant c_1 , note that the particle is initially at rest, so that $v = 0$ when $t = 0$. Substituting these values shows that $c_1 = 0$. Hence the velocity is:

$$v = 4t - \frac{t^2}{40} \text{ ms}^{-1}$$

The displacement of the particle can be found by integrating the velocity:

$$s = \int \left(4t - \frac{t^2}{40}\right) dt = 2t^2 - \frac{t^3}{120} + c_2$$

To find the constant c_2 , assume that the particle starts at the origin, so that $s = 0$ when $t = 0$. Hence $c_2 = 0$ and the displacement at time t is given by:

$$s = 2t^2 - \frac{t^3}{120} \text{ m}$$

To find the distance travelled substitute $t = 80$.

$$s = 2 \times 80^2 - \frac{80^3}{120} = 8530 \text{ m (to 3sf)}$$