33

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Improvement Test

Sub:	Engineering Mathematics - IV					Sub Code:	15MAT41	Branch:	EE/CS/CV
Date:	21.05.2018	Duration:	90 mins	Max Marks:	50	Sem / Sec:	IV/ EEE-A, CSE-A, CIV-A		OBE

	Question 1 is compulsory and answer any SIX questions from rest .	MARKS	CO	RBT
1.	The joint probability distribution of two discrete random variables X and Y is given by $f(x,y)=k(2x+y)$ where x and y are integers such that $0\le x\le 2$, $0\le y\le 3$. a) Find the constant k b)Find the marginal probability distributions of X and Y c) Show that the random variables X and Y are dependent d) Compute $E(X)$, $E(Y)$, $E(XY)$, $E(X^2)$, $E(Y^2)$, σ_X, σ_Y	[08]	CO6	L3
2.	Derive Mean and Variance of Binomial distribution.	[07]	CO4	L3
3.	a) The length of a telephone conversation has an exponential distribution with a mean of 3 minutes .Find the probability that a call i) ends in less than 3 minutes and ii) takes between 3 and 5 minutes. b)The probability that an individual suffers a bad reaction from a certain injection is 0.001.Using Poisson's distribution, determine the probability that out of 2000 individuals i) exactly 3 ii) more than 2 will suffer a bad reaction.	[07]	CO4	L3

<table>
<tr><td>4.</td><td>In a normal distribution ,31% of items are under 45 and 8% of items are over 64.Find the mean and standard deviation , given that A(0.5)=0.19 ,A(1.4)=0.42</td><td>[07]</td><td>CO4</td><td>L3</td></tr>
<tr><td>5.</td><td>From a sealed box containing a dozen apples it was found that 3 apples are perished . Obtain the probability distribution of the number of perished apples when 2 apples are drawn at random.Also find the mean and variance of this distribution.</td><td>[07]</td><td>CO4</td><td>L3</td></tr>
<tr><td>6.</td><td>Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is a regular stochastic matrix. Also find the corresponding unique fixed probability vector.</td><td>[07]</td><td>CO6</td><td>L3</td></tr>
<tr><td>7.</td><td>Three boys A,B,C are throwing ball to each other.A always throws the ball to B and B always throws the ball to C.C is just likely to throw the ball to B as to A.If C was the first person to throw the ball find the probabilities that after three throws
i) A has the ball ii)B has the ball iii) C has the ball.</td><td>[07]</td><td>CO6</td><td>L3</td></tr>
<tr><td>8.</td><td>Certain bulbs manufactured by a company have mean life time of 800 hrs and std deviation of 60 hrs.Find the probability that a random sample of 16 bulbs taken from the group will have a mean life time i) between 790 hrs and 810 hrs ii) less than785 hrs iii) more than 820 hrs iv) between 770 hrs and 830 hrs. Given A(0.67) = 0.2486 , A(1) = 0.3413 , A(1.33) = 0.4082 , A(2) = 0.4772</td><td>[07]</td><td>CO5</td><td>L3</td></tr>
<tr><td>9.</td><td>The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given below. Test the goodness of fit of Poisson distribution of fit the given data ($\chi^2_{0.05}$ = 9.49 for 4 d.f.)
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>f</td><td>173</td><td>168</td><td>37</td><td>18</td><td>3</td><td>1</td></tr></table></td><td>[07]</td><td>CO5</td><td>L3</td></tr>
</table>

Improvement Test – May 2018
Solution Manual
(EE-A, CSE-A, CIV-A)

1.

$X = x_i = \{0,1,2\}$, $Y = y_j = \{0,1,2,3\}$

$f(x,y) = k(2x+y)$ & the joint probability distribution table is formed as follows.

X\Y	0	1	2	3	Sum
0	0	k	2k	3k	6k
1	2k	3k	4k	5k	14k
2	4k	5k	6k	7k	22k
Sum	6k	9k	12k	15k	42k

1m

a. $42k = 1 \Rightarrow k = 1/42$ 1m

b. Marginal probability distribution is

x_i	0	1	2
$f(x_i)$	$1/7$	$1/3$	$11/21$

y_j	0	1	2	3
$g(y_j)$	$1/7$	$3/14$	$2/7$	$5/14$

1m

c. $f(x_i)\, g(y_j) \neq J_{ij}$

d. $E(X) = \sum_i x_i f(x_i) = 58/42 = 29/21$ ½m

$E(Y) = \sum y_j g(y_j) = 13/7 = 0 \times 6/42 + 1 \cdot 9/42 + 2 \cdot \frac{12}{42} + 3 \cdot \frac{15}{42}$ ½m

$E(XY) = \sum_{i,j} x_i y_j J_{ij} = 0 + \left(0 + 3/42 + 8/42 + 15/42\right) + \left(0 + 10/42 + 24/42 + 42/42\right)$

$= 102/42 = 17/7$ 1m

$E(X^2) = \sum_i x_i^2 f(x_i)$

$$E(X^2) = \sum x_i^2 f(x_i)$$

$$= 0 + 1 \times 14/42 + 4 \times 22/42 = 102/42 = 17/7$$ 1m

$$E(Y^2) = \sum_j y_j^2 g(y_j)$$

$$= 0 + 1 \times 9/42 + 4 \times \frac{12}{42} + 9 \times \frac{15}{42} = \frac{192}{42} = \frac{32}{7}$$

$$\sigma_x^2 = E(x^2) - \mu_x^2 = \frac{17}{7} - \left(\frac{29}{21}\right)^2 = \frac{230}{441}$$ 1m

$$\sigma_x = \frac{\sqrt{230}}{21}$$

$$\sigma_y^2 = E(Y^2) - \mu_y^2 = \frac{32}{7} - \left(\frac{39}{21}\right)^2 = \frac{495}{441} \Rightarrow \sigma_y = \frac{\sqrt{495}}{21}$$

$$\sigma_x = 0.72, \quad \sigma_y = 1.06.$$ 1m

2. $$p(x) = {}^nC_x \, p^x q^{n-x}$$ 1m

$$\text{Mean} = \sum_{x=0}^{n} x \, p(x).$$

$$\mu = \sum_{x=0}^{n} x \cdot {}^nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n(n-1)!}{x(x-1)!(n-x)!} p^x p$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!} p^{x-1} q^{(n-1)-(x-1)}$$

$$= np \sum_{x=1}^{n} {}^{n-1}C_{x-1} \, p^{x-1} q^{(n-1)-(x-1)}$$

$$\mu = np\,(p+q)^{n-1} = np$$ 2m

Variance : $V = \sum_{x=0}^{n} x^2 p(x) - \mu^2$ — (1)

$$\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} [x(x-1)+x] p(x)$$

$$= \sum_{x=0}^{n} x(x-1)p(x) + \sum_{x=0}^{n} x p(x)$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^nC_x p^x q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x} + np$$

$$= \sum_{x=0}^{n} \frac{n(n-1)(n-2)!}{(x-2)!(n-x)!} p^2 p^{x-2} \cdot q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)![(n-2)-(x-2)]!} p^{x-2} \cdot q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 \sum_{x=2}^{n} {}^{n-2}C_{x-2}\, p^{x-2} q^{(n-2)-(x-2)} + np$$

$$= n(n-1)p^2 (q+p)^{n-2} + np$$ 3m

$\sum x^2 p(x) = n(n-1)p^2 + np$ — (2)

using (2) in (1) we get:

$$V = n(n-1)p^2 + np - (np)^2$$

$$= np(1-p) = npq.$$ 1m

$\therefore V = npq.$

3\. Mean $\mu = 1/\alpha = 3$

a. x – length of the telephone conversation

$P(x) = \alpha e^{-\alpha x}$ 0.5m $x > 0$

$= \frac{1}{3} e^{-x/3}$

i) Probability that a call ends in < 3

$$P(x<3) = \frac{1}{3}\int_0^3 e^{-x/3}\,dx = \left[1 - e^{-3/3}\right] = 0.6321$$

ii) The probability that a call takes 1.5m
b/w 3 & 5 minutes is,

$$P(3<x<5) = \frac{1}{3}\int_3^5 e^{-x/3}\,dx = e^{-3/3} - e^{-5/3}$$

$= 0.179.$ 1.5 m

b. The probability that an individual Suffers a bad reaction is $p = 0.001$.

$n = 2000$, $\mu = np = 2000 \times 0.001 = 2$

$$p(x) = \frac{m^x e^{-m}}{x!} \quad \mu = m$$ 0.5 m

x - denotes the no of persons who suffer a bad reaction.

∴ out of 2000 persons

i) Prob that exactly 3 persons will suffer a bad reaction is

$$p(3) = \frac{e^{-2} \cdot 2^3}{3!} = \frac{4}{3}e^{-2} = \frac{4}{3} \times 0.1353$$ 1.5m

$= 0.1804.$

ii) The probability that more than 2 persons will suffer a bad reaction is

$$P(x>2) = 1 - P(x \le 2) = 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - \left\{ \frac{e^{-2} \cdot 2^0}{1!} + \frac{e^{-2} \cdot 2}{1!} + \frac{e^{-2} \cdot 2^2}{2!} \right\}$$

$$= 1 - [1+2+2]e^{-2} = 1 - 5 \times 0.13534$$

$$= 1 - 0.67667 = 0.3233$$ 1.5m

4. x - normal variate for the distribution being considered.

It is given that 31% are under 45 & 8% are over 64.

Let μ & σ be the mean & Std deviation of normal distribution.

$P(x<45) = 0.31$ & $P(x>64) = 0.08$. 1m

S.n.v $Z = \frac{x-\mu}{\sigma}$ 1m

$x = 45$, $Z = \frac{45-\mu}{\sigma} = z_1$, $x = 64$, $z = \frac{64-\mu}{\sigma} = z_2$

Thus we have

$P(z<z_1) = 0.31$ & $P(z>z_2) = 0.08$.

$0.5 + \phi(z_1) = 0.31$ $\quad$ $0.5 - \phi(z_2) = 0.08$.

$\phi(z_1) = -0.19$ 1m $\quad$ $\phi(z_2) = 0.42$. 1m

$\phi(0.5) = 0.19$ $\quad$ $\phi(1.4) = 0.42$

$z_1 = -0.5$ & $z_2 = 1.4$.

$\frac{45-\mu}{\sigma} = -0.5$, $\frac{64-\mu}{\sigma} = 1.4$

$\mu - 0.5\sigma = 45$ & $\mu + 1.4\sigma = 64$ 2m

By solving $\mu = 50, \sigma = 10$.

Mean = 50, Std deviation = 10. 1m

5. Let X be the no of perished apples

$\therefore$ 2 were drawn we have $x = 0, 1, 2, 2$

out of 12 can be selected in $12C_2$ ways.

9 are good apples & 3 are perished apples

$\therefore$ we have 1m

$P(x=0)$ = Prob of getting 0 perished apple

$= \frac{3C_0 \times 9C_2}{12C_2} = 6/11$ 1m

$P(x=1)$ = Prob of getting 1 perished apple

$= \frac{3C_1 \times 9C_1}{12C_2} = 9/22$ 1m

$P(x=2)$ = Prob of getting 2 perished apples

$= \frac{3C_2 \times 9C_0}{12C_2} = \frac{1}{22}$ 1m

The prob distribution is as follows

$X = x_i$	0	1	2	
$P(X) = p_i$	$6/11$	$9/22$	$1/22$	1m

Mean $\mu = \sum x_i\, p(x_i) = 0 + \frac{9}{22} + \frac{2}{22} = \frac{11}{22} = \frac{1}{2}$ 1m

Variance $V = \sum x_i^2 p_i - \mu^2$

$= \left(0 + \frac{9}{22} + \frac{4}{22}\right) - 1/4$

$= \frac{13}{22} - 1/4 = 15/44$. 1m

$\mu = 1/2$, $V = 15/44$

6. $P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$

$P^3 = P \cdot P^2 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$, $P^4 = P \cdot P^3 = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$

4M

$P^5 = P \cdot P^4 = \begin{bmatrix} 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ 1/8 & 3/8 & 1/2 \end{bmatrix}$ $\therefore$ P is a regular stochastic matrix.

To find $v = (a, b, c) \ni a + b + c = 1$

$\ni vP = v$

$$[a\ b\ c]\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = [a, b, c]$$

$$\left[\frac{c}{2}, a + \frac{c}{2}, b\right] = [a, b, c]$$

$c = 2a$, $b = c = 2a$. $a+b+c=1$.

$\therefore a = 1/5$, $b = c = 2/5$ 3m

$\therefore V = [1/5, 2/5, 2/5]$ is unique fixed prob vector of P.

7. State space = $\{A, B, C\}$ $\therefore$ The associated t.p.m is 1m

$$P = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & 0 & 0 & 1 \\ C & 1/2 & 1/2 & 0 \end{array}$$ 1m

If C has the ball. the associated initial prob vector is

$p^{(0)} = (0, 0, 1)$.

$\therefore$ the probabilities are desired after three throws we have to find $p^{(3)} = p^{(0)}p^{(3)}$

$$P^3 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$ 2m $$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$ 1m

$p^{(3)} = p^{(0)} \cdot P^3 = [\frac{1}{4} \;\; \frac{1}{4} \;\; \frac{1}{2}] = [P_A^{(3)}, P_B^{(3)}, P_C^{(3)}]$

Thus after three throws the prob that the ball is with A is $\frac{1}{4}$, B is $\frac{1}{4}$, C is $\frac{1}{2}$. 2m

8. $\mu = 800, \sigma = 60, n = 16$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 60/4 = 15$

$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{\bar{x} - 800}{15}$ 1m

a) $P(790 < \bar{x} < 810) = P$

$\bar{x} = 790, z = -0.67$

$\bar{x} = 810, z = 0.67$

$P(-0.67 < z < 0.67) = 2P(0 < z < 0.67)$

$= 2\phi(0.67)$

$= 2(0.2486) = 0.4972$

$P(790 < \bar{x} < 810) = 0.4972$ 1.5m

b) $P(\bar{x} < 785)$

$\bar{x} = 785, z = -1$

$P(z < -1) = P(z > 1)$

$= P(z > 0) - P(0 < z < 1)$

$= 0.5 - \phi(1)$

$= 0.5 - 0.3413 = 0.1587$ 1.5m

$P(\bar{x} < 785) = 0.1587.$

c) $P(\bar{x} > 820) = P(z > 1.33) = P(z > 0) - P(0 < z < 1.33)$

$= 0.5 - \phi(1.33)$ 1.5M

$= 0.5 - 0.4082 = 0.0918$

d) $P(770 < \bar{x} < 830) = P(-2 < z < 2) = 2P(0 < z < 2)$

$= 2\phi(2)$

$= 0.9544.$ 1.5M

9. $\mu = \frac{\Sigma fx}{\Sigma f} = \frac{0+168+74+54+12+5}{400} = 0.7825$

1M

$p(x) = \frac{m^x e^{-m}}{x!}$, $f(x) = 400\,p(x)$

$= \frac{400\,(0.7825)^x e^{-0.7825}}{x!}$

$f(x) = \frac{182.9\,(0.7825)^x}{x!}.$

Theoritical frequencies are got by subs $x = 0, 1, 2, 3, 4, 5$ in $f(x)$ they are 183, 143, 56, 15, 3, 0

O_i	173	168	37	18	4.	
E_i	183	143	56	15	3	4M

$$\chi^2 = \Sigma \frac{(O_i - E_i)^2}{E_i}$$

$$\chi^2 = \frac{100}{183} + \frac{625}{143} + \frac{361}{56} + \frac{9}{15} + \frac{1}{3} = 12.3$$

$\chi^2 = 12.3 > \chi^2_{0.05} = 9.49.$ 2M

The fitness is not good.

1M

The hypothesis that the fitness is good is rejected