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Internal Assessment Test 3 – May 2018

Sub:	Engineering Mathematics II					Sub Code:	17MAT21	
Date:	21/05/2018	Duration:	90 mins	Max Marks:	50	Sem / Sec:	II/J, L and M	OBE

Question 1 is compulsory and answer any SIX questions from the rest.

		MARKS	CO	RBT
1.	(a) Evaluate $L\left\{\frac{cos2t-cos3t}{t}\right\}$ (b) Evaluate $L\{t^2e^{-3t}sin2t\}$.	[08]	CO6	L3
2.	A periodic function of period 2a is defined by, $f(t)=\begin{cases} E & for\ 0\le t\le a \\ -E & for\ a<t\le 2a\end{cases}$ where E is a constant. Show that $L\{f(t)\}=\frac{E}{s}\ tanh\left(\frac{as}{2}\right)$.	[07]	CO6	L3
3.	Express $f(t)=\begin{cases} cost & 0<t<\pi \\ cos2t & \pi<t<2\pi \\ cos3t & t>2\pi\end{cases}$ in terms of unit step function and hence find its Laplace transform.	[07]	CO6	L3
4.	Solve $y''(t)-2y'(t)+y(t)=e^t$ subject to the conditions, y(0)=2, $y'(0)=-1$ by using Laplace transform.	[07]	CO6	L3
5.	Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+9)}\right\}$ by using Convolution theorem.	[07]	CO6	L3

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7. For $m > 0$ and $n > 0$, show that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. [7] CO5 L3

8. Show that $\int_{-1}^{1} (1+x)^{p-1}(1-x)^{q-1}\, dx = 2^{p+q-1}\beta(p,q)$ using Beta and Gamma functions. [7] CO5 L3

9. Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\, dxdy$ by changing into polar coordinates. [7] CO3 L3

10. (a) Find $L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\}$ (b) Find $L^{-1}\left\{\log\frac{s^2+1}{s(s+1)}\right\}$. [7] CO6 L3

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SOLUTIONS TO IAT- 03

10. (a) To find $L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\}$

$$\frac{4s+5}{(s+1)^2(s+2)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+2} \quad \text{——— (1)}$$

$$\Rightarrow 4s+5 = A(s+1)(s+2) + B(s+2) + C(s+1)^2$$

$$\Rightarrow 4s+5 = A(s^2+3s+2) + Bs + 2B + Cs^2 + 2Cs + C$$

Equating the coefficients of s^2, s and constants.

s^2 : $0 = A + C$

s : $4 = 3A + B + 2C$

Const : $5 = 2A + 2B + C$

$A = 3$, $B = 1$ and $C = -3$.

$$\therefore \frac{4s+5}{(s+1)^2(s+2)} = \frac{3}{s+1} + \frac{1}{(s+1)^2} - \frac{3}{s+2}. \quad \text{——— (2)}$$

$$\therefore L^{-1}\left\{\frac{4s+5}{(s+1)^2(s+2)}\right\} = 3L^{-1}\left(\frac{1}{s+1}\right) + L^{-1}\left(\frac{1}{(s+1)^2}\right) - 3L^{-1}\left(\frac{1}{s+2}\right)$$

$$= 3e^{-t} + te^{-t} - 3e^{-2t} \quad \text{——— (1)}$$

1. (a) To evaluate: $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$

$$L(\cos 2t - \cos 3t) = \frac{s}{s^2+4} - \frac{s}{s^2+9}. \quad \text{——— (1)}$$

$$\therefore L\left\{\frac{\cos 2t - \cos 3t}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+4} - \frac{s}{s^2+9}\right) ds \quad \text{——— (1)}$$

$$= \frac{1}{2}\log\left(\frac{s^2+9}{s^2+4}\right) \quad \text{——— (3)}$$

(b) To evaluate: $L\{t^2 e^{-3t} \sin 2t\}$

$$L(\sin 2t) = \frac{2}{s^2+4} \quad \text{--- (1)}$$

$$L(t^2 \sin 2t) = \frac{d^2}{ds^2}\left(\frac{2}{s^2+4}\right) = 2\frac{d}{ds}\left[\frac{-1}{(s^2+4)^2} \times 2s\right] \quad \text{--- (1)}$$

$$= -4\frac{d}{ds}\left\{\frac{s}{(s^2+4)^2}\right\}$$

$$= -4\left[\frac{(s^2+4)^2 - s \times 2(s^2+4) \times 2s}{(s^2+4)^4}\right]$$

$$= \frac{-4(s^2+4)}{(s^2+4)^4}\left\{s^2+4-4s^2\right\}$$

$$= \frac{-4}{(s^2+4)^3}(-3s^2+4) \quad \text{--- (1)}$$

$$L(e^{-3t} t^2 \sin 2t) = \frac{-4\left[-3(s+3)^2+4\right]}{\left[(s+3)^2+4\right]^3} \quad \text{--- (1)}$$

2. $$L\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt \quad \text{--- (1)}$$

$$= \frac{1}{1-e^{-2as}}\int_0^{2a} e^{-st} f(t)\,dt \quad \text{--- (1)}$$

$$= \frac{1}{1-e^{-2as}}\left[\int_0^a e^{-st} E\,dt + \int_a^{2a} e^{-st}(-E)\,dt\right] \quad \text{--- (1)}$$

$$= \frac{1}{1-e^{-2as}}\left[E\left(\frac{e^{-st}}{-s}\right)_0^a - E\left(\frac{e^{-st}}{-s}\right)_a^{2a}\right]$$

--- (1)

$$= \frac{E}{s} \times \frac{1}{1-e^{-2as}} \left[\left\{ e^{-st} \right\}_a^{2a} - \left\{ e^{-st} \right\}_0^{a} \right] \quad (2)$$

$$= \frac{E}{s} \times \frac{1}{1-e^{-2as}} \left[\left(e^{-2as} - e^{-as} \right) - \left(e^{-as} - 1 \right) \right]$$

$$= \frac{E}{s} \times \frac{1}{1-e^{-2as}} \left(e^{-2as} - 2e^{-as} + 1 \right)$$

$$= \frac{E}{s} \times \frac{1}{1-e^{-2as}} \left(e^{-as} - 1 \right)^2 \quad \text{--- (1)}$$

$$= \frac{E}{s} \times \frac{1}{(1-e^{-as})(1+e^{-as})} \left(e^{-as} - 1 \right)^2$$

$$= \frac{E}{s} \frac{(1-e^{-as})}{(1+e^{-as})} \times \frac{e^{as/2}}{e^{as/2}} = \frac{E}{s} \left\{ \frac{e^{as/2} - e^{-as/2}}{e^{as/2} + e^{-as/2}} \right\}$$

$$= \frac{E}{s} \tanh\left(\frac{as}{2}\right) \quad \text{--- (2)}$$

3. $f(t) = \cos t + (\cos 2t - \cos t)\, u(t-\pi) + (\cos 3t - \cos 2t)\, u(t-2\pi)$

$$L[f(t)] = L(\cos t) + L(\cos 2t - \cos t)\, u(t-\pi) + L(\cos 3t - \cos 2t)\, u(t-2\pi) \quad \text{--- (2)}$$

$$= L_1 + L_2 + L_3$$

L_1 : $L(\cos t) = \frac{s}{s^2+1}$ --- (1)

L_2 : $L(\cos 2t - \cos t)\, u(t-\pi)$

$F(t) = \cos 2t - \cos t$

$t \to t + \pi$

$F(t+\pi) = \cos 2(t+\pi) - \cos(t+\pi) = \cos 2t + \cos t$ --- (2)

$\therefore f(t) = \cos 2t + \cos t$

$\Rightarrow \bar{f}(s) = \frac{s}{s^2+4} + \frac{s}{s^2+1}.$

$\therefore L_2 = \left(\frac{s}{s^2+4} + \frac{s}{s^2+1}\right) e^{-\pi s}$

$L_3 : L(\cos 3t - \cos 2t)\, u(t - 2\pi)$

$F(t) = \cos 3t - \cos 2t$

$t \to t + 2\pi$

$\therefore f(t) = \cos 3(t+2\pi) - \cos 2(t+2\pi)$

$= \cos 3t - \cos 2t$

$\bar{f}(s) = \frac{s}{s^2+9} - \frac{s}{s^2+4}$

$L_3 = \left(\frac{s}{s^2+9} - \frac{s}{s^2+4}\right) e^{-2\pi s}$

$\therefore L[f(t)] = \frac{s}{s^2+1} + \left(\frac{s}{s^2+4} + \frac{s}{s^2+1}\right) e^{-\pi s}$

$+ \left(\frac{s}{s^2+9} - \frac{s}{s^2+4}\right) e^{-2\pi s}$ —— (2)

4. $y''(t) - 2y'(t) + y(t) = e^t$

$L(y'') - 2L(y') + L(y) = L(e^t)$ —— (1)

$\Rightarrow s^2\bar{y}(s) - sy(0) - y'(0) - 2\{s\bar{y}(s) - y(0)\} + \bar{y}(s) = \frac{1}{s-1}$ —— (1)

$\Rightarrow (s^2 - 2s + 1)\bar{y}(s) - 2s + 1 + 4 = \frac{1}{s-1}$

$\Rightarrow (s^2 - 2s + 1)\bar{y}(s) - 2s + 5 = \frac{1}{s-1}$

$\Rightarrow (s^2 - 2s + 1)\bar{y}(s) = \frac{1}{s-1} + (2s - 5)$ —— (1)

$\Rightarrow$

$$\bar{y}(s) = \frac{1}{(s-1)^3} + 2\frac{s}{(s-1)^2} - \frac{5}{(s-1)^2} \quad \text{—— (1)} \quad (3)$$

$$y(t) = L^{-1}\left[\frac{1}{(s-1)^3}\right] + 2L^{-1}\left[\frac{s-1+1}{(s-1)^2}\right] - L^{-1}\left[\frac{5}{(s-1)^2}\right]$$

$$= e^t \times \frac{t^2}{2} + 2L^{-1}\left[\frac{1}{s-1}\right] + 2L^{-1}\left[\frac{1}{(s-1)^2}\right] - e^t t \times 5$$

$$= \frac{t^2 e^t}{2} + 2e^t - 3te^t \quad \text{—— (3)}$$

5. To find $L^{-1}\left\{\frac{1}{(s+1)(s^2+9)}\right\}$

$\bar{f}(s) = \frac{1}{s+1}$ $\qquad \bar{g}(s) = \frac{1}{s^2+9}$ —— (1)

$\Rightarrow f(t) = e^{-t}$ $\qquad g(t) = \frac{1}{3}\sin 3t$ —— (1)

By Convolution theorem,

$$L^{-1}\{\bar{f}(s)\,\bar{g}(s)\} = \int_{u=0}^{t} f(u)\, g(t-u)\, du \quad \text{—— (1)}$$

$$= \frac{1}{3}\int_{u=0}^{t} e^{-u} \sin 3(t-u)\, du \quad \text{—— (1)}$$

$$= \frac{1}{3}\int_{u=0}^{t} e^{-u} \sin(3t-3u)\, du$$

$$= -\frac{1}{3}\int_{u=0}^{t} e^{-u} \sin(3u-3t)\, du \qquad a=-1,\ b=3$$

$$= -\frac{1}{3}\left[\frac{1}{1+9} e^{-u}\left(-\sin(3u-3t) - 3\cos(3u-3t)\right)\right]_0^t \quad \text{—— (5)}$$

$$= +\frac{1}{30}\left[e^{-u}\left\{\sin(3u-3t) + 3\cos(3u-3t)\right\}\right]_0^t$$

$$= \frac{1}{30}\left(e^{-t} + \sin 3t - 3\cos 3t\right) \quad \text{—— (1)}$$

$$= \frac{1}{30}\left(e^{-t} + \sin 3t - 3\cos 3t\right). \quad \text{— (1)}$$

6. $$\int_{z=-c}^{c}\int_{y=-b}^{b}\int_{x=-a}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$$

$$= \int_{-c}^{c}\int_{-b}^{b}\left(\frac{x^3}{3} + xy^2 + xz^2\right)_{-a}^{a} dy\,dz \quad \text{— (1)}$$

$$= \int_{-c}^{c}\int_{-b}^{b}\left[\left\{\frac{a^3}{3} + ay^2 + az^2\right\} - \left\{-\frac{a^3}{3} - ay^2 - az^2\right\}\right] dy\,dz.$$

$$= 2\int_{-c}^{c}\int_{-b}^{b}\left(\frac{a^3}{3} + ay^2 + az^2\right) dy\,dz \quad \text{— (1)}$$

$$= 2\int_{-c}^{c}\left(\frac{a^3 y}{3} + \frac{ay^3}{3} + ayz^2\right)_{-b}^{b} dz \quad \text{— (1)}$$

$$= 2\int_{-c}^{c}\left[\left\{\frac{a^3 b}{3} + \frac{ab^3}{3} + abz^2\right\} - \left\{-\frac{a^3 b}{3} - \frac{ab^3}{3} - abz^2\right\}\right] . dz$$

$$= 4\int_{-c}^{c}\left(\frac{a^3 b}{3} + \frac{ab^3}{3} + abz^2\right) dz \quad \text{— (1)}$$

$$= 4\left[\frac{a^3 b}{3}z + \frac{ab^3}{3}z + ab\frac{z^3}{3}\right]_{-c}^{c} \quad \text{— (1)}$$

$$= 4\left[\left\{\frac{a^3 bc}{3} + \frac{ab^3}{3}c + \frac{abc^3}{3}\right\} - \left\{-\frac{a^3 bc}{3} - \frac{ab^3 c}{3} - \frac{abc^3}{3}\right\}\right] \quad \text{— (1)}$$

$$= \frac{8abc}{3}\left(a^2+b^2+c^2\right) \quad \text{— (1)}$$

7) WKT

$$\Gamma(m) = 2\int_0^\infty e^{-x^2} x^{2m-1}\, dx$$

$$\Gamma(n) = 2\int_0^\infty e^{-y^2} y^{2n-1}\, dy$$

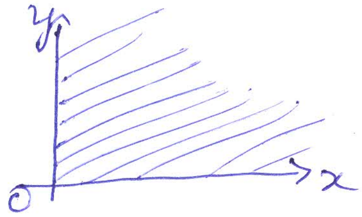
$$\Gamma(m)\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2m-1}\, dx \times 2\int_0^\infty e^{-y^2} y^{2n-1}\, dy$$

$$= 4\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} x^{2m-1} y^{2n-1}\, dx\, dy$$

Take $x = r\cos\theta$, $y = r\sin\theta$

$\theta : 0 \to \pi/2$

$r : 0 \to \infty$



$$\therefore \Gamma(m)\Gamma(n) = 4\int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} e^{-r^2} (r\cos\theta)^{2m-1} (r\sin\theta)^{2n-1}\, r\, dr\, d\theta$$

$$= 2\int_{r=0}^{\infty} e^{-r^2} r^{2(m+n)-1}\, dr \times 2\int_{\theta=0}^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta\, d\theta$$

$$= \Gamma(m+n) \times \beta(n,m)$$

$$= \Gamma(m+n) \times \beta(m,n)$$

$$\therefore \beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

6) $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$

$= \int_0^{\pi/2} \sin^{-1/2}\theta \cdot d\theta \times \int_0^{\pi/2} \sin^{1/2}\theta \cdot d\theta$

$p = -1/2,\ q = 0 \qquad p = 1/2,\ q = 0$

$= \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right) \times \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$

$= \frac{1}{2}\beta\left(\frac{1}{4}, \frac{1}{2}\right) \times \frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{2}\right)$

$= \frac{1}{4} \frac{\Gamma(1/4)\Gamma(1/2)}{\Gamma(3/4)} \frac{\Gamma(3/4)\Gamma(1/2)}{1/4\,\Gamma(1/4)}$

$= \sqrt{\pi} \times \sqrt{\pi}$

$= \pi$

8) Consider

$\int_{-1}^{1} (1+x)^{p-1} (1-x)^{q-1}\, dx$

Take $x = \cos 2\theta \quad \Rightarrow dx = -2\sin 2\theta\, d\theta$

when $x = -1, \quad 2\theta = \pi \quad \Rightarrow \theta = \pi/2$

when $x = 1, \quad 2\theta = 0 \quad \Rightarrow \theta = 0$

$\therefore LHS = \int_{\pi/2}^{0} (1+\cos 2\theta)^{p-1} (1-\cos 2\theta)^{q-1} (-2) \sin 2\theta\, d\theta$

$$= 2\int_0^{\pi/2} (2\cos^2\theta)^{p-1} (2\sin^2\theta)^{q-1}\, 2\sin\theta\cos\theta\, d\theta$$

$$= 2\times 2^{p-1}\times 2^{q-1}\times 2\int_0^{\pi/2} \cos^{2p-2}\theta\ \sin^{2q-2}\theta \cdot \sin\theta\cos\theta\, d\theta$$

$$= 2^{p+q-1}\times 2\int_0^{\pi/2} \sin^{2q-1}\theta\cos^{2p-1}\theta \cdot d\theta$$

$$= 2^{p+q-1}\times \beta(q,p)$$

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$= RHS$