SOLUTION AND SCHEME OF EVALUATION : IAT-II

Analysis of Determinate structures. : 15CV42 :

1(a) state moment area theorem for slope and deflection [4]

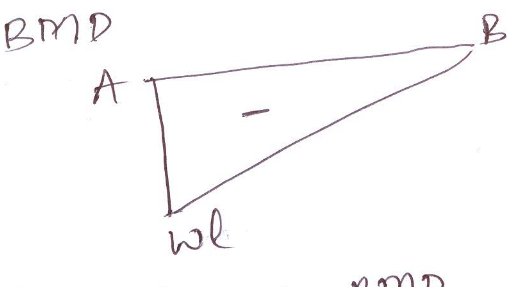
Ans: For slope :- The change in slope between any two points in a straight member is given by the are of M/EI diagram between those two points (02)

$$\theta = \frac{\text{Area of BMD}}{EI}$$

For deflection :- Deflection at any point can be calculated by taking the moment of $\frac{M}{EI}$ diagram about the point where deflection is required. (02)

1 (b) Determine the slope at the free end of a cantilever beam of span 'l' when it is subjected to point load of magnitude 'w' at the free end. [06]

Soln:



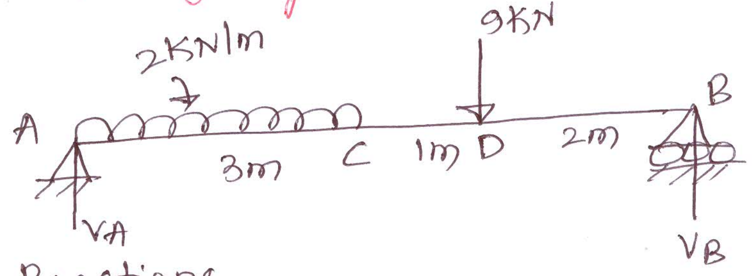
(03)

$$\theta_B = \frac{\text{Area of BMD}}{EI}$$

$$= \frac{\frac{1}{2} \times l \times (-wl)}{EI}$$ (03)

$$= -wl^2/2EI$$

1(c) Determine the maximum deflection for the s/s beam shown in fig. by moment area method.



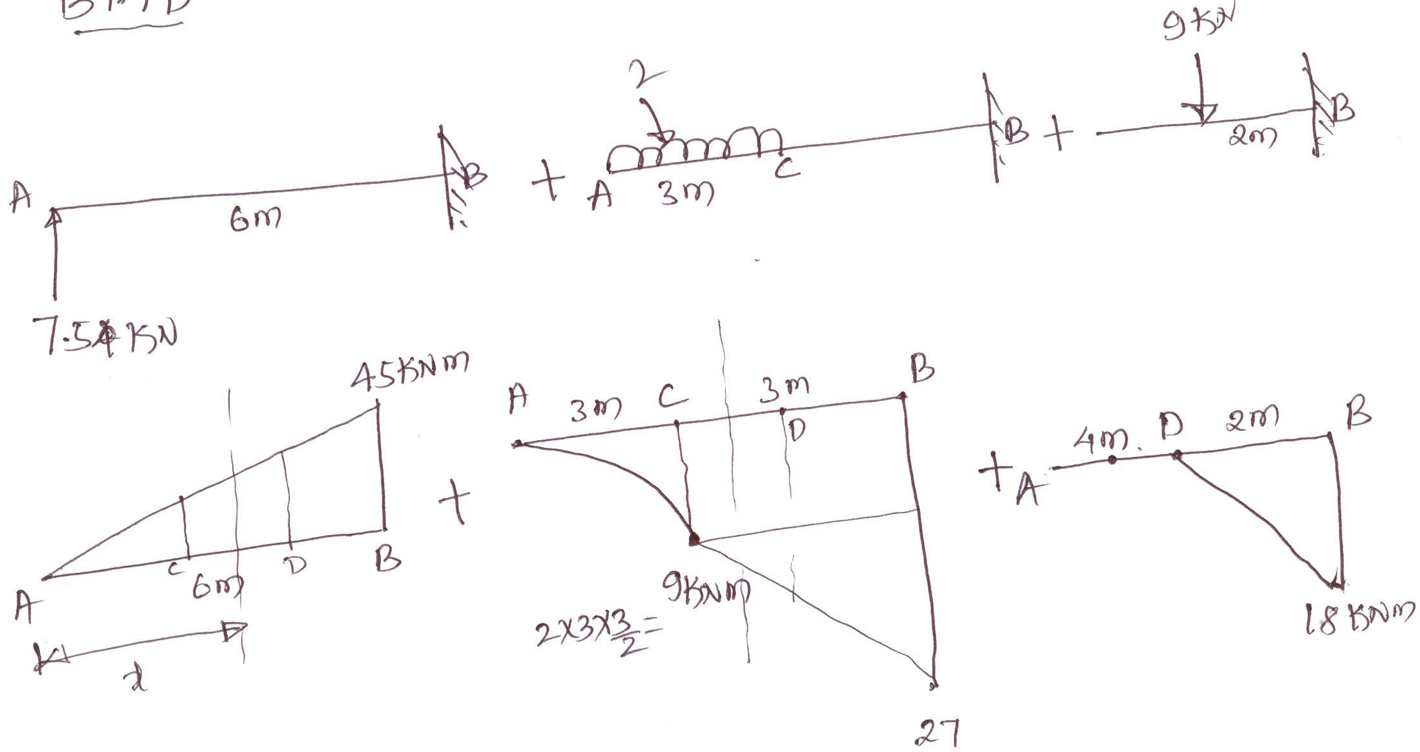
Soln: Reactions

$\Sigma F_y = 0 \quad V_A + V_B = 9 + 2 \times 3$

$\Sigma M_A = 0 \quad 2 \times 3 \times \frac{3}{2} + 9 \times 4 - V_B \times 6 = 0$ (03)

$V_B = 7.5\,KN \quad V_A = 7.5\,KN.$

BMD



$t_{B/A} = \left(\frac{1}{2} \times 6 \times \frac{45}{EI}\right)\left(\frac{1}{3} \times 6\right) - \left(\frac{1}{3} \times \frac{3 \times 9}{EI}\right)\left(3 + \frac{1 \times 3}{4}\right) - \frac{(3 \times 9)}{EI}\left(\frac{3}{2}\right)$

$- \left(\frac{1}{2} \times 3 \times \frac{18}{EI}\right)\left(\frac{1}{3} \times 3\right) - \left(\frac{1}{2} \times 2 \times \frac{18}{EI}\right)\left(\frac{1}{3} \times 2\right)$ (08)

$t_{B/A} = \frac{270}{EI} - \frac{33.75}{EI} - \frac{40.5}{EI} - \frac{27}{EI} - \frac{12}{EI}$

$= \frac{156.75}{EI}$

$$\theta_A = \frac{t_{B/A}}{l} = \frac{156.75}{EI\,6}$$

$$\theta_A = \frac{26.125}{EI}$$

Assuming max defⁿ btⁿ C & D

$$\theta_A - \theta_x = \frac{\frac{1}{2} \times x \times 7.5x}{EI} - \frac{1}{3} \times 3 \times \frac{9}{EI} - \frac{9(x-3)}{EI}$$

$$\frac{156.75}{EI} - 0 = \frac{3.75x^2}{EI} - \frac{9}{EI} - \frac{9x}{EI} + \frac{27}{EI}$$

$$3.75x^2 - 9x - 138.75 = 0$$

$3 \times x \times \frac{1}{2}$

$$x = 7.4\,m.$$

Assuming max defⁿ btⁿ 'AC'

$$\theta_A - \theta_x = \left(\frac{\frac{1}{2} \times x \times 7.5x}{EI}\right) - \left(\frac{1}{3} \times x \times \frac{3}{2} \times x^2\right)$$

OK

$$156.75 = 3.75x^2 - 0.5x^3$$

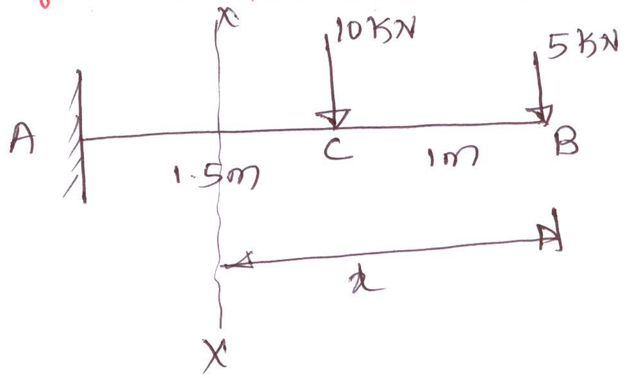
$$x = \underline{\underline{2.9\,m}}$$

∴ Δ_{max} will occur at 2.9m from 'A'

$$\Delta_{max} = \left(\frac{\frac{1}{2} \times 2.9 \times 7.5 \times 2.9}{EI}\right)\left(\frac{1}{3} \times 2.9\right) - \left(\frac{\frac{1}{3} \times 3 \times 8.41}{EI}\right)\left(\frac{1}{4} \times 2.9\right)$$

$$\Delta_{max} = \frac{24.36}{EI.}$$

2(a) Calculate the maximum slope and maximum deflection for the cantilever beam shown in fig.



Sol^n:

$$M_{xx} = 5x - 10(x-1)$$

$$EI \frac{d^2y}{dx^2} = -5x - 10(x-1)$$

$$EI.\frac{dy}{dx} = C_1 \pm 5.\frac{x^2}{2} - \frac{10(x-1)^2}{2}$$

$$EI.\frac{dy}{dx} = C_1 \pm 2.5x^2 - 5(x-1)^2 \quad \text{—— (1)}$$

$$EI.y = C_2 \pm C_1 x \pm \frac{2.5}{3}x^3 - \frac{5}{3}(x-1)^3$$

$$EI.y = C_2 \pm C_1 x \pm 0.833x^3 - 1.66(x-1)^3 \quad \text{—— (2)}$$

05

@ $x = 2.5m$. $\frac{dy}{dx} = 0$

11.25

$$0 = C_1 \mp 2.5(2.5)^2 - 5(2.5-1)^2$$

$$C_1 = \underline{\underline{26.875}}$$

@ $x = 2.5m$. $y = 0$

$$0 = C_2 + 26.875(2.5) \mp 0.833(2.5)^3 - 1.66(2.5-1)^3$$

$$C_2 = -48.56$$

$\therefore EI.\frac{dy}{dx} = 26.875 - 2.5x^2 - 5(x-1)^2$ —— (3)

$EI.y = -48.56 + 26.875x - 0.833x^3 - 1.66(x-1)^3$ —— (4)

$\therefore$ slope at 'B' $x=0$ in (3)

$EI.\theta_B = 26.875$

$\theta_B = \frac{26.875}{EI.}$ 0.5

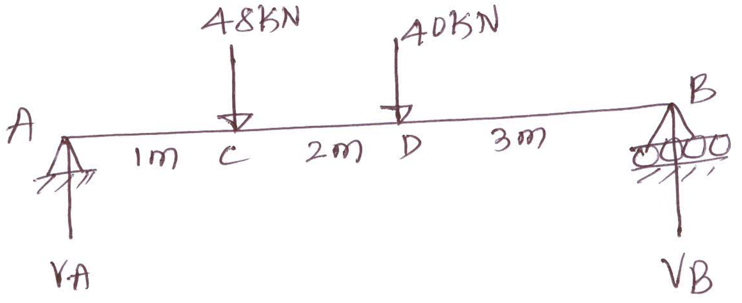
deflection at 'B' $x=0$ in eqn (4)

$EI.y_B = -48.56 + 26.875 \times 0$

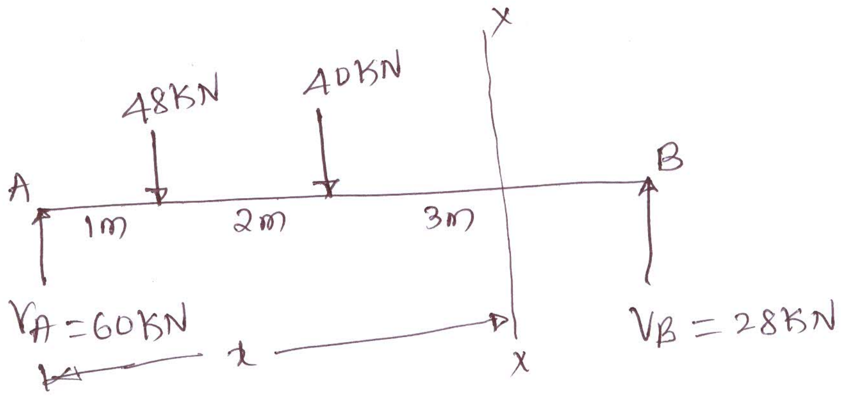
$y_B = \frac{-48.56}{EI}$

2(b) Determine the slope at the supports and deflection under the point load for the s/s beam shown in fig

$E = 2 \times 10^5$ MPa & $I = 85 \times 10^6 mm^4$.



Sol?



$\Sigma F_y = 0 \qquad V_A + V_B = 48 + 40 \quad \text{—(i)}$

$\Sigma M_A = 0 \qquad 48 \times 1 + 40 \times 3 - V_B \times 6 = 0$

$V_B = 28KN, \quad V_A = 60KN$

$M_{xx} = 60x - 48(x-1) - 40(x-3)$

$$EI.\frac{d^2y}{dx^2} = 60x - 48(x-1) - 40(x-3)$$ (05)

$$EI.\frac{dy}{dx} = C_1 + 30x^2 - 24(x-1)^2 - 20(x-3)^2$$

$$EI.y = C_2 + C_1 x + 10x^3 - 8(x-1)^3 - 6.66(x-3)^3$$

@ $x = 0$, $y = 0$

$C_2 = 0$

@ $x = 6m, \; y = 0$

$$0 = 0 + C_1 6 + 10(6)^3 - 8(6-1)^3 - 6.66(6-3)^3$$

$$C_1 = -163.63$$

Final slope and deflection eq^ns

$$EI.\frac{dy}{dx} = -163.63 + 30x^2 - 24(x-1)^2 - 20(x-3)^2 \quad \text{--- (3)}$$

$$EI.y = 0 - 163.66 + 10x^3 - 8(x-1)^3 - 6.66(x-3)^3 \quad \text{--- (4)}$$ (05)

Deflections: at 'C' $x = 1$ m in eq^n (4)

$$EI.y_C = -163.66 + 10(1)^3$$

$$y_C = -\frac{153.66}{EI}$$

at 'D' $x = 2m$ in eq^n (4)

$$EI.y_D = -163.66 + 10(2)^3 - 8(2-1)^3$$

$$y_D = \frac{-91.633}{EI}$$

EI

$= 2 \times 10^5 \times 85 \times 10^6 \; N\,mm^2$

$= \frac{2 \times 10^{5} \times 85 \times 10^{6}}{1000\,(1000)^2}$

$= 17000 \; KNm^2$

$$y_C = -\frac{153.66}{17000} \times 1000 = 9.003\,mm$$ (05)

$$y_D = \frac{-91.33}{17000} \times 1000 = -5.37\,mm$$