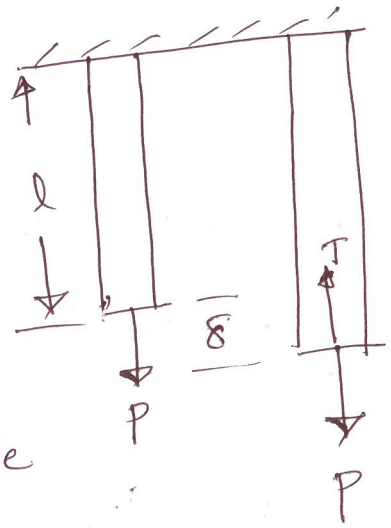
SOLUTION AND SCHEME OF EVALUATION

Analysis of determinate structures.

Improvement test: Sub code: 15CV42

1 (a) Derive an expression for strain energy stored in a member due to axial loading [06].

Ans: consider an elastic member of length 'l' c/s area 'A'. Let w be the gradually applied load. Let 'δ' be the deflection.



$\therefore$ External work done $= \frac{1}{2} W\delta = We$

Let energy stored in the member be Wi

WKT $We = Wi$

Let 'T' be the tension in the member

for equilibrium $T = P$

06

tensile stress $= f = \frac{T}{A}$

$\therefore$ tensile strain $= e = \frac{f}{E} = \frac{T}{AE}$

δ = strain x original length

$\delta = el$

$\delta = \frac{T.l}{AE}$

Strain energy stored = Work done

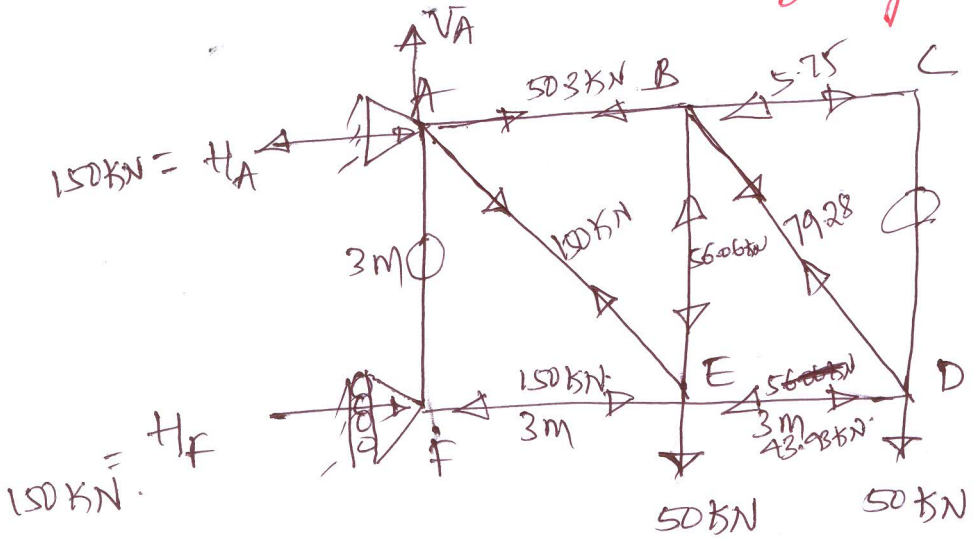
$$= \frac{1}{2} \cdot W \cdot \delta$$

$$= \frac{1}{2} \cdot T \cdot \frac{T \cdot l}{AE}$$

$$= \frac{T^2 l}{AE}$$

$$\text{Strain energy stored} = \frac{W^2 l}{AE}$$

1 (b) Determine the vertical deflection at point 'E' for the truss shown in fig by strain energy method.



Soln:

$\Sigma F_x = 0$

$H_A + H_F = 0$

$\Sigma F_y = 0$

$V_A = 50 + 50$

$V_A = 100$ kN

$\Sigma M_A = 0$

$-H_F \times 3 + 50 \times 3 + 50 \times 6 = 0$

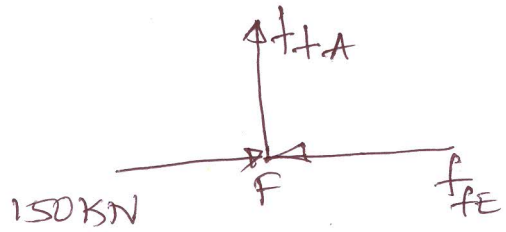
$H_F = 150$ kN

$\therefore H_A = -150$ kN

$H_A = 150$ kN ($\leftarrow$)

Member forces due to applied loads

Joint 'F'



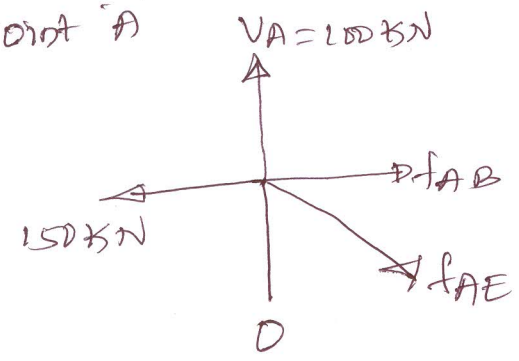
$\Sigma F_y = 0$

$f_{FA} = 0$

$\Sigma f_x = 0$

$f_{FE} = 150 KN \cdot (C)$

Joint 'A'



$\Sigma F_y = 0$

$-f_{AE} \sin 45 + 100 = 0$

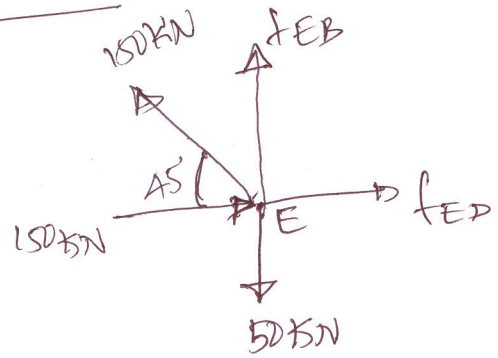
$f_{AE} = 141.42 KN (T)$

$\Sigma F_x = 0$

$-150 + 141.42 \cos 45 + f_{AB} = 0$

$f_{AB} = 50.3 KN (T)$

Joint 'E'



$\Sigma f_x = 0$

$f_{ED} + 150 - 150 \cos 45 = 0$

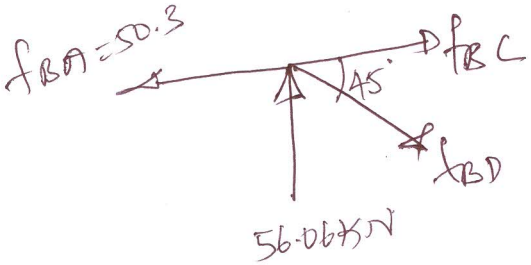
$f_{ED} = -43.93 KN (C)$

$\Sigma F_y = 0$

$-50 + f_{EB} + 150 \sin 45 = 0$

$f_{ED} = -56.066 KN (C)$

Joint B :-



$\Sigma F_y = 0$

$56.06 - f_{BD} \sin 45 = 0$

$f_{BD} = 79.28 KN$

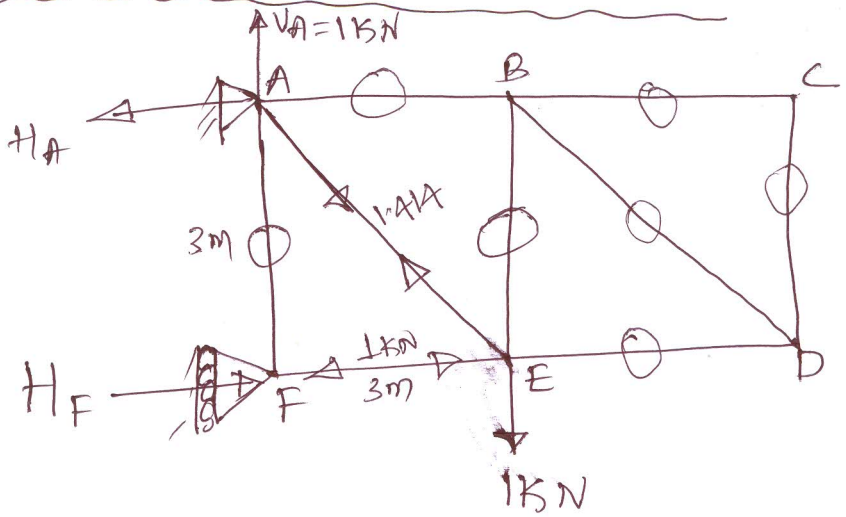
$\Sigma F_x = 0$

$-50.3 + f_{BC} + 79.28 \cos 45 = 0$

$f_{BC} = -5.75 KN (C)$

08

member forces due to unit load



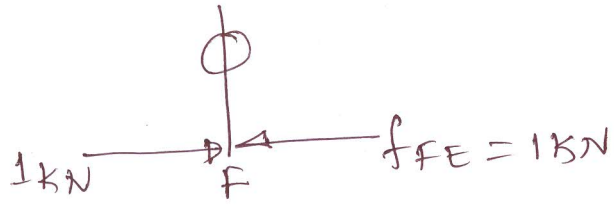
$\Sigma f_y = 0 \quad V_A = 1KN$

$\Sigma f_x = 0 \quad H_A = H_F$

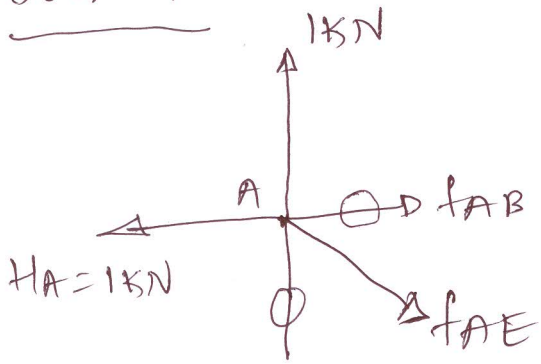
$\Sigma M_A = 0 \quad -H_F \times 3 + 1 \times 3 = 0$

$H_F = 1KN \quad \therefore H_A = 1KN$

Joint F:



Joint 'A'



$\Sigma f_y = 0$

$1 - f_{AE} \sin 45 = 0$

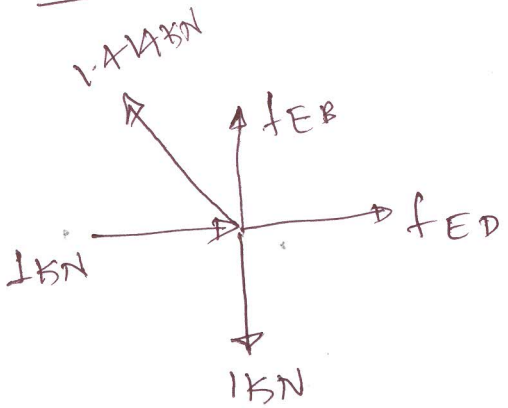
$f_{AE} = 1.414\ KN$

$\Sigma f_x = 0$

$-1 + f_{AB} + 1.414 \cos 45 = 0$

$f_{AB} = 0$

Joint 'E'



$\Sigma F_x = 0$

$+1 + f_{ED} - 1.414 \cos 45 = 0$

$f_{ED} = 0$

$\Sigma F_y = 0$

$-1 + f_{EB} + 1.41 \sin 45 = 0$

$f_{EB} = 0$

08

member	length (m)	P (KN)	K	$\frac{PKl}{AE}$
AB	3	50.3	0	0
BC	3	5.75	0	0
FE	3	150	1	$\frac{45000}{AE}$
ED	3	43.93	0	0
FA	3	0	0	0
EB	3	56.06	0	0
DC	3	0	0	0
AE	4.24	150	1.414	$\frac{894304}{AE}$
BD	4.24	79.28	0	0

$$\therefore \Delta_E = \sum \frac{PKl}{AE}$$

$$= \frac{94430.4}{AE} //$$

03

2 (a) Derive an expression for strain energy stored in a member due to bending

Ans:

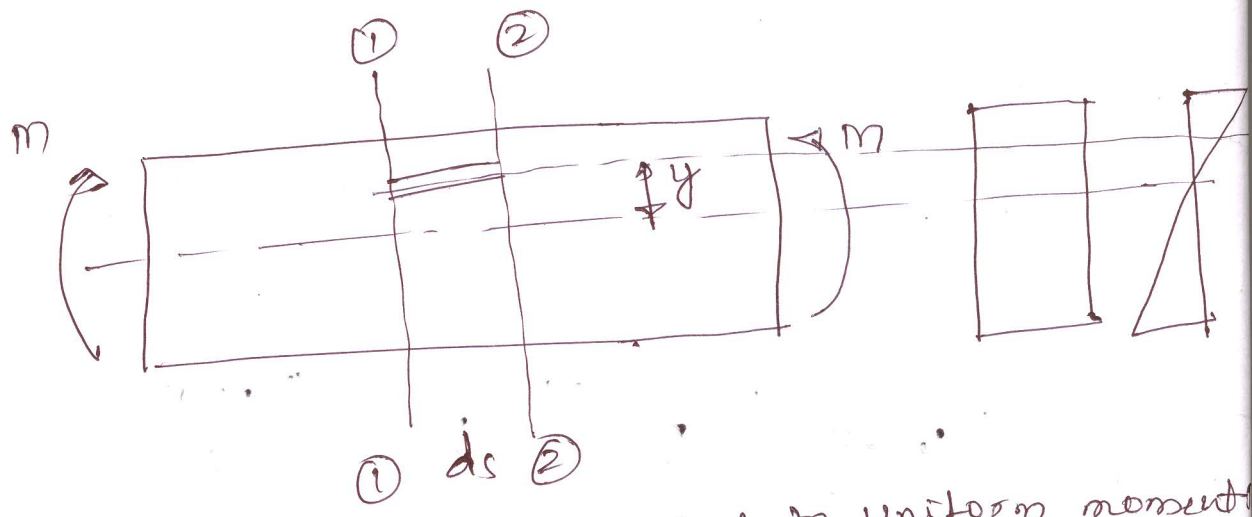


Fig shows a beam subjected to uniform moment

Intensity of stress in elemental area $= f = \frac{M}{I} \cdot y$

∴ Energy stored by the elemental cylinder $= \frac{f^2}{2E} \cdot da \cdot ds$ (07)

$= \frac{1}{2E} \left(\frac{M}{I} \cdot y\right)^2 \cdot da \cdot ds$

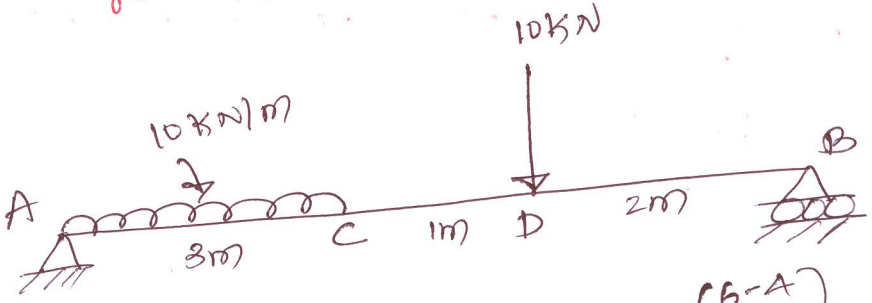
$= \frac{M^2}{2EI^2} \cdot ds \cdot da \cdot y^2$

Energy stored b/w ① - ② $= \frac{M^2}{2EI} \cdot ds \sum da \cdot y^2$

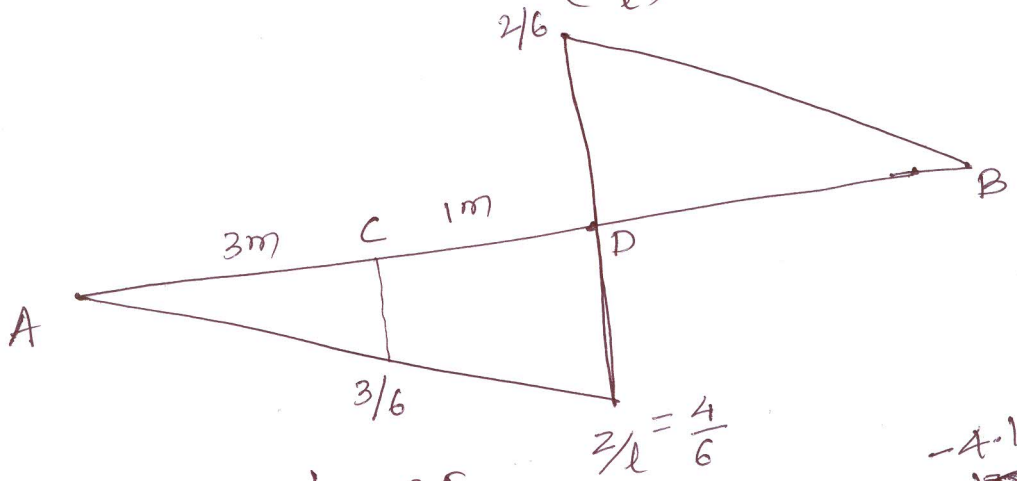
∴ Energy stored by ds length $= \frac{M^2 ds}{2EI}$

∴ Total strain energy stored $= \int \frac{M^2 \cdot ds}{2EI}$

2 (b) For the simply supported beam shown in Fig. determine SF & BM at a distance of 4m from the left support. [8]



$\left(\frac{l-z}{l}\right) = \left(\frac{6-4}{6}\right)$



ILD for S.F

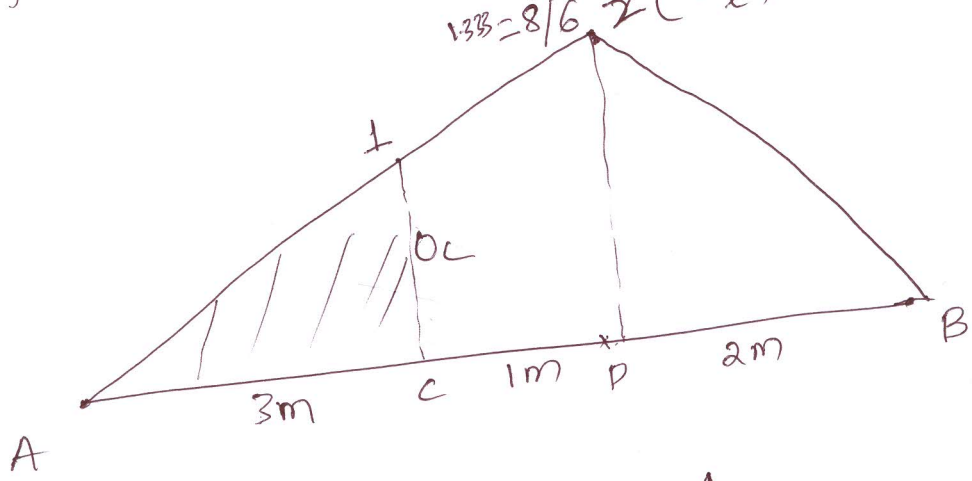
SF @ D $= \left(\frac{1}{2} \times 3 \times \frac{3}{6}\right) \times 10 - 10 \times \frac{2}{6} =$ ~~-4.17~~ KN

& SF @ D $= -\left(\frac{1}{2} \times 3 \times \frac{3}{6}\right)^{\times 10} + 10 \times \frac{2}{6} =$ ~~KN~~ -1.66 KN

04

$\left(\frac{l-z}{l}\right)$ A $\left(\frac{6-4}{6}\right)$

1.33 = 8/6 z



04

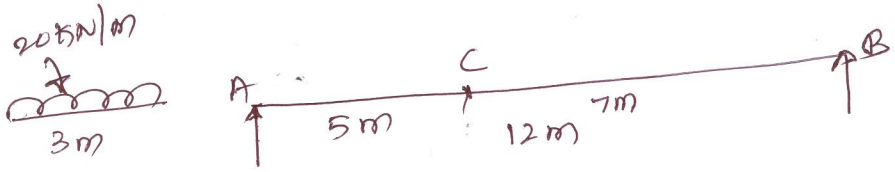
$\frac{4}{1.333} = \frac{3}{O_L} \Rightarrow O_L = 1$

BM @ D $= \left(\frac{1}{2} \times 3 \times 1\right) 10 + 10 \times 1.333$

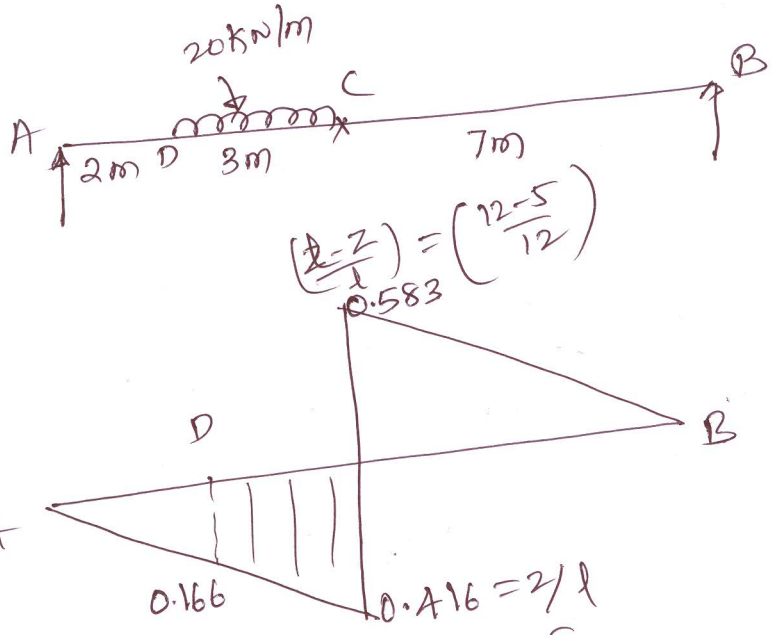
$= 28.33$ KNm.

2(c) A S/S beam has a span of 12m, a udl of 20KN/m and length 3m is moving from left to right. Determine max values of S.F & BM at a distance of 5m from left support.

Soln:



(i) For max -ve S.F Load position is as shown below



$\left(\frac{l-z}{l}\right) = \left(\frac{12-5}{12}\right)$

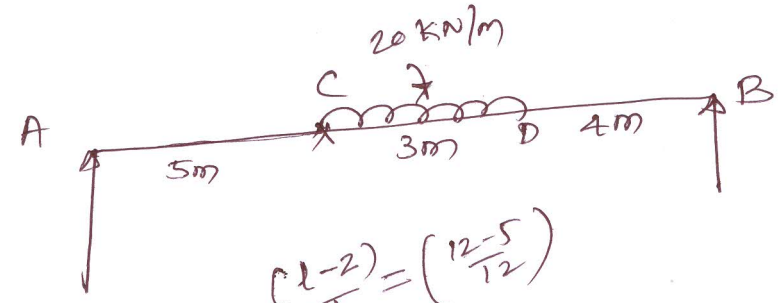
ILD for S.F

03

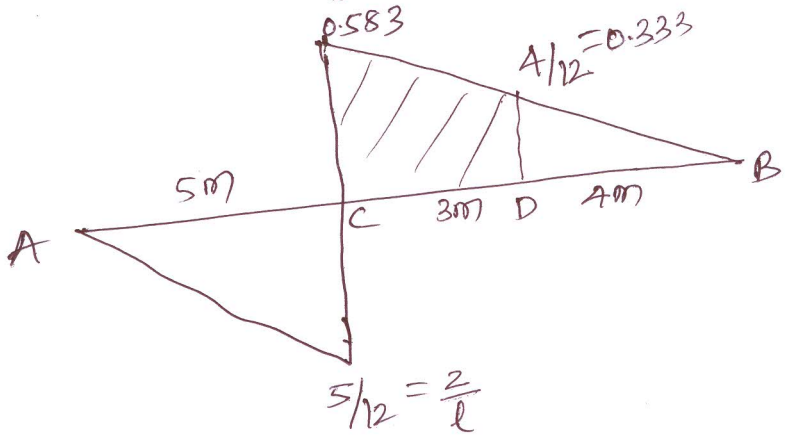
Max -ve. S.F $= -\frac{(0.166 + 0.416) \times 3 \times 20}{2}$

$= -17.478$ KN

(ii) For max +ve S.F load position is as shown below



$\frac{(l-2)}{l} = \left(\frac{12-5}{12}\right)$



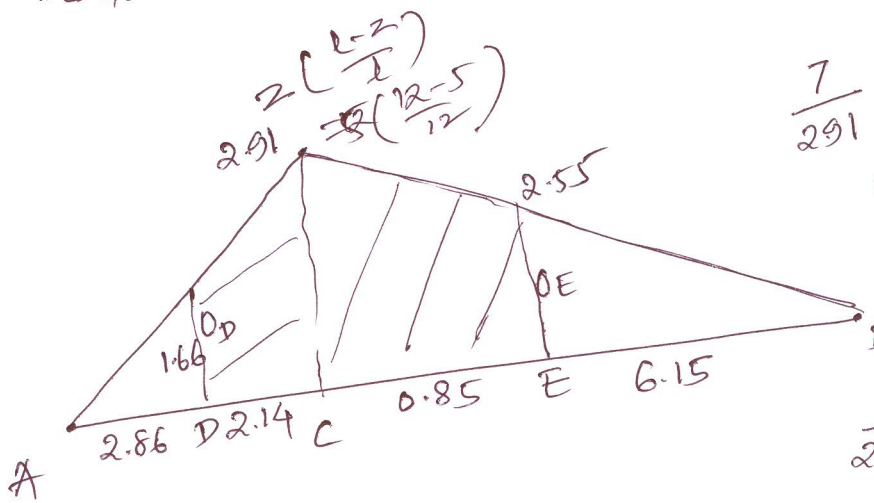
03

$5/12 = \frac{2}{l}$

Max +ve S.F $= \left(\frac{0.583+0.333}{2}\right) \times 3 \times 20$

$= 27.48$ KN.

(iii) For max B.M load position is as shown below

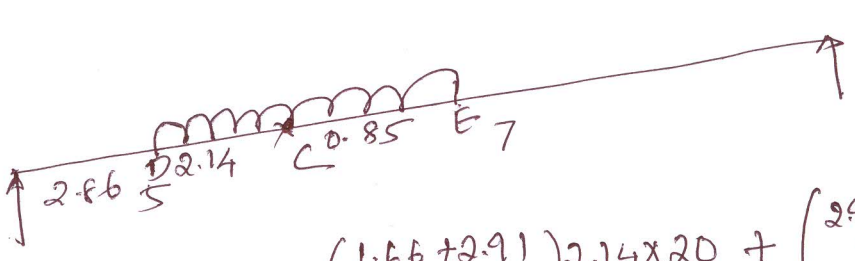


$\frac{7}{291} = \frac{6.15}{OE}$

$OE = 2.55$

$\frac{5}{2.91} = \frac{2.86}{OD}$

$OD = 1.66$



04

$\therefore$ Max B.M $= \left(\frac{1.66+2.91}{2}\right) 2.14 \times 20 + \left(\frac{2.91+2.55}{2}\right) \times 0.85 \times 20$

$= 97.798 + 46.41 = 144.208$ KNM.