AH - IAT - 3
Scheme & Solution

1) Longport sketch. — (4)

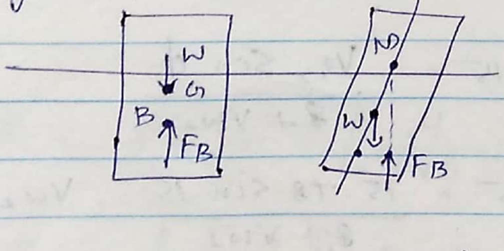
Functions of components - Dam.
Penstock.
Turbines.
Tail race.
} - (3)

Heads - Gross head
Net head.
} - (2)

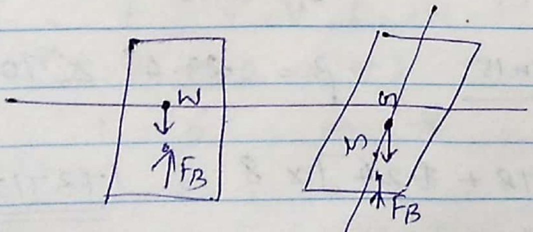
2) Centre of bouyancy = Pt. through which force of buoyancy acts. Acts through C.G. of fluid displaced. — (2)

Metacentre = Pt. ab. which a body starts oscillating when body tilted by small angle. The pt. @ which line of action of buoyant force will meet normal axis of body when body given a small displacement. → (2)

Floating body



M above G -
Stable equilibrium.
Anticlockwise couple developed — (5)



M below G -
unstable equilibrium.
Clockwise moment developed

3.) Geometric - Linear dimensions in model & prototype same. — (3)

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r.$$

Kinematic - ratios of velocity & acceleration @ corresponding pts in model & prototype same. — (3)

$$\frac{V_{p1}}{V_{m1}} = \frac{V_{p2}}{V_{m2}} = V_r; \quad \frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} = a_r :$$

Dynamic similarity - Similarity of forces b/n model & prototype.

$$\frac{F_{p1}}{F_{m1}} = \frac{F_{p2}}{F_{m2}} = F_r .$$ — (3)

Part B

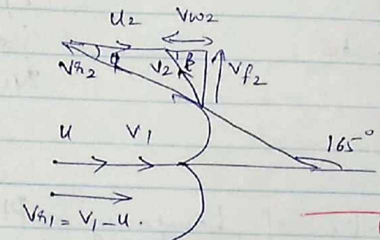
4.) $u = 8\,m/s$

$Q = .7\,m^3/s$.

$H = 30m$.

$\phi = 180 - 165 = 15°$.

$C_v = .98$.



— (2)

$V_1 = C_v\sqrt{2gH} = .98\sqrt{2g \times 30} = \underline{23.78\,m/s}$. — (1)

$V_{r1} = 23.78 - 8 = \underline{15.78\,m/s}$

$V_{r1} = V_{r2} = \underline{15.78\,m/s}$ $\quad u_1 = u_2 = \underline{8\,m/s}$. — (1)

$W = \rho a v_1 [V_{w1} \pm V_{w2}] u$. — (1)

$V_{w1} = V_1 = \underline{23.78\,m/s}$

$\tan\phi = \frac{V_{f2}}{u_2 + V_{w2}}$ $\quad \tan 15 = \frac{V_{r2}\sin\phi}{8 + V_{w2}}$.

$\tan 15 = \frac{15.78\sin 15}{8 + V_{w2}}$ $\quad V_{w2} = \underline{7.24\,m/s}$ — (1)

$\tan\beta = \frac{V_{f2}}{V_{w2}} = \frac{15.78\sin 15}{7.42}$ $\quad \beta = 29.4° < 90°$

$W = 1000 \times .7 [23.78 + 7.24] \times 8 = \underline{173712\,W}$ — (1)

$\eta = \frac{173712}{\frac{1}{2}\rho a v_1^3} = \frac{173712}{197920.94} \times 100 = \underline{87.77\%}$ — (1)

Prototype	Model
$D_p = 2m$.	$D_m = .15m$.
$S_p = .9$	$V_m = ?$
$\rho_p = 900\,kg/m^3$.	$Q_m = ?$
$\mu_p = 3\times10^{-2}$ Poise.	$\mu_m = .01$ Poise.
$Q_p = 5000\,L/s$.	$\rho_m = 1000\,kg/m^3$

— (1)

$V_p = \frac{Q_p}{A_p} = \frac{5}{\frac{\pi}{4}\times2^2} = \underline{1.59\,m/s}$. — (1)

Reynold's model law since pipe flow.

$$\left[\frac{\rho V D}{\mu}\right]_p = \left[\frac{\rho V D}{\mu}\right]_m$$ — (2)

$$\frac{900 \times 1.59 \times 2}{3\times10^{-2}} = \frac{1000 \times V_m \times .15}{0.01}$$

$V_m = \underline{6.36\,m/s}$. — (2)

$Q_m = V_m \cdot A_m = 6.36 \times \frac{\pi}{4} \times .15^2 = \underline{0.1123\,m^3/s}$.

$$R = \rho V^2 D^2 \phi\left[\frac{\mu}{\rho V D}, \frac{H}{D}\right]$$

Dimension

$R = MLT^{-2}$ $\quad D = L$

$\rho = ML^{-3}$ $\quad \mu = ML^{-1}T^{-1}$

$V = LT^{-1}$ $\quad H = L$

$R = f[\rho, V, D, \mu, H]$

$f'(R, \rho, V, D, \mu, H) = 0$.

$n = 6$; $m = 3$; No of π terms $= 6 - 3 = 3$. — (1)

$f(\pi_1, \pi_2, \pi_3) = 0$.

Repeating variables => D, V, ρ.

$\pi_1 = D^{a_1} V^{b_1} \rho^{c_1} R$ — (1)

$\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} \mu$ — (2)

$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} H$. — (3)

} (2)

$\pi_1 = D^{a_1} V^{b_1} \rho^{c_1} R$

$M^0 L^0 T^0 = L^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1} MLT^{-2}$

$0 = c_1 + 1 \; ; \; c_1 = -1$

$0 = a_1 + b_1 - 3c_1 + 1 \; ; \; a_1 = -2$

$0 = -b_1 - 2 \; ; \; b_1 = -2$

$\pi_1 = D^{-2} V^{-2} \rho^{-1} R$

$\pi_1 = \dfrac{R}{\rho D^2 V^2}$ — (1)

$\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} \mu$

$M^0 L^0 T^0 = L^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2} ML^{-1}T^{-1}$

$0 = c_2 + 1 \; ; \; c_2 = -1$

$0 = a_2 + b_2 - 3c_2 - 1 \; ; \; a_2 = -1$

$0 = -b_2 - 1 \quad b_2 = -1$

$\pi_2 = D^{-1} V^{-1} \rho^{-1} \mu$

$\pi_2 = \dfrac{\mu}{\rho V D}$ — (1)

$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} H$

$M^0 L^0 T^0 = L^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} L$

$0 = c_3$

$0 = a_3 + b_3 - 3c_3 + 1 \quad a_3 = -1$

$0 = -b_3$

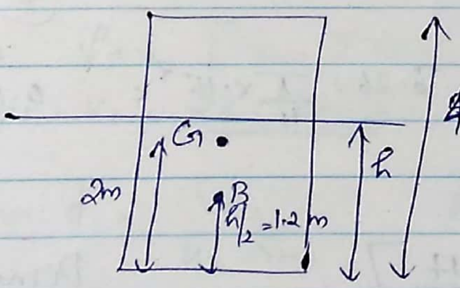
$\pi_3 = D^{-1} V^0 \rho^0 H = H/D$ — (1)

$f\left(\dfrac{R}{\rho V^2 D^2}, \dfrac{\mu}{\rho V D}, \dfrac{H}{D}\right) = 0$

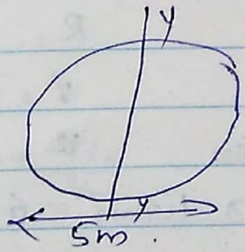
$\dfrac{R}{\rho V^2 D^2} = \phi\left[\dfrac{\mu}{\rho V D}, \dfrac{H}{D}\right] = 0$

$R = \rho V^2 D^2 \phi\left[\dfrac{\mu}{\rho V D}, \dfrac{H}{D}\right] = 0$

7.)



— (2)



$GM = \dfrac{I}{V} - BG$ — (1)

$h = ?$

$W = F_B$

$\left(\dfrac{\pi}{4} \times 5^2 \times 4\right) \cdot 6 \times 9810 = \dfrac{\pi}{4} \times 5^2 \times h \times 9810$

$h = 2.4\,m$ — (2)

$BG = 2 - 1.2 = .8\,m$ — (1)

$GM = \dfrac{\dfrac{\pi D^4}{64}}{\dfrac{\pi}{4} \times 5^2 \times 2.4} - .8 = -.1489\,m$ — (1)

$\therefore$ Unstable. — (1)

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