USN

CMRIT

Internal Assessment Test III– May 2018

Sub	OPERATIONS RESEARCH			Sub Code:	15CS653	Branch:	CSE/ISE
Date:	23.05.2018	Duration: 90 min's	Max Marks: 50	Sem/Sec:	BE 6thsem		

Answer any FIVE FULL Questions

		MARKS	CO	RBT
1	Solve for the optimal solution, the following transportation problem that minimizes the transportation cost for the following transportation problem	[10]	CO2	L3

	D1	D2	D3	D4	Supply
O1	10	2	20	11	10
O2	12	7	9	20	20
O3	4	14	16	18	10
Demand	5	15	15	15	

2	Solve for the optimal distribution for the company so as to maximize the total profit for the following transportation problem	[10]	CO2	L3

	A	B	C	D	Supply
P	2	3	11	7	6
Q	1	0	6	1	1
R	5	8	15	9	10
Demand	7	5	3	2	

3	Solve for an optimal assignment schedule that minimizes the assignment cost for the following problem	[10]	CO2	L3

	A	B	C	D	E
M1	4	6	10	5	6
M2	7	4	-	5	4
M3	-	6	9	6	2
M4	9	3	7	2	3

4	Solve for an optimal assignment schedule that maximizes the assignment profit for the following problem	[10]	CO2	L3

	P	Q	R	S
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

5	Use the principle of dominance to solve the game	[10]	CO2	L3

PAYOFFS	PLAYER B		
PLAYER A	3	-2	4
	-1	4	2
	2	2	6

6 Explain the nature of Metaheuristics. [10] [illegible]

7 Explain about Tabu search algorithm.. [10] [illegible]

TRANSPORTATION PROBLEM

Pr Find the optimal solution to the following transportation problem that minimizes the transportation cost

Destination

		D_1	D_2	D_3	D_4		
origin	O_1	10	2	20	11	10	Supply
	O_2	12	7	9	20	20	
	O_3	4	14	16	18	10	
		5	15	15	15		

Demand

Sol The given problem is of minimization and unbalanced. Total supply $= 10+20+10 = 40$
$< 50 = 5+15+15+15 =$ Total demand

Let O_3 be a dummy origin at which the supply is 10 and we assume that the associated cost from O_3 to any of the destinations is zero (To make this problem a balanced one)

	D_1	D_2	D_3	D_4	
O_1	10	2	20	11	10
O_2	12	7	9	20	20
O_3	4	14	16	18	10
O_4	0	0	0	0	10
	5	15	15	15	

we apply VAM to find initial basic feasible solution to the above problem.

	D_1	D_2	D_3	D_4		
O_1	10	2	20	11	10	(10-2=8)
O_2	12	7	9	20	20	(9-7=2)
O_3	4	14	16	18	10	(14-4=10)
O_4	0	0	0	$\boxed{10}$ 0	10	(0-0=0)
	5	15	15	15		
	(4-0=4)	(20-0=2)	(9-0	(11-0=11)		

$x_{44} = 10$

	D_1	D_2	D_3	D_4		
O_1	10	2	20	11	10	(10-2=8)
O_2	12	7	9	20	20	(9-7=2)
O_3	$\boxed{5}$ 4	14	16	18	10	(14-4=10)
	5	15	15	5		
	(10-4=6)	(7-2=5)	(16-9=7)	(18-11=7)		

$x_{31} = 5$

	D_2	D_3	D_4		
O_1	$\boxed{10}$ 2	20	11	10	(11-2=9)
O_2	7	9	20	20	(9-7=2)
O_3	14	16	18	5	(16-14=2)
	15	15	5		
	(7-2=5)	(16-9=7)	(18-11=7)		

$x_{12} = 10$

	D_2	D_3	D_4		
O_2	7	$\boxed{15}$ 9	20	20	(9-7=2)
O_3	14	16	18	5	(16-14=2)
	5	15	5		
	(14-7=7)	(16-9=7)	(20-18=2)		

$x_{23} = 15$

	D_2	D_4		
O_2	$\boxed{5}$ 7	20	5	(20-7=13)
O_3	14	18	5	(18-14=4)
	5	5		
	(14-7=7)	(20-18=2)		

$x_{22} = 5$

	D_4	
O_3	$\boxed{5}$ 18	5
	5	

$x_{34} = 5$

Now the initial basic feasible solution obtained by VAM is given by

	D_1	D_2	D_3	D_4	
O_1	10	$\boxed{10}$ 2	20	11	10
O_2	12	$\boxed{5}$ 7	$\boxed{15}$ 9	20	20
O_3	$\boxed{5}$ 4	14	16	$\boxed{5}$ 18	10
O_4	0	$\boxed{\cdot}$ 0	0	$\boxed{10}$ 0	10
	5	15	15	15	

→ This is choosen as basic cell.

Here No of origins $= m = 4$ and No of Destinations $= n = 4$

$\therefore$ The no of basic variables $= m+n-1 = 7$

Thus the initial basic feasible solution is degenerate.

Optimality condition (for minimization) for this transportation problem is

'$u_i + v_j - c_{ij} \leq 0$' for all (i,j) nonbasic cell

Here the multipliers u_i, v_j are chosen such that $u_i + v_j - c_{ij} = 0$ for all (i,j) basic

	D_1	D_2	D_3	D_4		
O_1	10	$\boxed{10}$ 2	20	11	$u_1 = 0$	10
O_2	12	$\boxed{5}$ 7	$\boxed{15}$ 9	20	$u_2 = 5$	20
O_3	$\boxed{5}$ 4	14	16	$\boxed{5}$ 18	$u_3 = 16$	10
O_4	0	$\boxed{0}$ 0	0	$\boxed{10}$ 0	$u_4 = -2$	10
	$v_1 = -12$	$v_2 = 2$	$v_3 = 4$	$v_4 = 2$		
	5	15	15	15		

	D_1	D_2	D_3	D_4	
O_1	10	[10] 2	20	11	10
O_2	12	[5] 7	[15] 9	20	20
O_3	[5] 4	[θ] 14	16	[5-θ] 18	10
O_4	0	[0-θ] 0	0	[10+θ] 0	10
	5	15	15	15	

Non basic cell	$u_i+v_j-c_{ij}$
(1,1)	0-12-10 = -22
(1,3)	0+4-20 = -16
(1,4)	0+2-11 = -9
(2,1)	5-12-12 = -19.
(2,4)	5+2-20 = -13
(3,2)	16+2-14 = 4
(3,3)	16+4-16 = 4
(4,1)	-2-12-0 = -14
(4,3)	-2+4-0 = 2

To move towards optimality the cell (3,2) becomes basic by replacing the basic cell (4,2). This results in the following table, for which we have to perform the optimality test.

	D_1	D_2	D_3	D_4		
O_1	10	[10] 2	20	11	10	$u_1 = 0$
O_2	12	[5] 7	[15] 9	20	20	$u_2 = 5$
O_3	[5] 4	[0] 14	16	[5] 18	10	$u_3 = 12$
O_4	0	0	0	[10] 0	10	$u_4 = -6$
	5	15	15	15		

$v_1 = -8$ $v_2 = 2$ $v_3 = 4$ $v_4 = 6$

Non basic cell	(1,1)	(1,3)	(1,4)	(2,1)	(2,4)	(3,3)	(4,1)
$u_i+v_j-c_{ij}$	0-8-10 = -18	0+4-20 = -16	0+6-11 = -5	5-8-12 = -15	5+6-20 = -9	12+4-16 = 0	-8-6-0 = -14

Non basic	(4,2)	(4,3)
$u_i+v_j-c_{ij}$	-6+2-0 = -4	-6+4-0 = -2

Thus, the solution associated with the above solution is optimal and the associated transportation cost is $z = \sum_{j=1}^{4} \sum_{i=1}^{4} c_{ij} x_{ij} = 300$

6 Marks

TRASPORTATION PROBLEM

(Pr) The profit per truck load of cement (in hundred of rupees) from each plant to each project site are as follow

		A	B	C	D	
Plants	P	2	3	11	7	6
	Q	1	0	6	1	1
	R	5	8	15	9	10
		7	5	3	2	

(Project sites: A, B, C, D)

Determine the optimal distribution for the company so as to maximize the total profit.

Sol) This is a maximization problem and associated balanced transportation profits are denoted by f_{ij}. To find the optimal solution, it is enough to find optimal solution that minimize the following transportation problem.

	A	B	C	D	
P	-2	-3	-11	-7	6
Q	-1	0	-6	-1	1
R	-5	-8	-15	-9	10
	7	5	3	2	

The transportation cost associated with this is denoted by w we will find initial basic feasible solution.

by using VAM

	A	B	C	D		
P	-2	-3	-11	-7	6	(-7-(-11)=4)
Q	-1	0	-6	-1	1	(-1-(-6)=5)
R	-5	-8	[3] -15	-9	10	(-9-(-15)=6) ✓
	7	5	3	2		

$x_{33}=3$

(-2-(-5)=3)

(-3-(-8)=5)

(-11-(-15)=4)

(-7-(-9)=2)

	A	B	D		
P	-2	-3	-7	6	(-3-(-7)=4)
Q	-1	0	-1	1	(-1-(-1)=0)
R	-5	[5] -8	-9	7	(-8-(-9)=1)
	7	5	2		

$x_{32}=5$

(-2-(-5)=3)

(-3-(-8)=5) ✓

(-7-(-9)=2)

	A	D	
P	-2	[2] -7	6 $(-2-(-7)=5)$
Q	-1	-1	1 $(-1-(-1)=0)$
R	-5	-9	2 $(-5-(-9)=4)$
	7	2	
	$(-2-(-5))=3$	$(-7-(-9)=2)$	

$x_{14}=2$

	A	
P	[4] -2	4
Q	[1] -1	1
R	[2] -5	2
	7	

$x_{11}=4$

$x_{21}=1$

$x_{31}=2$

Thus the initial basic feasible solution obtained by VAM is

	A	B	C	D	
P	[4] -2	-3	-11	[2] -7	6
Q	[1] -1	0	-6	-1	1
R	[2] -5	[5] -8	[3] -15	-9	10
	7	5	3	2	

Here, the no of origins $=3=m$ and the no of destinations $=n=4$

$\therefore$ No of basic variables (cells) $= m+n-1=3+4-1=6$

The optimality (to minimize the transportation cost) condition is '$u_i + v_j - c_{ij} \le 0$' $\forall$ nonbasic cells (i,j).

The multipliers u_i, v_j can be found by solving $u_i + v_j = c_{ij}$ $\forall (i,j)$ basic

	A	B	C	D		
P	$\boxed{4}$ −2	−3	−11	$\boxed{2}$ −7	6	$u_1 = 0$
Q	$\boxed{1}$ −1	0	−6	−1	1	$u_2 = 1$
R	$\boxed{2}$ −5	$\boxed{5}$ −8	$\boxed{3}$ −15	−9	10	$u_3 = -3$
	7	5	3	2		
	$v_1 = -2$	$v_2 = -5$	$v_3 = -12$	$v_4 = -7$		

Non basic cells	$u_i + v_j - c_{ij}$
(1,2)	$0 - 5 - (-3) = -2$
(1,3)	$0 - 12 - (-11) = -1$
(2,2)	$1 + (-5) - 0 = -4$
(2,3)	$1 - 12 - (-6) = -5$
(2,4)	$1 - 7 - (-1) = -5$
(3,4)	$-3 - 7 - (-9) = -1$

$\therefore$ The basic feasible solution associated with the above table is optimal, and the optimal (minimum) value is

$$w = \sum_{j=1}^{4} \sum_{i=1}^{3} c_{ij} x_{ij} = -118.$$

$\therefore$ Maximum profit $= -w = -(-118)$
$= 118$,

which is attained at

$x_{11} = 4,\ x_{14} = 2,\ x_{21} = 1,\ x_{31} = 2,\ x_{32} = 5,\ x_{33} = 3$

$x_{12} = x_{13} = x_{22} = x_{23} = x_{24} = x_{34} = 0.$

ASSIGNMENT PROBLEM

Pb 3) Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limiting space Machine M_2 cannot be placed at C and M_3 cannot be placed at A. C_{ij} the assignment cost of machine i to place J in rupees is shown below.

	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	—	5	4
M_3	-	6	9	6	2
M_4	9	3	7	2	3

Find the optimal assignment shedule.

Sol As it is not possible to assign M_2 to C and M_3 to A, one can assume the associated assignment costs to be very large M (say) > 0. Further, the problem is not balanced as the number of machines are less than

Ans
$M_1 \to A$
$M_2 \to B$
$M_3 \to E$
$M_4 \to D$
Cost = 12

the number of places. We need a dummy Machine M_5 for which the installing in a place is assumed to be zero. This results in a balanced assignment problem whose matrix is shown in the left

$$\begin{array}{c|ccccc|c} & A & B & C & D & E & \\ M_1 & 4 & 6 & 10 & 5 & 6 & (4) \\ M_2 & 7 & 4 & M & 5 & 4 & (4) \\ M_3 & M & 6 & 9 & 6 & 2 & (2) \\ M_4 & 9 & 3 & 7 & 2 & 3 & (2) \\ M_5 & 0 & 0 & 0 & 0 & 0 & \end{array}$$

↳ row minimum

Subtracting row minimum from element of the associated row results in the following matrix

(5 Mark)

$$\begin{pmatrix} 0 & 2 & 6 & 1 & 2 \\ 3 & 0 & M & 1 & 0 \\ M & 4 & 7 & 4 & 0 \\ 7 & 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The minimum number of lines that covers every zero of this matrix is 5, which is same as the order $n=5$ of this matrix. So optimal assignment is possible.

(5 Mark)

	A	B	C	D	E
M_1	$\boxed{0}$				
M_2		$\boxed{0}$			$\cancel{0}$
M_3					$\boxed{0}$
M_4				$\boxed{0}$	
M_5	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

The optimal assignment that minimizes the total placement charge is

$M_1 \to A$

$M_2 \to B$

$M_3 \to E$

$M_4 \to D$

$\boxed{M_5 \to C}$

↳ not required

and the associated cost is $c_{11}+c_{22}+c_{35}+c_{44}+c_{53}=12$

ASSIGNMENT PROBLEM

P04 A company has a team of four Salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesman and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as below.

	D1	D2	D3	D4
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

Find the assignment of Salesmen to various districts which will yield maximum profit.

$A \to D1$
$B \to D3$
$C \to D2$
$D \to D4$
Max Profit $= 61$

Sol The given problem is balanced as no of sales persons = no of districts = 4 = the order of the associated matrix.

As the problem is of maximization, it is enough to minimize the following assignment problem

$$\begin{pmatrix} -16 & -10 & -14 & -11 \\ -14 & -11 & -15 & -15 \\ -15 & -15 & -13 & -12 \\ -13 & -12 & -14 & -15 \end{pmatrix}$$

subtracting row minimum from each of the element of associated row results in

$$\begin{pmatrix} 0 & 6 & 2 & 5 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{pmatrix}$$

small

Now this matrix has atleast one row in every row/column. and the minimum no of lines that covers every zero of this matrix is 4, which is the order of the matrix. so optimal assignment is possible

small

	D1	D2	D3	D4
A	$\boxed{0}$			
B			$\boxed{0}$	$\cancel{0}$
C	$\cancel{0}$	$\boxed{0}$		
D				$\boxed{0}$

$\therefore$ The optimal assignment that minimizes the cost of this problem is $A \to D_1$; $B \to D3$, $C \to D_2$; $D \to D4$. The associated cost is $c_{11} + c_{23} + c_{32} + c_{44}$

$$= (-16) + (-15) + (-15) + (-15)$$

$$= -61$$

$\therefore$ Maximum profit is $-(-61) = 61$ and this is possible for the assignment

$A \to D1$, $B \to D3$; $C \to D2$; $D \to D4$

(P1) Use the principle of dominance to solve the game

$$
\begin{array}{cc}
 & \text{Player B} \\
\text{Player A} &
\begin{array}{c|ccc}
 & B_1 & B_2 & B_3 \\
\hline
A_1 & 3 & -2 & 4 \\
A_2 & -1 & 4 & 2 \\
A_3 & 2 & 2 & 6
\end{array}
\end{array}
$$

Sol: The payoff matrix associated with the game is

$$
\begin{pmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{pmatrix}
\begin{matrix} (-2) \\ (-1) \\ (2) \end{matrix}
\rightarrow \text{Row minimum}
$$

$$(3) \quad (4) \quad (6)$$

Column Maximums

Maxmin value = Maximum of row minimums $= 2$

Minimum of column maximums = Minmax value

$3 =$ Min max value $\neq$ Max min value $= 2$

The Game has no saddle point. We have to use mixed strategies.

Notice that, in the payoff matrix every element of column 3 is greater than equal to every element of column 1. Thus by principle of dominance one can exclude the dominated column (column 3 is dominated column and column 1 is dominating column). The resulting payoff matrix is

(3Mark)

$$\begin{array}{c|cc} & B_1 & B_2 \\ A_1 & 3 & -2 \\ A_2 & -1 & 4 \\ A_3 & 2 & 2 \end{array}$$

Startogy B_3 is dominated by the strategy B_1

Let the mixed strategy for player A is $S_A = \begin{pmatrix} A_1 & A_2 & A_3 \\ p_1 & p_2 & p_3 \end{pmatrix}$

$\boxed{p_1, p_2, p_3 \geq 0,\ p_1+p_2+p_3 = 1}$

and the mixed strategy for Player B is

$S_B = \begin{pmatrix} B_1 & B_2 \\ q_1 & q_2 \end{pmatrix}$ $\boxed{q_1+q_2 = 1,\ q_1, q_2 \geq 0}$

For the player A,

expected payoff for the strategy $A_1 = 3q_1 - 2q_2$

$= 3q_1 - 2(1-q_1)$

$= 5q_1 - 2$

" " " " " $A_2 = -q_1 + 4q_2$

$= -q_1 + 4(1-q_1)$

$= -5q_1 + 4$

" " " " " $A_3 = 2q_1 + 2q_2$

$= 2q_1 + 2(1-q_1)$

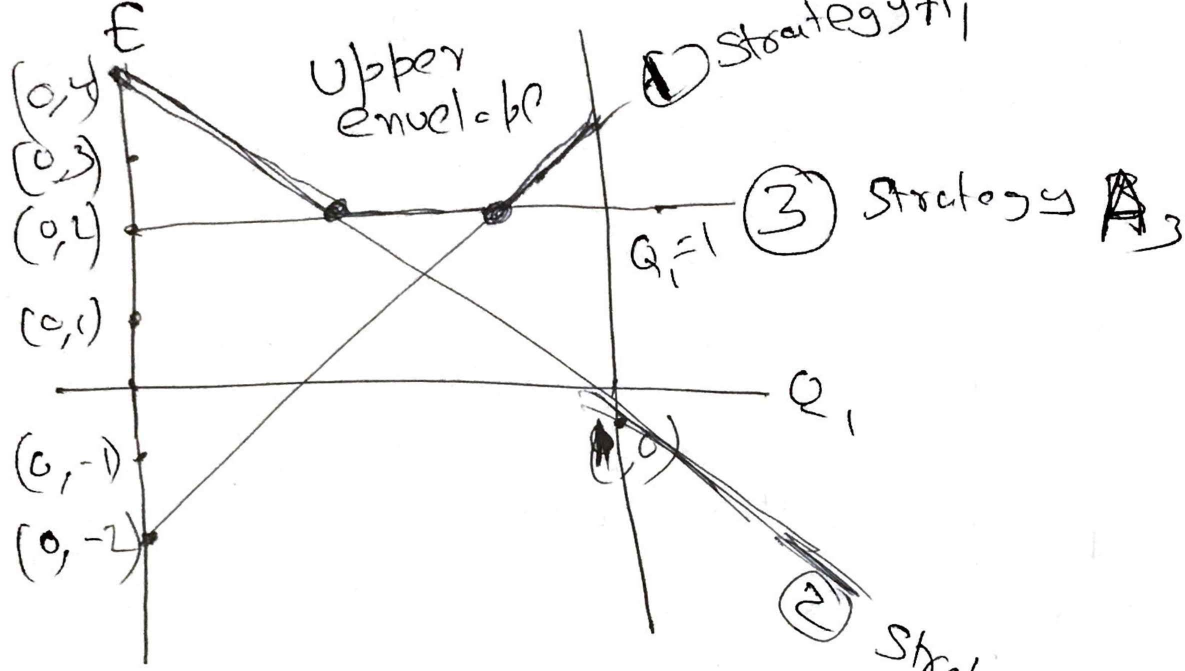
$= 2$

Thus,

for the strategy A_1, the line is $E(Q_1) = 5Q_1 - 2$ → (1)

" " " A_2, " " " $E(Q_1) = -5Q_1 + 4$ → (2)

" " " A_3, " " " $E(Q_1) = 2$ → (3)



So, we can either choose A_1, A_3 or A_2, A_3.

If we choose A_2, A_3, the resultant payoff matrix is $\begin{pmatrix} -1 & 4 \\ 2 & 2 \end{pmatrix}$

Here $a = -1$, $b = 4$, $c = 2$, $d = 2$

Now $S_A = \begin{pmatrix} A_1 & A_2 & A_3 \\ 0 & p_2 & p_3 \end{pmatrix}$, Here $p_1 = 0$

$S_B = \begin{pmatrix} B_1 & B_2 \\ Q_1 & Q_2 \end{pmatrix}$.

$$p_2 = \frac{c-d}{(a+d)-(b+c)} = \frac{2-2}{(-1+2)-(2+4)} = 0$$

$$p_3 = 1-(p_2+p_3) = 1.$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{2-4}{(-1+2)-(2+4)} = \frac{-2}{-5}$$

$$= \frac{2}{5}.$$

$$\therefore q_2 = 1-q_1 = \frac{3}{5}.$$

value of the game $= \dfrac{ad-bc}{(a+d)-(b+c)}$

$$= \frac{-1\times 2-2\times 4}{(-1+2)-(2+4)} = \frac{-10}{-5} = 2$$

3Marks

$$S_A = \begin{pmatrix} A_1 & A_2 & A_3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S_B = \begin{pmatrix} B_1 & B_2 \\ \frac{2}{5} & \frac{3}{5} \end{pmatrix}.$$

Nature of Metaheuristics

10 marks

1. Metaheuristics is a general kind of Solution method that orchestrates the interaction between local improvement procedures and higher level of strategies to create a process that is capable of escaping from local optima and performing a robust seach of the feasible region.

2. A key feature of the metaheuristic is its ability to escape from a local optimum

3. After reaching (or nearly reaching) a local optimum different metaheuristics execute this escape in different ~~types~~ ways

4. However, a common characteristic is that the trial Solution that immediately follow a local optimum are allowed to be inferior to this local optimum.

(5) The advantage of a well designed metaheuristic is that it tends to move relatively quickly towards very good solutions. So it provides a very efficient way of dealing with large and complicated problems

(6) The disadvantage is that there is no guarantee that the best solution found will be optimum solution or even a near optimal solution

Outline of a Basic Tabu Search Algorithm

1 mark. **Initialization:-** Start with a feasible initial trial solution.

Iteration:- Use an appropriate local search procedure to define the feasible moves into the local neighborhood of the current solution.

Eliminate from consideration any move on the current tabu list unless the move would result in a better solution than the best trial solution found so far.

Determine which of the remaining moves provides the best solution.

Adopt this solution as the next trial solution regardless of whether it is better or worse than the current trial solution.

Update the tabu list to forbid cycling back to what had been the current trial solution.

If the tabu list already had been full,
delete the oldest member of the tabu list
to provide more flexibility for future
moves

Stopping rule :- Use any stopping criteria
such as fixed number of iterations,
a fixed amount of CPU time, a fixed
number of consecutive iterations without
an improvement in the best objective function
value.
Also stop at any iteration where
there are no feasible moves in the
local neighbourhood of the current trial
solution.
Accept the best trial solution found on
any iteration as the final solution.