14/03/18

Control Systems

VI Sem, Dept. of EEE

15EE61

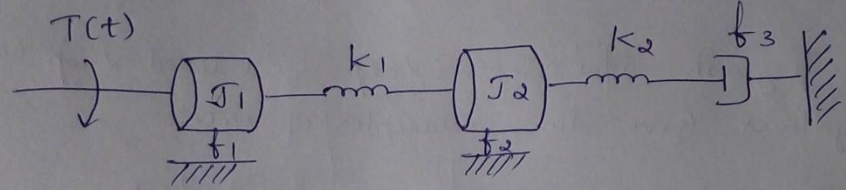
Internal Assessment Test - 1

1

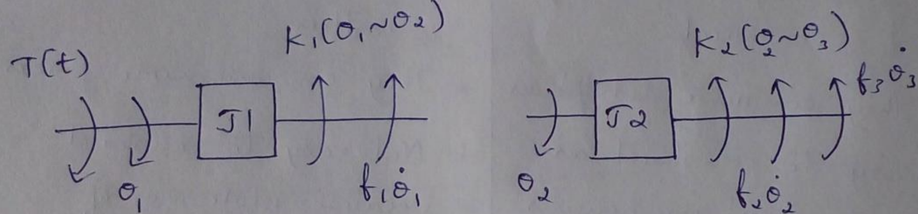
1. (a) Distinguish b/w Open-Loop & Closed-Loop Control Systems. Give two examples of each.

Open-Loop Control S/m	Closed-Loop Control S/m
* These s/ms are simple & economical.	* These s/ms are complex & costlier.
* They consume less power.	* They consume more power.
* Easy to construct as less no. of components required.	* Not easy to construct because more no. of components are required.
* They are inaccurate & unreliable.	* They are accurate & reliable.
Eg: Washing machine & traffic control s/m.	Eg: Washing machine can be a closed loop s/m, if the level of cleanliness can be measured & compared with the desired cleanliness & the difference is used to control the washing time of the machine. The Traffic control s/m can be a closed loop if the time slots of the signals are based on the density of traffic.

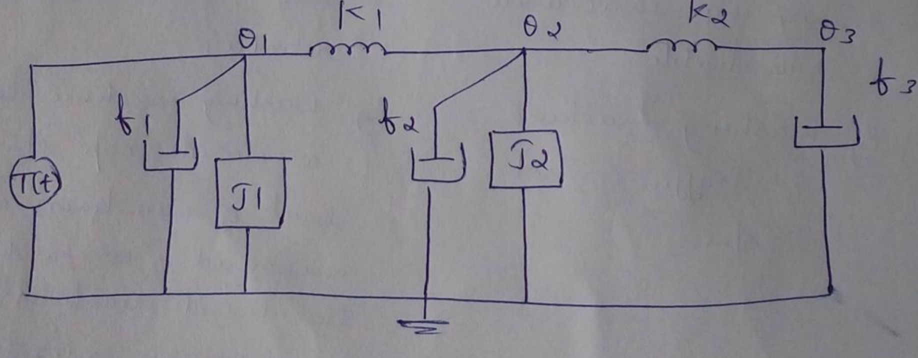
1. (b) Write the differential equations for the mechanical rotational s/m shown in fig. below. Obtain the torque-current analogy of the s/m. List all the analogous quantities.



Free-body diagram



Mechanical N/w



$$T = k_1(\theta_1 - \theta_2) + f_1\dot{\theta}_1 + J_1\ddot{\theta}_1$$

$$-f_2\dot{\theta}_2 - k_1(\theta_2 - \theta_1) - k_2(\theta_2 - \theta_3) = J_2\ddot{\theta}_2$$

$$-k_2(\theta_3 - \theta_2) - f_3\dot{\theta}_3 = 0$$

Torque - current analogous quantities

$T \rightarrow i$

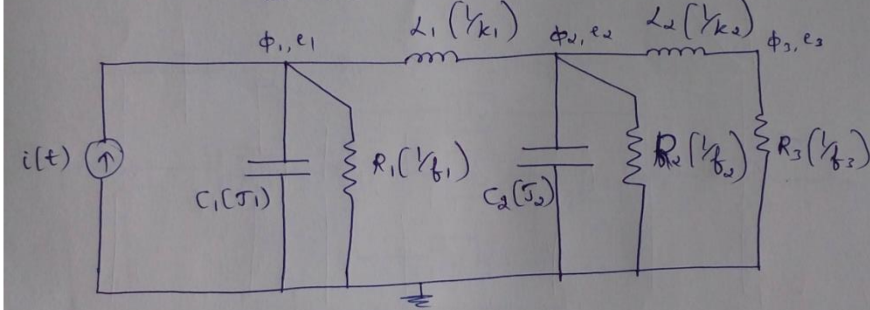
$J \rightarrow C$

$f \rightarrow 1/R$

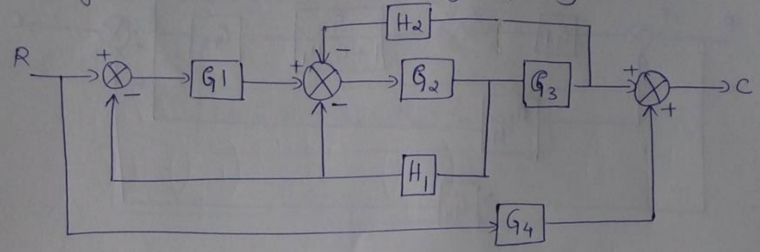
$k \rightarrow 1/L$

$\theta \rightarrow \phi$

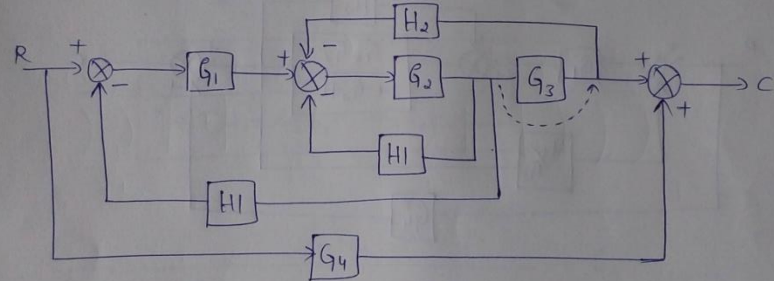
$\omega \rightarrow e$



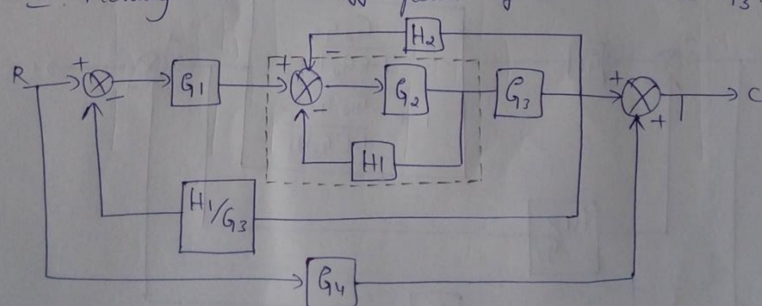
2. Obtain the overall transfer function C/R, by the block diagram reduction technique by referring the fig below



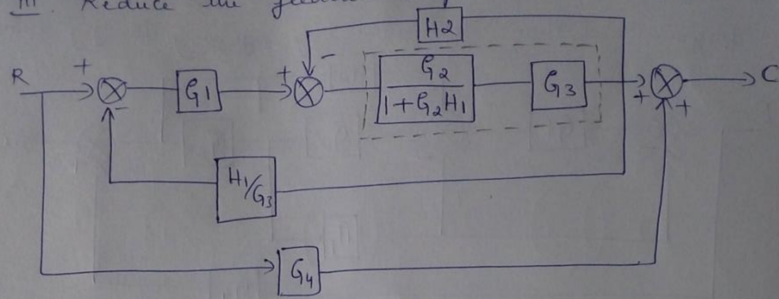
I. Reconstructing the block diagram,



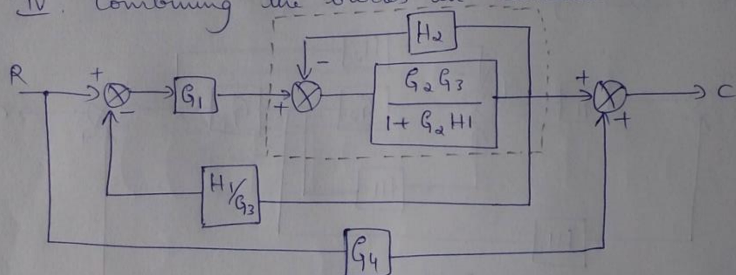
II. Moving the take-off point after the block G_3.



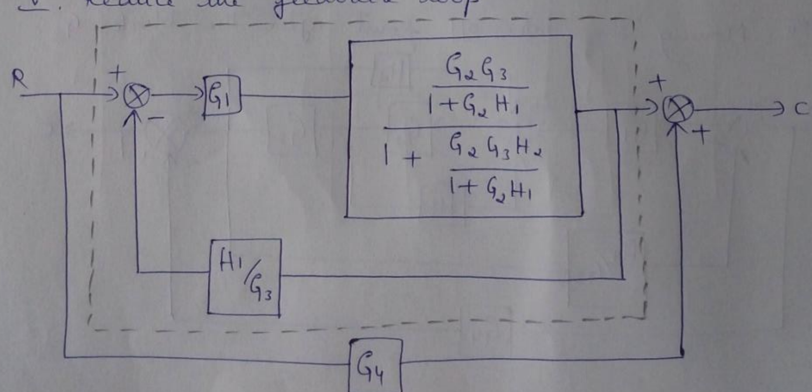
III. Reduce the feedback loop.



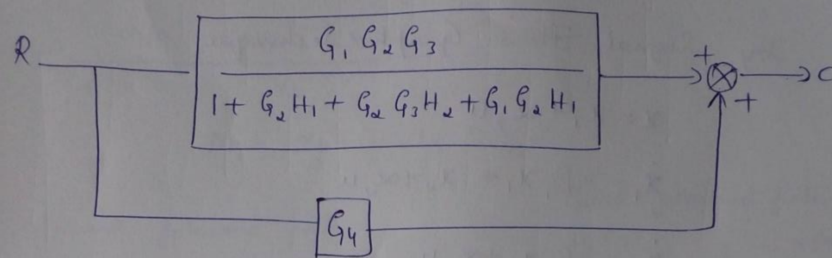
IV. Combining the blocks in cascade.



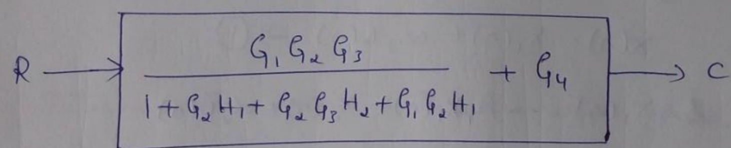
V. Reduce the feedback loop



VI. Combining the blocks in cascade & reducing the feedback loop.



VII. Combining the blocks in cascade



$$\frac{C}{R} = \frac{G_1 G_2 G_3}{1 + G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 H_1} + G_4$$

3. For the system represented by the following equations, find the transfer function $X(s)/U(s)$ by Signal Flow Graph technique.

$$x = x_1 + \alpha_3 u$$

$$\dot{x}_1 = -\beta_1 x_1 + x_2 + \alpha_2 u$$

$$\dot{x}_2 = -\beta_2 x_1 + \alpha_1 u$$

Applying Laplace Transform to the given equations,

$$X(s) = X_1(s) + \alpha_3 U(s) \rightarrow ①$$

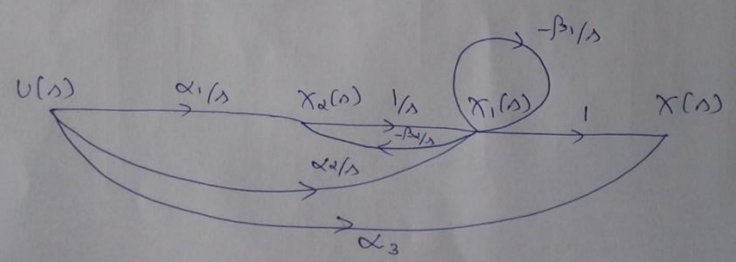
$$sX_1(s) = -\beta_1 X_1(s) + X_2(s) + \alpha_2 U(s)$$

$$\Rightarrow X_1(s) = \frac{-\beta_1}{s} X_1(s) + \frac{1}{s} X_2(s) + \frac{\alpha_2}{s} U(s) \rightarrow ②$$

$$sX_2(s) = -\beta_2 X_1(s) + \alpha_1 U(s)$$

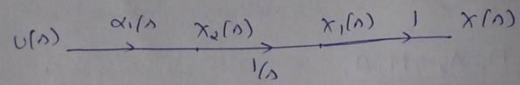
$$\Rightarrow X_2(s) = \frac{-\beta_2}{s} X_1(s) + \frac{\alpha_1}{s} U(s) \rightarrow ③$$

Using equations ①, ② & ③,



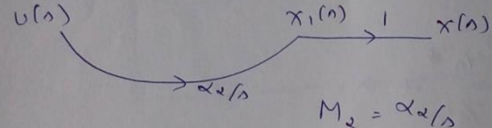
I. Determine the forward paths & their respective gains.

First forward path



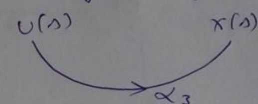
$$M_1 = \frac{\alpha_1}{s} \times \frac{1}{s} \times 1 = \alpha_1/s^2$$

Second forward path



$$M_2 = \alpha_2/s$$

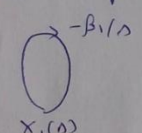
Third forward path



$$M_3 = \alpha_3$$

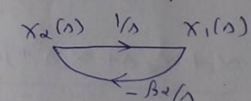
II. Determine the loops & their respective gains.

First loop



$$L_1 = -\beta_1/s$$

Second loop



$$L_2 = -\beta_2/s^2$$

III. Determine the determinants.

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\Delta_3 = 1 - (L_1 + L_2) = 1 - \left(-\beta_1/s - \beta_2/s^2\right) = 1 + \beta_1/s + \beta_2/s^2$$

IV. Applying Mason's Gain Formula,

$$T = \frac{\sum M_k \Delta_k}{\Delta}$$

$$= \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta}$$

$$= \frac{\left(\alpha_1/s^2 \times 1\right) + \left(\alpha_2/s \times 1\right) + \left(\alpha_3 \times \left(1 + \beta_1/s + \beta_2/s^2\right)\right)}{1 + \beta_1/s + \beta_2/s^2}$$

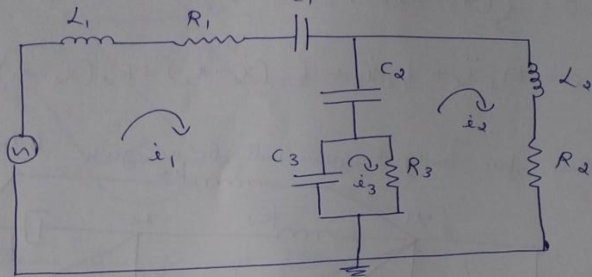
$$= \frac{\alpha_1/s^2 + \alpha_2/s}{1 + \beta_1/s + \beta_2/s^2} + \alpha_3$$

$$= \frac{\frac{\alpha_1 + \alpha_2 s}{s^2}}{\frac{s^2 + \beta_1 s + \beta_2}{s^2}} + \alpha_3 = \frac{\alpha_1 + \alpha_2 s}{s^2 + \beta_1 s + \beta_2} + \alpha_3$$

$$= \frac{\alpha_1 + \alpha_2 s + \alpha_3 s^2 + \beta_1 \alpha_3 s + \beta_2 \alpha_3}{s^2 + \beta_1 s + \beta_2}$$

$$\frac{X(s)}{U(s)} = \frac{\alpha_3\left(s^2 + \beta_1 s + \beta_2\right) + \alpha_2 s + \alpha_1}{s^2 + \beta_1 s + \beta_2}$$

4. The force-voltage analogy of a mechanical translational s/m is given in the figure below, obtain its analogous mechanical system & its differential equations.



Apply KVL to all three loops,

$$e = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1\, dt + \frac{1}{C_2}\int (i_1 - i_2)\, dt + \frac{1}{C_3}\int (i_1 - i_3)\, dt$$

$$0 = R_3 (i_3 - i_2) + \frac{1}{C_3}\int (i_3 - i_1)\, dt$$

$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + R_3 (i_2 - i_3) + \frac{1}{C_2}\int (i_2 - i_1)\, dt$$

For force-voltage analogy,

F	e
M	L
f	R
K	$1/C$
$v\,(dx/dt)$	$i\,(dq/dt)$
$x\,(\int v\, dt)$	$q\,(\int i\, dt)$

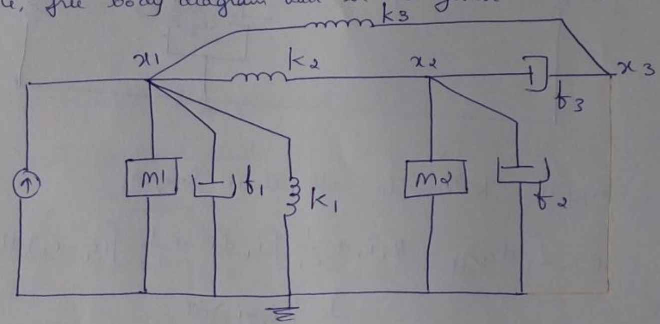
$\therefore$ we have,

$$F = M_1\ddot{x}_1 + f_1\dot{x}_1 + K_1x_1 + K_2(x_1 - x_2) + K_3(x_1 - x_3)$$

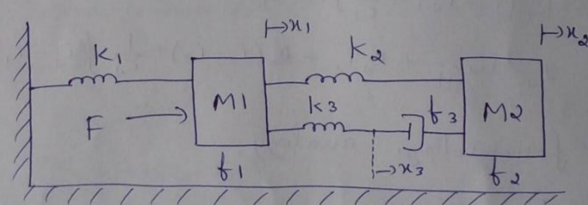
$$0 = f_3(\dot{x}_3 - \dot{x}_2) + K_3(x_3 - x_1)$$

$$0 = M_2\ddot{x}_2 + f_2\dot{x}_2 + f_3(\dot{x}_2 - \dot{x}_3) + K_2(x_2 - x_1)$$

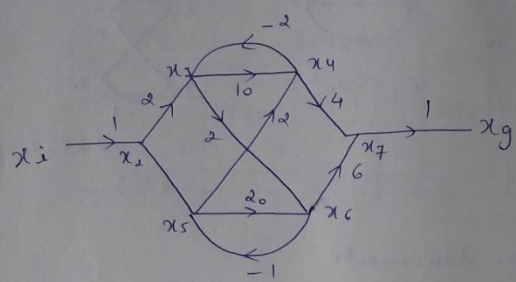
Hence, free body diagram will be as given below,



Mechanical s/m,

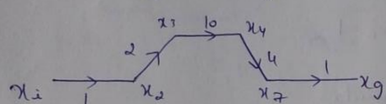


5. Describe the gain x_9/x_i of the s/m described by the signal flow graph given in the figure below.



I. Determine the forward paths.

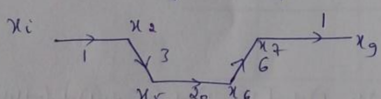
First forward path



$M_1 = 1 \times 2 \times 10 \times 4 \times 1$

$M_1 = 80$

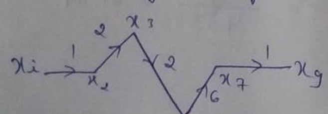
Second forward path



$M_2 = 1 \times 3 \times 20 \times 6 \times 1$

$M_2 = 360$

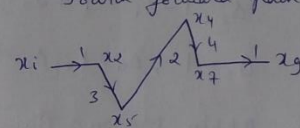
Third forward path



$M_3 = 1 \times 2 \times 2 \times 6 \times 1$

$M_3 = 24$

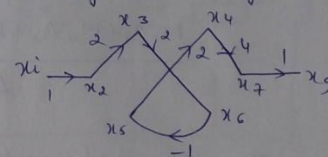
Fourth forward path



$M_4 = 1 \times 3 \times 2 \times 4 \times 1$

$M_4 = 24$

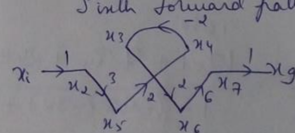
Fifth forward path



$M_5 = 1 \times 2 \times 2 \times -1 \times 2 \times 4 \times 1$

$M_5 = -32$

Sixth forward path

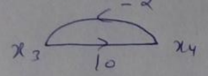


$M_6 = 1 \times 3 \times 2 \times -2 \times 2 \times 6 \times 1$

$M_6 = -144$

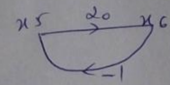
II. Determine the loops

First loop



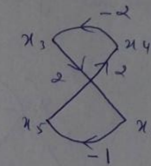
$L_1 = -20$

Second loop



$L_2 = -20$

Third loop



$L_3 = -2 \times 2 \times 2 \times -1$

$L_3 = 8$

III. Determine the determinants

$\Delta_1 = 1 - L_2 = 1 + 20 = 21$

$\Delta_2 = 1 - L_1 = 1 + 20 = 21$

$\Delta_3 = 1 \; ; \Delta_4 = 1 \; ; \Delta_5 = 1 \; ; \Delta_6 = 1$

IV. Apply Mason's Gain Formula

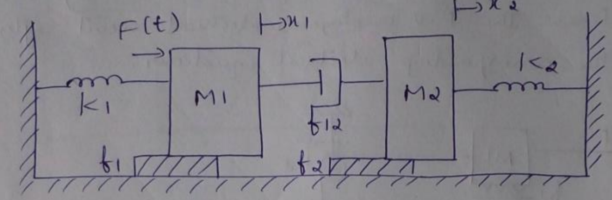
$$T = \frac{\Sigma M_k \Delta_k}{\Delta}$$

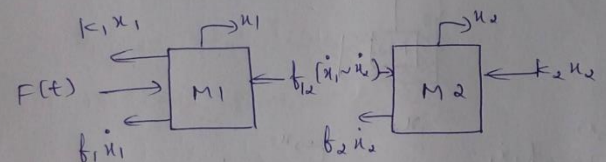
$$= \frac{M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3 + M_4 \Delta_4 + M_5 \Delta_5 + M_6 \Delta_6}{\Delta}$$

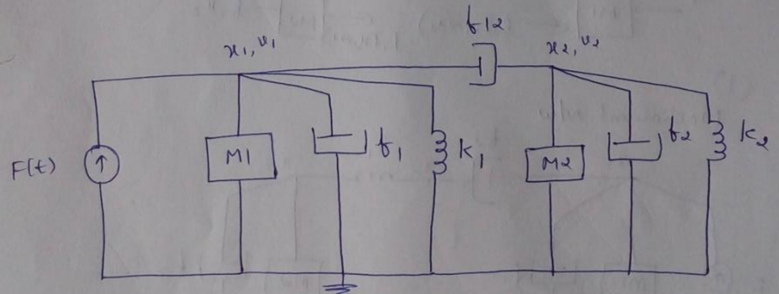
$$\frac{x_9}{x_i} = \frac{(80 \times 21) + (360 \times 21) + (24 \times 1) + (24 \times 1) + (-32 \times 1) + (-144 \times 1)}{1 - (-20 - 20 + 8) + (-20)(-20)}$$

$$\frac{x_9}{x_i} = \frac{1680 + 7560 + 24 + 24 - 32 - 144}{1 - (-32) + 400} = \frac{9112}{433} = 21.04$$

7. Obtain the differential equations of the mechanical system given in the figure below







$$F - k_1 x_1 - f_1 \dot{x}_1 - f_{12}(\dot{x}_1 - \dot{x}_2) = M_1 \ddot{x}_1$$

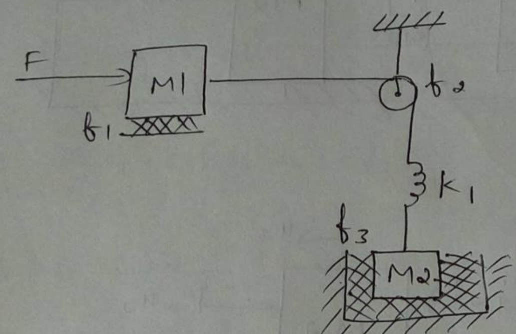
$$-f_{12}(\dot{x}_2 - \dot{x}_1) - f_2 \dot{x}_2 - k_2 x_2 = M_2 \ddot{x}_2$$

(6) For the mechanical n/w shown in the fig below,

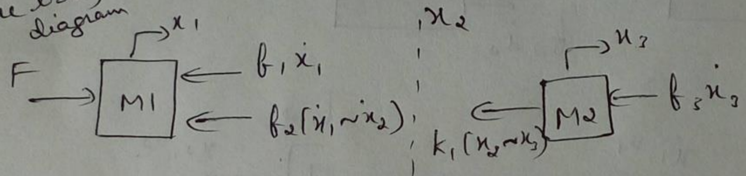
(i) Draw the mechanical n/w.

(ii) Write the differential equations describing the s/m.

(iii) Draw the F-V analogous electrical circuit after writing the corresponding electrical equations.

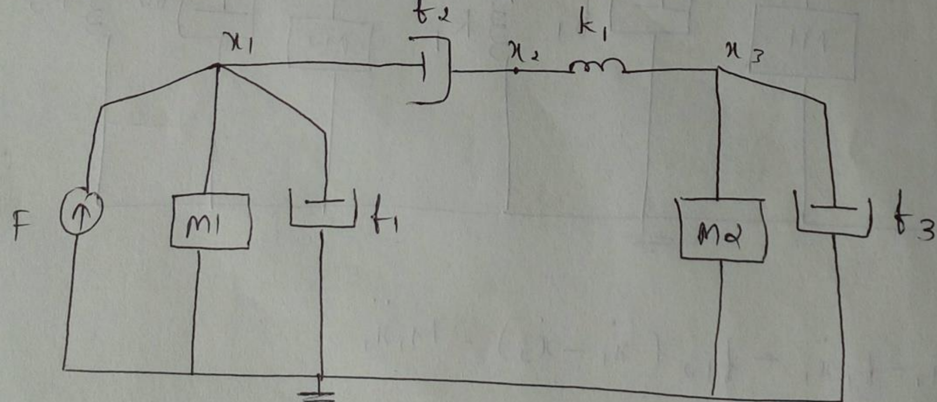


Free body diagram



(i)

Mechanical N/W

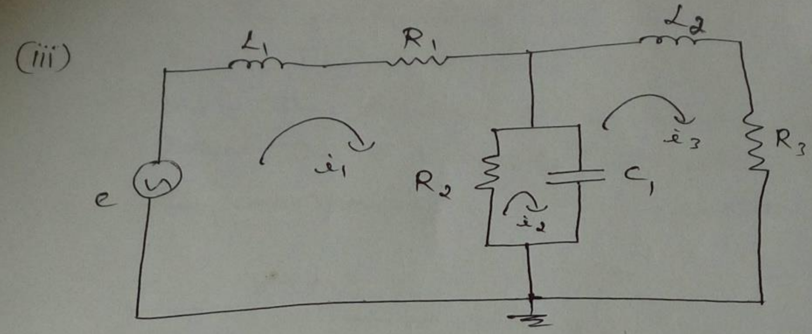


(ii) $F - f_1\dot{x}_1 - f_2(\dot{x}_1 - \dot{x}_2) = M_1\ddot{x}_1$

$f_2(\dot{x}_2 - \dot{x}_1) + k_1(x_2 - x_3) = 0$

$-f_3\dot{x}_3 - k_1(x_3 - x_2) = M_2\ddot{x}_3$

(iii)



$$e - R_1 i_1 - R_2 (i_1 - i_2) = L_1 \frac{di_1}{dt}$$

$$R_2 (i_2 - i_1) + \frac{1}{C_1} \int (i_2 - i_3)\, dt = 0$$

$$-R_3 i_3 - 1/C_1 \int (i_3 - i_2)\, dt = L_2\, di_3/dt$$