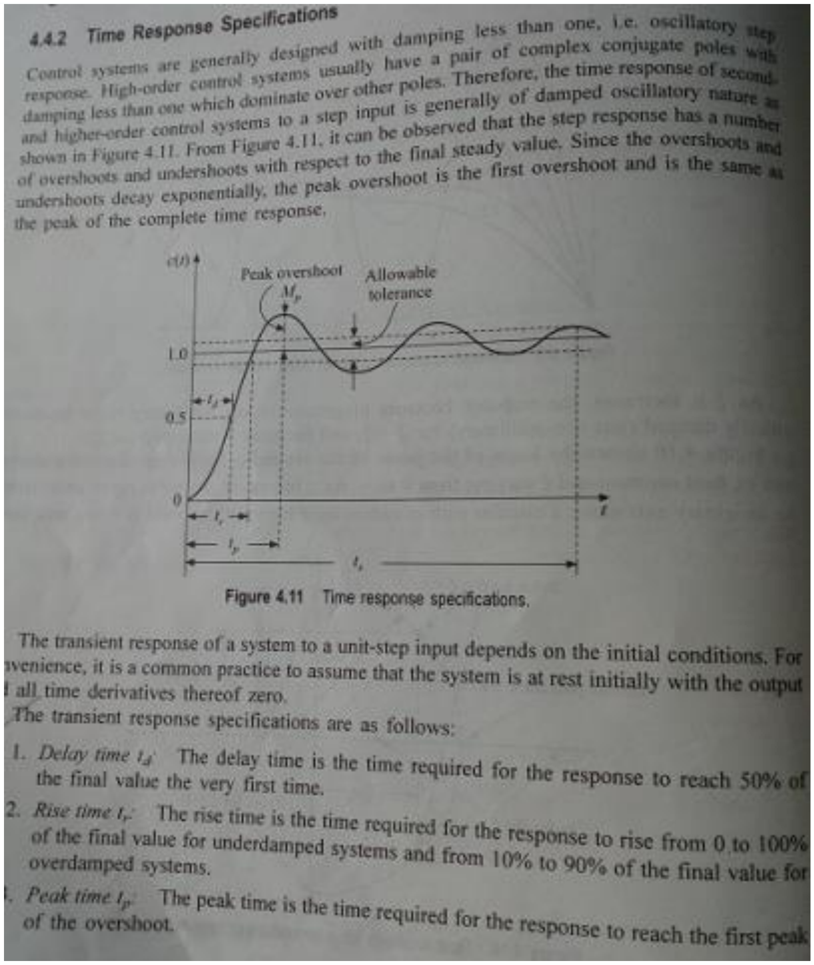
CS IAT 2 QUESTION PAPER AND SOLUTION

1 Explain the time domain specifications of a second order system, with neat sketch i) Peak Time ii) Delay Time iii) Rise Time iv) Maximum Overshoot v) Settling Time vi) Steady State Error



4.4.2 Time Response Specifications

Control systems are generally designed with damping less than one, i.e. oscillatory step response. High-order control systems usually have a pair of complex conjugate poles with damping less than one which dominate over other poles. Therefore, the time response of second- and higher-order control systems to a step input is generally of damped oscillatory nature as shown in Figure 4.11. From Figure 4.11, it can be observed that the step response has a number of overshoots and undershoots with respect to the final steady value. Since the overshoots and undershoots decay exponentially, the peak overshoot is the first overshoot and is the same as the peak of the complete time response.

Figure 4.11 Time response specifications.

The transient response of a system to a unit-step input depends on the initial conditions. For venience, it is a common practice to assume that the system is at rest initially with the output all time derivatives thereof zero.

The transient response specifications are as follows:

1. *Delay time t_d*: The delay time is the time required for the response to reach 50% of the final value the very first time.
2. *Rise time t_r*: The rise time is the time required for the response to rise from 0 to 100% of the final value for underdamped systems and from 10% to 90% of the final value for overdamped systems.
3. *Peak time t_p*: The peak time is the time required for the response to reach the first peak of the overshoot.

2 For a system described by the following equation, evaluate the response and maximum output for a step of 2.5 units.

$$d^2y/dt^2 + 8dy/dt + 25y(t) = 50x(t)$$

2. For a s/m described by the foll. eq^n, evaluate the response of max. o/p for a step i/p of 2.5 units.

$$d^2y/dt^2 + 8\,dy/dt + 25\,y(t) = 50\,x(t)$$

(ii) Taking Laplace Transform on both sides of eq^n.

$$s^2 Y(s) + 8sY(s) + 25Y(s) = 50X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{50}{s^2+8s+25}$$

Comparing the above equation with the standard form of the transfer function of a second order s/m,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\omega_n^2 = 25$

$\omega_n = \sqrt{25} = 5$ rad/s

$2\zeta\omega_n = 8$

$\zeta = \frac{8}{2\omega_n} = 0.8$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 3$ rad/s

$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = 0.643$ rad/s

Rise time, $t_r = \frac{\pi-\theta}{\omega_d} = 0.832$ s

Percentage overshoot, $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100$

$= 1.519\%$

Peak overshoot for a unit-step i/p,

0.0151

Peak overshoot for a step i/p of 2.5 units,

$0.0151 \times 2.5 = 0.037$

Peak time, $t_p = \pi/\omega_d = 1.04$ s

Settling time, $t_s = \frac{4}{\zeta\omega_n}$ (for 2% tolerance) = 1 s

$t_s = \frac{3}{\zeta\omega_n}$ (for 5% tolerance) = 0.75 s

3. A feedback system has an open loop transfer function,

$$G(s)H(s) = \frac{\mathrm{K}e^{-s}}{s(s^2+5s+9)}$$

Determine by use of Routh Criterion, the range of values of K for the closed loop system to be stable. [Hint: For low frequencies $e^{-s} \approx$(1-s)]

Example 5.14 A ...

$$G(s)H(s) = \frac{Ke^{-s}}{s(s^2+5s+9)}$$

Determine by use of the Routh criterion, the maximum value of K for the closed-... to be stable.

[*Hint:* For low frequencies $e^{-s} \approx (1-s)$]

Solution: The characteristic equation of the system is

$$1 + G(s)H(s) = 0$$

i.e.

$$1 + \frac{K(1-s)}{s(s^2+5s+9)} = 0$$

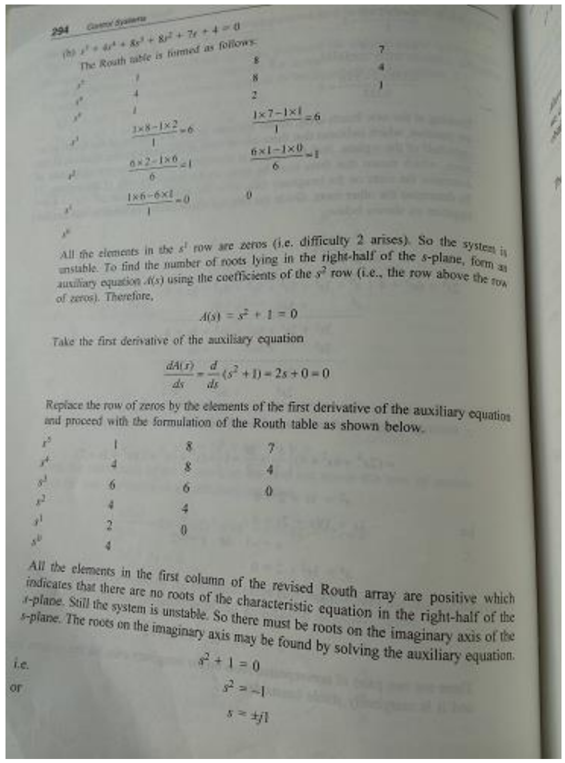
$$s^3 + 5s^2 + (9-K)s + K = 0$$

Forming the Routh array, we have

s^3	1	$9-K$
s^2	5	K
s^1	$\frac{5\times(9-K)-1\times K}{5}$	0
s^0	K	

4. By means of Routh Criterion, determine the stability of the system represented by the following characteristic equation. If the system is found to be unstable, determine the number of roots of the characteristic equation in the right half of the s-plane.

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$



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(b) $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

The Routh table is formed as follows:

s^5	1	8	7
s^4	4	8	4
	1	2	1
s^3	$\frac{1\times 8-1\times 2}{1}=6$	$\frac{1\times 7-1\times 1}{1}=6$	
s^2	$\frac{6\times 2-1\times 6}{6}=1$	$\frac{6\times 1-1\times 0}{6}=1$	
s^1	$\frac{1\times 6-6\times 1}{1}=0$	0	
s^0			

All the elements in the s^1 row are zeros (i.e. difficulty 2 arises). So the system is unstable. To find the number of roots lying in the right-half of the s-plane, form an auxiliary equation $A(s)$ using the coefficients of the s^2 row (i.e., the row above the row of zeros). Therefore,

$$A(s) = s^2 + 1 = 0$$

Take the first derivative of the auxiliary equation

$$\frac{dA(s)}{ds} = \frac{d}{ds}(s^2+1) = 2s + 0 = 0$$

Replace the row of zeros by the elements of the first derivative of the auxiliary equation and proceed with the formulation of the Routh table as shown below.

s^5	1	8	7
s^4	4	8	4
s^3	6	6	0
s^2	4	4	
s^1	2	0	
s^0	4		

All the elements in the first column of the revised Routh array are positive which indicates that there are no roots of the characteristic equation in the right-half of the s-plane. Still the system is unstable. So there must be roots on the imaginary axis of the s-plane. The roots on the imaginary axis may be found by solving the auxiliary equation.

i.e. $$s^2 + 1 = 0$$

or $$s^2 = -1$$

$$s = \pm j1$$

5. The characteristic equation of a feedback control system is given by

$$s^4 + 20s^3 + 15s^2 + 2s + K = 0$$

(a) Determine the range of values of K for the system to be stable.

(b) Can the system be marginally stable? If so, find the required value of K and the frequency of sustained oscillations.

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Since K must be a real positive ... system is always unstable.

Example 5.10 The characteristic equation of a feedback control system is given by

$$s^4 + 20s^3 + 15s^2 + 2s + K = 0$$

(a) Determine the range of values of K for the system to be stable.

(b) Can the system be marginally stable? If so, find the required value of K and the frequency of sustained oscillations.

Solution:

(a) The Routh table for the given characteristic equation is formed as follows:

s^4	1	15	K
s^3	20	2	
s^2	$\frac{20\times15-1\times2}{20}=14.9$	$\frac{20\times K-1\times0}{20}=K$	
s^1	$\frac{14.9\times2-20\times K}{14.9}$	0	
s^0	K		

For the system to be stable, all the elements in the first column of the Routh array must be positve. Therefore, from the s^0 row

$$K > 0$$

and from the s^1 row

$$2 \times 14.9 - 20K > 0$$

i.e.

$$K < 1.49$$

So the range of values of K for stability is $0 < K < 1.49$.

(b) Yes, the system can be marginally stable, for the marginal value of K, i.e. for $2 \times 14.9 - 20K = 0$, i.e. for $K = 1.49$. To obtain the frequency of sustained oscillations, substitute this critical value of K in the auxiliary equation formed by using the coefficients of the row just above the row which gives the critical value of K and solve it, i.e.

i.e.

$$A(s) = 14.9s^2 + K = 0$$

i.e.

$$14.9s^2 + 1.49 = 0$$

$$s = \pm j\sqrt{\frac{1.49}{14.9}} = \pm j\sqrt{0.1} = \pm j0.316$$

Therefore, the frequency of sustained oscillations is 0.316 rad/s.

6. A second order control system is represented by a transfer function given below

$$\frac{Q(s)}{T(s)} = \frac{1}{Js^2 + fs + K}$$

where Q(s) is the proportional output and T is the input torque. A step input 10 N-m is applied to the system and the test results are given below.

(a) Peak Overshoot M_p=6%

(b) Peak Time t_p=1s

(c) The steady state output of the system is 0.5 radians.

Determine the values of J, K and f.

That is the time for peak overshoot is $t_p = 0.363$s.

Example 4.9 A second-order control system is represented by a transfer function given below

$$\frac{Q_0(s)}{T(s)} = \frac{1}{Js^2 + fs + K}$$

where $Q_0(s)$ is the proportional output and T is the input torque. A step input 10 N-m is applied to the system and the test results are given below.

(a) Peak overshoot $M_p = 6\%$

(b) Peak time $t_p = 1$s

(c) The steady-state output of the system is 0.5 radian.

Determine the values of J, K and f.

Solution: Given, the input torque T is a step of 10 N-m. Therefore,

$$T(s) = \frac{10}{s}$$

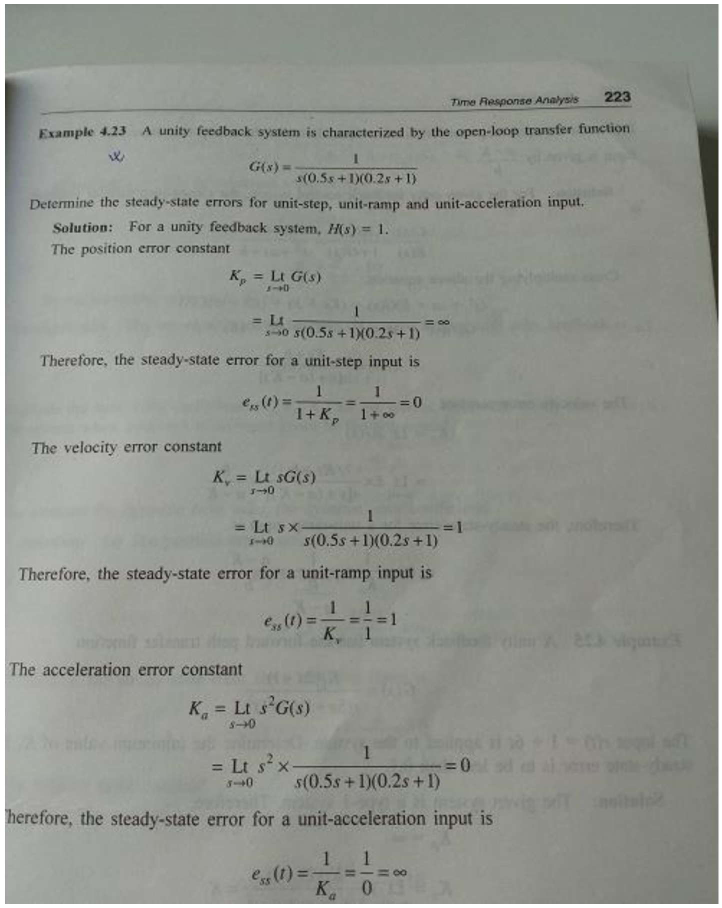
$$Q_0(s) = \frac{T(s)}{Js^2 + fs + K} = \frac{10}{s(Js^2 + fs + K)}$$

The steady-state value of output $= \underset{s\to0}{\text{Lt}}\ sQ_0(s)$. Therefore,

$$0.5 = \underset{s\to0}{\text{Lt}}\ s \cdot \frac{10}{s(Js^2 + fs + K)}$$

7. A unity feedback control system is characterized by the open-loop transfer function, determine the steady state errors for unit step, unit ramp and unit acceleration input.

$$G(s) = \frac{1}{s(0.5s+1)(0.2s+1)}$$



Example 4.23 A unity feedback system is characterized by the open-loop transfer function

$$G(s) = \frac{1}{s(0.5s+1)(0.2s+1)}$$

Determine the steady-state errors for unit-step, unit-ramp and unit-acceleration input.

Solution: For a unity feedback system, $H(s) = 1$.

The position error constant

$$K_p = \operatorname*{Lt}_{s\to 0} G(s)$$

$$= \operatorname*{Lt}_{s\to 0} \frac{1}{s(0.5s+1)(0.2s+1)} = \infty$$

Therefore, the steady-state error for a unit-step input is

$$e_{ss}(t) = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

The velocity error constant

$$K_v = \operatorname*{Lt}_{s\to 0} sG(s)$$

$$= \operatorname*{Lt}_{s\to 0} s \times \frac{1}{s(0.5s+1)(0.2s+1)} = 1$$

Therefore, the steady-state error for a unit-ramp input is

$$e_{ss}(t) = \frac{1}{K_v} = \frac{1}{1} = 1$$

The acceleration error constant

$$K_a = \operatorname*{Lt}_{s\to 0} s^2 G(s)$$

$$= \operatorname*{Lt}_{s\to 0} s^2 \times \frac{1}{s(0.5s+1)(0.2s+1)} = 0$$

herefore, the steady-state error for a unit-acceleration input is

$$e_{ss}(t) = \frac{1}{K_a} = \frac{1}{0} = \infty$$

8. Determine the range of values of K such that the characteristic equation given below has roots more negative than **s = -1**.

$$s^3 + 3(K+1)s^2 + (7K+5)s + (4K+7) = 0$$

Example 5.23 Determine the range of values of $K(K > 0)$ such that the characteristic equ

$$s^3 + 3(K + 1)s^2 + (7K + 5)s + (4K + 7) = 0$$

has roots more negative than $s = -1$.

Solution: To determine the range of values of K such that the given charact equation has roots more negative than −1, shift the origin of the s-plane to $s = -1$ by substi $s = z - 1$ in the characteristic equation and apply Routh's test. The characteristic equa the new variable z is

$$(z - 1)^3 + 3(K + 1)(z - 1)^2 + (7K + 5)(z - 1) + (4K + 7) = 0$$

$$(z^3 - 3z^2 + 3z - 1) + 3(K + 1)(z^2 - 2z + 1) + (7K + 5)(z - 1) + 4K + 7 = ($$

$$z^3 + z^2(3K) + z(K + 2) + 4 = 0$$

Forming the Routh array, we have

z^3	1	$K + 2$
z^2	$3K$	4
z^1	$\dfrac{3K(K+2)-4}{3K}$	
z^0	4	