

Internal Assessment Test - II

Sub:	Power System Analysis	Code:	15EE62
Date:	16/04/2018	Duration:	90 mins
		Max Marks:	50
		Sem:	6
		Branch:	EEE
Answer Any FIVE FULL Questions			

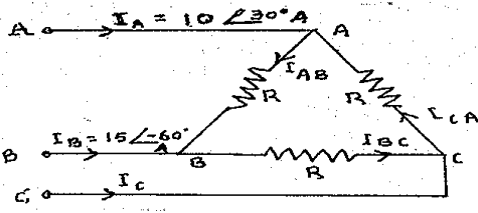
	Marks	OBE	
		CO	RBT
1a	[5]	CO3	L2
1b	[5]	CO3	L2
2	[10]	CO4	L3
3	[10]	CO3	L3
4	[10]	CO3	L3

1a Prove that a balanced set of 3 phase voltages will have only positive sequence components of voltages.

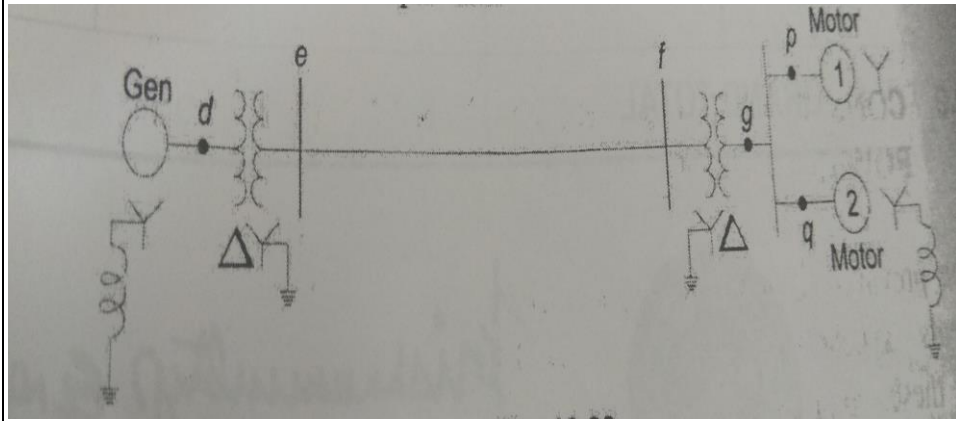
1b With the help of relevant vector diagrams for voltages and currents, establish the phase shift of symmetrical components in Y Δ transformers.

2 Draw the interconnected sequence networks for L-G fault in an unloaded generator with and without fault impedance  $Z_f$  clearly indicating positive, negative and zero sequence impedance, symmetrical components of voltages and currents. Also write the expressions for the fault current

3 A delta connected balanced resistive load is connected across an unbalanced 3 phase supply as shown in fig. With currents in the line A and B specified, find the symmetrical components of line currents. Also find the symmetrical components of delta currents.



4 25 MVA, 11 kV, three phase generator has a sub transient reactance of 20%. The generator supplies two motors over a transmission line with transformers at both ends as shown in single line diagram. The motors have rated inputs of 15 and 7.5 MVA, both 10 kV with 25% sub transient reactance. The three phase transformers are both rated 30 MVA, 10.8/121 kV, connection Δ Y with leakage reactance of 10 % each. The series reactance of the line is 100 ohms.



Zero sequence reactance for the generator and motors are 0.06 p.u. Current limiting reactors of 2.5 ohms each are connected in the neutral of the generator and motor 2. The zero sequence reactance of the transmission line is 300 ohms. Draw the positive sequence, negative sequence and zero sequence networks of the system with reactances marked in per unit.

5a Derive an expression for 3 phase power in terms of symmetrical components. [5]

5b Derive phase currents of unbalanced system in terms of sequence currents. [5]

Draw and explain about the positive, negative and zero sequence network of an unloaded synchronous generator.

CO3	L2
CO3	L3
CO3	L2

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Relate the power system network to network topology.	2	-	-	-	-	-	-	-	-	1	-	-
CO2:	Recognize the network and form the matrix.	3	-	-	-	-	-	-	-	-	1	-	-
CO3:	Use the algorithms to calculate the load flow in the power system.	3	-	1	-	-	-	-	-	-	1	-	-
CO4:	Analyse the different algorithms for the load flow in the power system.	3	-	-	-	-	-	-	-	-	1	-	-
CO5:	Apply the economic scheduling algorithm for the load dispatch in power system.	3	-	-	-	-	-	-	-	-	1	-	-
CO6:	Apply different mathematical methods to solve the swing equation.	3	-	-	-	-	-	-	-	-	1	-	-

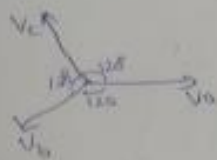
Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 - *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

Solutions:

1a)

Prove that a balanced set of three phase voltages will have only positive sequence components of voltages only.



$V_a, V_b, V_c$  - phase voltages of balanced 3 $\phi$  system

$$V_b = a^2 V_a$$

$$V_c = a V_a$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ a^2 V_a \\ a V_a \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} V_a + a^2 V_a + a V_a \\ V_a + a^3 V_a + a^3 V_a \\ V_a - a^4 V_a + a^2 V_a \end{bmatrix} = \begin{bmatrix} 0 \\ 3V_a \\ 0 \end{bmatrix} =$$

$$ii) \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

ii)  $V_{a0} = 0$   
 $V_{a1} = V_a$   
 $V_{a2} = 0$

Proves that a balanced set of three phase voltages will have only positive sequence voltages. Negative and zero sequence components are always absent in a balanced system.  
 It holds good for a balanced

1b)

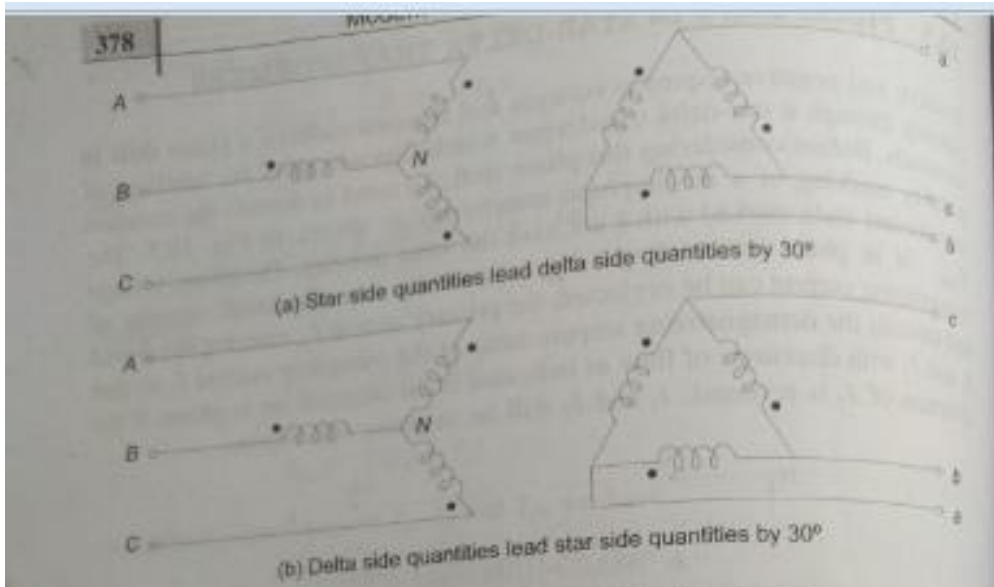


Fig. 10.6 Labelling of star/delta transformer

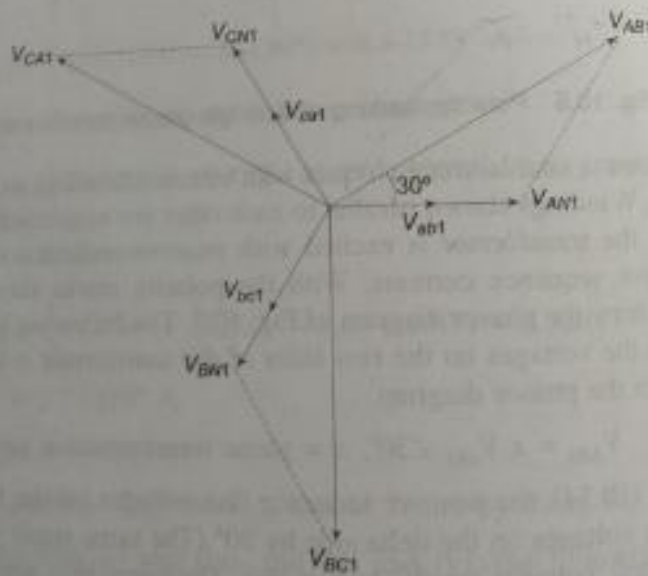


Fig. 10.7 Positive sequence voltages on a star/delta transformer

Instead, if the transformer of Fig. 10.6(a) is now excited by negative sequence voltages and currents, the voltage phasor diagram will be as in Fig. 10.8. The phase shift in comparison to the positive sequence case now reverses, i.e., the star side quantities lag the delta side quantities by 30°. The result for Fig. 10.6(b) also correspondingly reverses.

It shall from now onwards be assumed that a star/delta transformer is so labelled that the positive sequence quantities on the HV side lead those

1b)

corresponding positive sequence quantities on the LV side by  $30^\circ$ . The reverse is the case for negative sequence quantities wherein HV quantities lag the corresponding LV quantities by  $30^\circ$ .

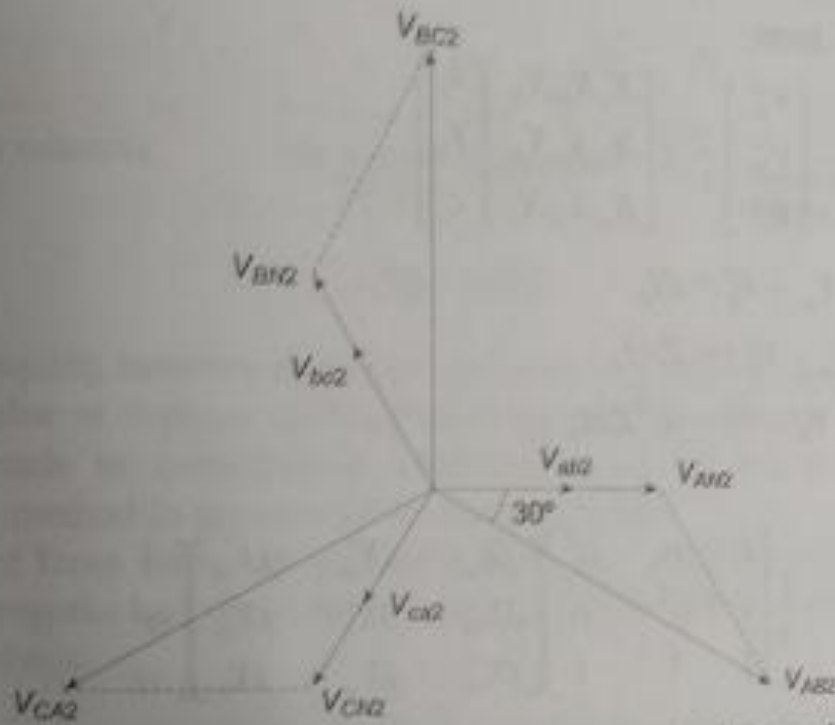
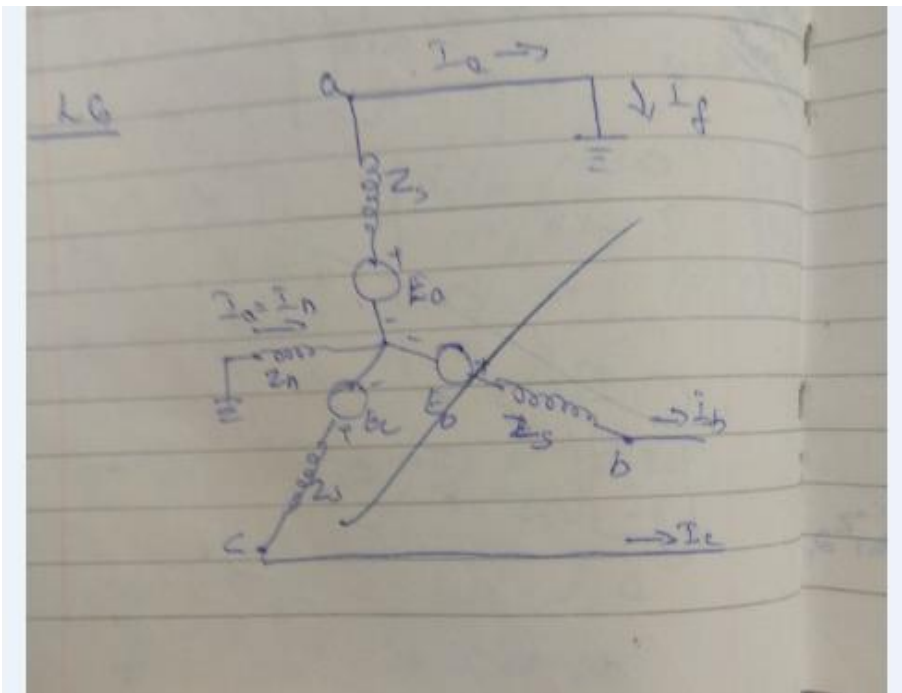


Fig. 10.8 Negative sequence voltages on a star/delta transformer

2)



circuit model for a single line to ground fault on an unloaded Y connected generator with its neutral grounded through a reactance. Here phase a is shorted to ground. The fault current  $I_f = I_a$  since the generator is unloaded the current in other phases are zero -  $I_b = 0, I_c = 0, V_a = 0$

sym components

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

sub.  $I_b = I_c = 0$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$I_{a0} = I_{a1} = I_{a2} = I_a/3$

From sequence  $\pi/\omega$  of the generator we get

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

sub  $I_{a0} = I_{a1}$ ,  $I_{a2} = I_{a1}$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a1} \\ I_{a1} \end{bmatrix}$$

$$V_{a0} = -Z_0 I_{a1}$$

$$V_{a1} = E_a - Z_1 I_{a1}$$

$$V_{a2} = -Z_2 I_{a1}$$

on adding

$$V_{a0} + V_{a1} + V_{a2} = -I_{a1} Z_0 + E_a - Z_1 I_{a1} - I_{a1} Z_2$$



Phase a is shorted to ground.

$$V_a = V_0 + V_{a1} + V_{a2} = 0$$

$$-I_{a1}Z_0 + E_a - I_{a1}Z_1 - I_{a2}Z_2 = 0$$

$$\therefore I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

From this eqn ckt can be drawn like this

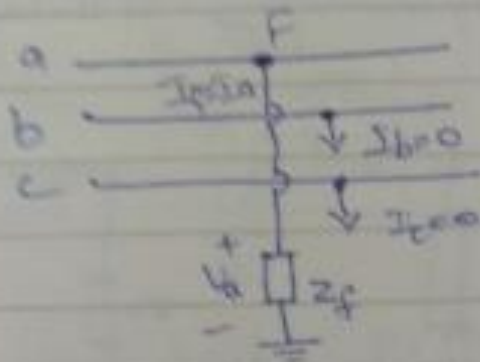


If the neutral of the generator is not grounded so zero sequence  $n_0$  is open circuited and  $Z_0$  is infinite. Then  $I_{a0} = I_{a1} = I_{a2} = 0$ . No path exists for the flow of current in the fault unless the generator neutral is grounded.

$$I_f = I_a = 3I_{a1}$$

# Faults on a Power System

## single line to ground fault



$$I_b = 0, I_c = 0, V_a = I_a Z_f$$

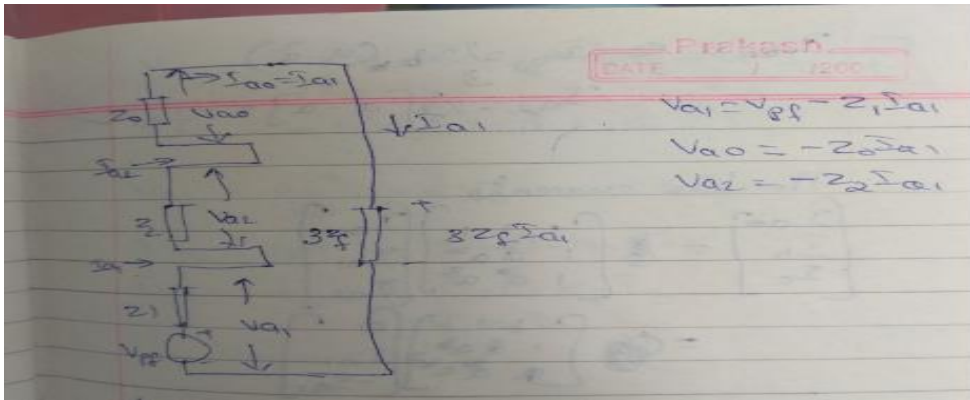
$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_b \\ 0 \\ 0 \end{bmatrix}$$

$$I_a = 3I_{a1}$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = Z_f I_a = 3Z_f I_{a1}$$

$$I_{a1} = \frac{V_{f1}}{(Z_0 + Z_1 + Z_2) + 3Z_f}$$

$$I_f = I_a = 3I_{a1} = \frac{3V_{f1}}{(Z_1 + Z_2 + Z_0) + 3Z_f}$$



3)

$$= \frac{1}{3} (I_a + I_b + I_c)$$

$$= \frac{1}{3} [I_c - I_b + I_a + I_c + I_b + I_a]$$

$$= 0$$

2) A delta connected balanced resistive load is connected across an unbalanced 3φ supply as shown in fig. Find the sym. components of line currents and delta current.

$I_{a1} + I_{b1} + I_{c1} = 0$   
 $I_c = -(I_{a1} + I_{b1}) = 12 \angle 150^\circ \text{ A}$

$$I_{a1} = \frac{1}{3} [I_{a1} + I_{b1} + I_{c1}]$$

$$= 13.9 \angle 41.8^\circ \text{ A}$$

$$I_{a2} = \frac{1}{3} [I_{a2} + I_{b2} + I_{c2}] = 4.6 \angle 240^\circ \text{ A}$$

$$I_{a0} = \frac{1}{3} [I_{a0} + I_{b0} + I_{c0}] = 0$$

$$I_{a1} = \sqrt{3} I_{A1} \quad \therefore I_{A1} = \frac{I_{a1}}{\sqrt{3}} = 8 \angle -48.1^\circ \text{ A}$$

$$I_{a2} = -j\sqrt{3} I_{A2} \quad \therefore I_{A2} = \frac{I_{a2}}{-j\sqrt{3}} = 2.6 \angle 338^\circ \text{ A}$$

$$I_{A0} = 0$$

4)

11 kV in the generator circuit requires a 25 MVA base in all other circuits and the following voltage bases.

Transmission line voltage base =  $11 \times \frac{121}{10.8} = 123.2 \text{ kV}$

Motor voltage base =  $123.2 \times \frac{10.8}{121} = 11 \text{ kV}$

The reactances of transformers, line and motors are converted to pu values on appropriate bases as follows:

Transformer reactance =  $0.1 \times \frac{25}{30} \times \left(\frac{10.8}{11}\right)^2 = 0.0805 \text{ pu}$

Line reactance =  $\frac{100 \times 25}{(123.2)^2} = 0.164 \text{ pu}$

Reactance of motor 1 =  $0.25 \times \frac{25}{15} \times \left(\frac{10}{11}\right)^2 = 0.345 \text{ pu}$

Reactance of motor 2 =  $0.25 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2 = 0.69 \text{ pu}$

The required positive sequence network is presented in Fig. 10.23.

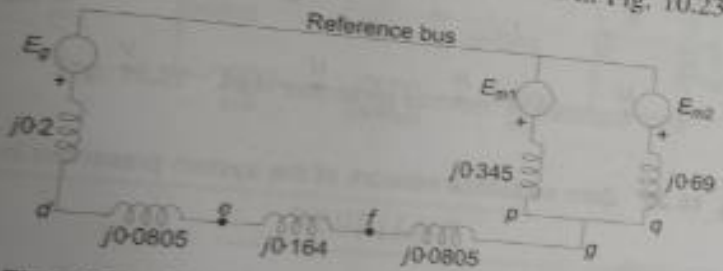


Fig. 10.23 Positive sequence network for Example 10.3

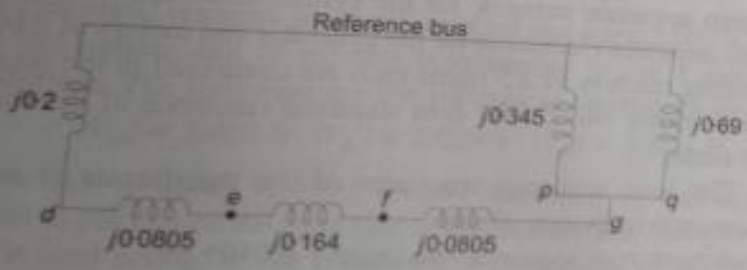


Fig. 10.24 Negative sequence network for Example 10.3

Since all the negative sequence reactances of the system are equal to the positive sequence reactances, the negative sequence network is identical to the

generator and motor No. 2. The zero sequence reactance of the transmission line is 300 ohms.

**Solution** The zero sequence reactance of the transformer is equal to its positive sequence reactance. Hence

$$\text{Transformer zero sequence reactance} = 0.0805 \text{ pu}$$

$$\text{Generator zero sequence reactances} = 0.06 \text{ pu}$$

$$\begin{aligned} \text{Zero sequence reactance of motor 1} &= 0.06 \times \frac{25}{15} \times \left(\frac{10}{11}\right)^2 \\ &= 0.082 \text{ pu} \end{aligned}$$

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Zero sequence reactance of motor 2 =  $0.06 \times \frac{25}{7.5} \times \left(\frac{10}{11}\right)^2 = 0.164 \text{ pu}$

Reactance of current limiting reactors =  $\frac{2.5 \times 25}{(11)^2} = 0.516 \text{ pu}$

Reactance of current limiting reactor included in zero sequence network =  $3 \times 0.516 = 1.548 \text{ pu}$

Zero sequence reactance of transmission line =  $\frac{300 \times 25}{(123.2)^2} = 0.494 \text{ pu}$

The zero sequence network is shown in Fig. 10.27.

Reference bus

Fig. 10.27 Zero sequence network of Example 10.5

**PROBLEMS**

5a)

Power in terms of symmetrical components

Total complex power into a 3 $\phi$  circuit is

$$S = P + jQ$$

$$= V_a I_a^* + V_b I_b^* + V_c I_c^*$$

$$S = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$= \bar{V}_{12} \bar{I}_1 + \bar{V}_{21} \bar{I}_2 + \bar{V}_{11} \bar{I}_1 + \bar{V}_{22} \bar{I}_2$$

$$\bar{A} \bar{A}^+ = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar{a} = 3 \begin{bmatrix} \bar{V}_{00} \\ \bar{V}_{01} \\ \bar{V}_{02} \end{bmatrix} \begin{bmatrix} 1 \\ -a^2 \\ -a \end{bmatrix}$$

Sum of powers of 3 symmetrical components equals the sum of powers of 3 unbalanced components.

$$3(\bar{V}_a \bar{I}_a + \bar{V}_b \bar{I}_b + \bar{V}_c \bar{I}_c)$$

Stevenson  
Pg 280

$$\begin{matrix} a^+ = a \\ (a^2)^+ = a \end{matrix}$$

5b)

Symmetrical components of unbalanced current vectors

$\bar{I}_a, \bar{I}_b, \bar{I}_c$  - Unbalanced current vectors.

$\bar{I}_{a1}, \bar{I}_{b1}, \bar{I}_{c1}$  - +ve sequence components with abc

$\bar{I}_{a2}, \bar{I}_{b2}, \bar{I}_{c2}$  - -ve sequence components with acb

$\bar{I}_{a0}, \bar{I}_{b0}, \bar{I}_{c0}$  - zero sequence components

$$\bar{I}_a = \bar{I}_{a0} + \bar{I}_{a1} + \bar{I}_{a2}$$

$$\bar{I}_b = \bar{I}_{b0} + \bar{I}_{b1} + \bar{I}_{b2}$$

$$\bar{I}_c = \bar{I}_{c0} + \bar{I}_{c1} + \bar{I}_{c2}$$

$$\bar{I}_a = \bar{I}_{a0} + \bar{I}_{a1} + \bar{I}_{a2}$$

$$\bar{I}_b = \bar{I}_{b0} + a^2 \bar{I}_{b1} + a \bar{I}_{b2}$$

$$\bar{I}_c = \bar{I}_{c0} + a \bar{I}_{c1} + a^2 \bar{I}_{c2}$$

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \bar{I}_{a0} \\ \bar{I}_{a1} \\ \bar{I}_{a2} \end{bmatrix}$$

$$\bar{I}_0 = \bar{A} \bar{I}_{01}$$

6)

### Positive Sequence Impedance and Network

Since a synchronous machine is designed with symmetrical windings, it induces emfs of positive sequence only, i.e. no negative or zero sequence voltages are induced in it. When the machine carries positive sequence currents only, this mode of operation is the balanced mode discussed at length in Chapter 9. The armature reaction field caused by positive sequence currents rotates at synchronous speed in the same direction as the rotor, i.e., it is stationary with respect to field excitation. The machine equivalently offers a direct axis reactance whose value reduces from subtransient reactance ( $X''_d$ ) to transient reactance ( $X'_d$ ) and finally to steady state (synchronous) reactance ( $X_d$ ), as the short circuit transient progresses in time. If armature resistance is assumed negligible, the positive sequence impedance of the machine is

$$Z_1 = jX''_d \text{ (if 1 cycle transient is of interest)} \quad (10.46)$$

$$= jX'_d \text{ (if 3-4 cycle transient is of interest)} \quad (10.47)$$

$$= jX_d \text{ (if steady state value is of interest)} \quad (10.48)$$

If the machine short circuit takes place from unloaded conditions, the terminal voltage constitutes the positive sequence voltage; on the other hand, if

\*This can be shown to be so by synchronous machine theory [5].

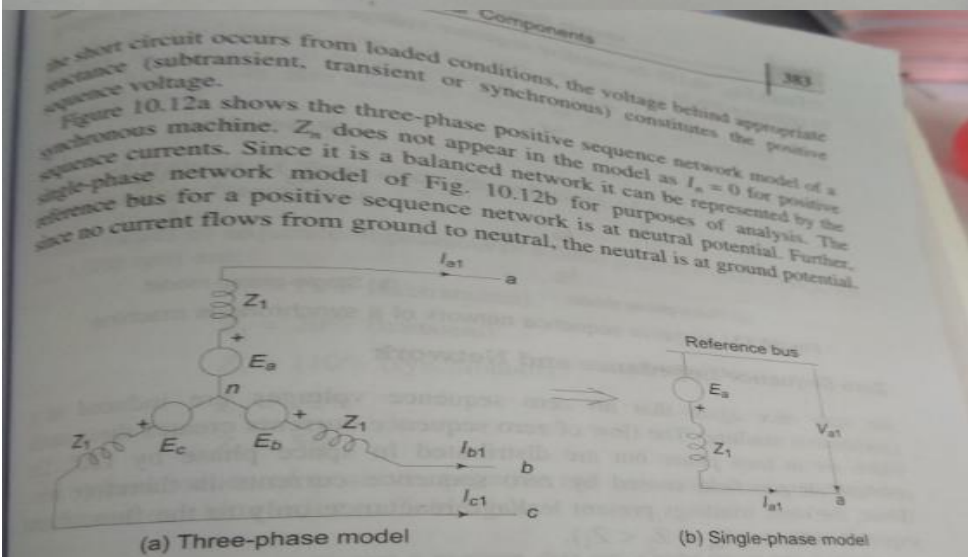


Fig. 10.12 Positive sequence network of synchronous machine

With reference to Fig. 10.12b, the positive sequence voltage of terminal a with respect to the reference bus is given by

$$V_{a1} = E_a - Z_1 I_{a1} \quad (10.49)$$

### Negative Sequence Impedance and Network

It has already been said that a synchronous machine has zero negative sequence induced voltages. With the flow of negative sequence currents in the stator a rotating field is created which rotates in the opposite direction to that of the positive sequence field and, therefore, at double synchronous speed with respect to rotor. Currents at double the stator frequency are therefore induced in rotor field and damper winding. In sweeping over the rotor surface, the negative sequence mmf is alternately presented with reluctances of direct and quadrature axes. The negative sequence impedance presented by the machine with consideration given to the damper windings, is often defined as

$$Z_2 = j \frac{X''_q + X''_d}{2}; |Z_2| < |Z_1| \quad (10.50)$$

Negative sequence network models of a synchronous machine, on a three-phase and single-phase basis are shown in Figs. 10.13a and b, respectively. The reference bus is of course at neutral potential which is the same as ground potential.

From Fig. 10.13b the negative sequence voltage of terminal *a* with respect to reference bus is

$$V_{a2} = -Z_2 I_{a2} \quad (10.51)$$

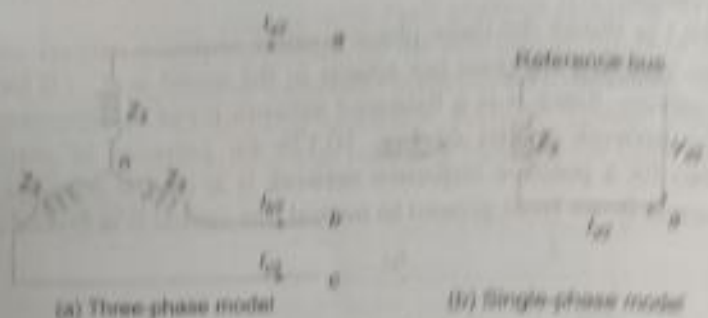
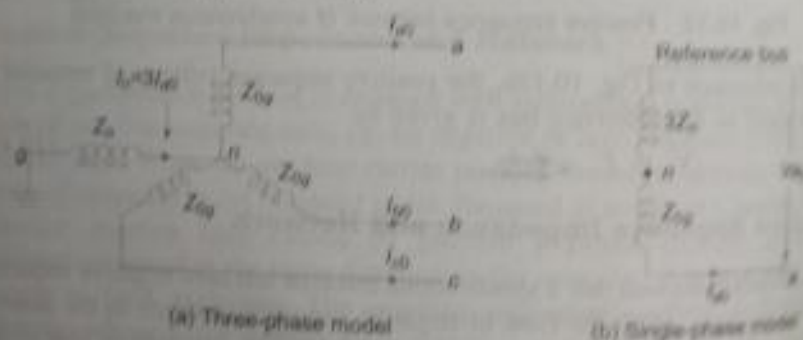


Fig. 10.13 Negative sequence network of a synchronous machine

### Zero Sequence Impedance and Network

We state once again that no zero sequence voltages are induced in a synchronous machine. The flow of zero sequence currents creates three fields which are in time phase but are distributed in space phase by 120°. The resultant air gap field caused by zero sequence currents is therefore zero. Hence, the rotor windings present leakage reactance only to the flow of zero sequence currents ( $Z_{0r} < Z_2 < Z_1$ ).



10.14 Zero sequence network of a synchronous machine

Zero sequence network models on a three- and single-phase basis are shown in Figs. 10.14a and b. In Fig. 10.14a, the current flowing in the impedance  $Z_0$  between neutral and ground is  $I_n = 3I_{a0}$ . The zero sequence voltage of terminal *a* with respect to ground, the reference bus, is therefore

$$V_{a0} = -3Z_n I_{a0} - Z_{0r} I_{a0} = -(3Z_n + Z_{0r}) I_{a0} \quad (10.52)$$

where  $Z_{0r}$  is the zero sequence impedance per phase of the machine. Since the single-phase zero sequence network of Fig. 10.14b carries only per phase zero sequence current, its total zero sequence impedance must be

$$Z_0 = 3Z_n + Z_{0r} \quad (10.53)$$

in order for it to have the same voltage from *a* to reference bus. The reference bus here is, of course, at ground potential.

From Fig. 10.14b zero sequence voltage of point *a* with respect to the reference bus is

$$V_{a0} = -Z_0 I_{a0} \quad (10.54)$$